4 Motion in Two Dimensions

Chapter Objectives

- Solve motion problems in two dimensions.
- Solve projection problems.
- Analyze projectile motion with and without air resistance. (Labs 4A and 4B)

Chapter Overview

Chapter 4 is a foundational chapter that combines students' knowledge of one-dimensional kinematics and vectors and relates it to two-dimensional applications. The chapter continues building the foundational concepts and skills essential to success with later chapters. Students should practice until they can quickly and easily resolve kinematic vector quantities. They will use this skill in later chapters to solve mechanics problems.

Airdrop

The chapter opener photo shows equipment being airdropped from a US Air Force C-17 Globemaster III. The combination of horizontal and vertical motion results in two-dimensional motion. The aircrew must conceptually understand the physics behind projectile motion as well as how the air drag of the parachute will affect the cargo's motion. Delivery of the cargo could be the difference between life and death for those on the ground.

SECTION 4.1 *OVERVIEW*

How does a pilot correct for the wind?

OBJECTIVES

» **4.1.1 Describe two-dimensional positions and motion using vectors.**

» **4.1.2 Solve kinematic problems in two dimensions using vectors.**

PRINTED RESOURCES

Careers: *Serving as a Humanitarian Engineer*

Section 4.1 Review

Section 4.1 Quiz

DIGITAL RESOURCES

Web Link: *Engineers Without Borders*

OVERVIEW

Section 4.1 brings together what students learned in Chapter 2 (kinematics) with what they learned in Chapter 3 (vectors) to introduce two-dimensional motion. Students should recognize that the equations of motion work in any direction. We will often solve problems by analyzing the motion in each direction (*x*, *y*) and then using vector mathematics to combine the component vectors into a final answer.

ENGAGE

Two-Dimensional Motion

Use the Chapter Opener photo as a **visual analysis** to get students thinking two-dimensionally.

Identify different vectors in the photo. *Students should identify displacements, velocities, and accelerations. There are also forces (e.g., gravity, tension, and air drag).*

In which direction do you think the different objects' velocity vectors are pointing? *aircraft: in its direction of flight; crate: in the direction of the aircraft and downward*

If a crate were falling straight down when its parachute deployed, what would be the directions of its velocity and acceleration vectors? Would it be speeding up or slowing down? *Its velocity vector would be pointing straight down, and its acceleration would be pointing straight up. It would be slowing down.*

If the parachute on one of the crates failed to open, what shape path do you think the crate would follow to the ground? *It would follow an essentially parabolic path. (An arc would be an acceptable answer.)*

4.1 Kinematics of Two-Dimensional Motion

Areas inhabited by people with the greatest needs are often the most inaccessible. Populated locales devastated and isolated by war or large-scale natural disasters quickly become uninhabitable. Homes are destroyed and utility networks are disrupted. Electricity and clean water supplies are in short supply, and sewage can't be disposed of properly. Roads, bridges, and airfields may be inaccessible, cutting off areas from the outside world. Governmental authority wanes, and local medical establishments may be overwhelmed with patients. The fabric of civilized living unravels. People surviving in such circumstances urgently need water, food, shelter, and medical care. The lack of transportation and communication amplifies these problems. How can humanitarian aid be delivered to such isolated and needy people? As we'll see later in this chapter, physics can help answer that question.

How does a pilot correct for the wind?

QUESTIONS

- How can we identify the positions of objects?
- How are vectors used to solve problems in kinematics?

TERMS

none

ConnectConcepts

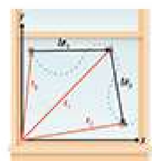
In this chapter we will be connecting the concepts of kinematics from Chapter 2 with the vector operations from Chapter 3.

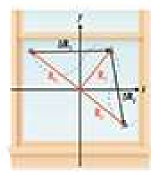
Position Vectors in Two Dimensions

Analyzing position and motion in two dimensions is only slightly more complicated than analyzing one-dimensional motion. Positions, displacements, velocities, and accelerations in two dimensions are all vector quantities that can be resolved into their components. Components can be analyzed using the equations of motion derived in Chapter 2.

In straight-line motion, we located an object by its position on a one-dimensional number line using coordinates. In two-dimensional motion position is typically determined using the rectangular Cartesian coordinate system. By convention, the x-axis is horizontal and the y-axis is vertical.

Position in two dimensions is determined by the position vector (**r**). The tail of the position vector is at the origin of the coordinate system, and the head is at the object's location. The diagram at right shows the path of a fly on a window pane. At time t_0, the fly is at the tip of $\mathbf{r}_0$; at time t_1, the fly is at the tip of $\mathbf{r}_1$; at time t_2, the fly is at the tip of $\mathbf{r}_2$. The origin is at the lower left corner of the window. The next figure shows the same path of the fly, but now the origin is at the center of the window. Although the positions of the fly are identical, the position vectors are different when measured from the different reference points. Recall from Chapter 2 that a *frame of reference* is simply the coordinate system within which motion is measured or observed. In theory, an infinite number of reference frames is possible.





Displacement in Two Dimensions

The displacement of an object is its change in position represented by the difference vector $\Delta\mathbf{r}$. The displacement for an interval of time is

$$\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i,$$

where $\mathbf{r}_f$ is the final position vector and $\mathbf{r}_i$ is the initial position vector. For instance, the fly's displacement ($\Delta\mathbf{r}_1$) for the interval t_0 to t_1 in the first figure is

$$\Delta\mathbf{r}_1 = \mathbf{r}_1 - \mathbf{r}_0;$$

for the second it is

$$\Delta\mathbf{R}_1 = \mathbf{R}_1 - \mathbf{R}_0.$$

The second question leads students to information that they will learn in this chapter.

INSTRUCT

Two-Dimensional Displacement

Use the two images on the reduced student page as part of your **direct instruction** to demonstrate that a net displacement for an object moving between three or more positions can be determined by finding the change in position or by summing the series of displacement vectors. Mathematically,

$$\Delta\mathbf{x} = \mathbf{x}_f - \mathbf{x}_i = \Sigma(\Delta\mathbf{x}).$$

This can be shown with the two images.

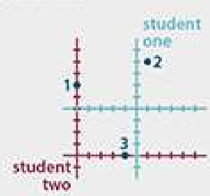
$$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_0 = \Sigma(\Delta\mathbf{r}) = \Delta\mathbf{r}_1 + \Delta\mathbf{r}_2$$

$$\Delta\mathbf{R} = \mathbf{R}_2 - \mathbf{R}_0 = \Sigma(\Delta\mathbf{R}) = \Delta\mathbf{R}_1 + \Delta\mathbf{R}_2$$

Point out that since these two images depict the same motion, $\Delta\mathbf{r} = \Delta\mathbf{R}$ even though the images use different frames of reference.

Motion and Reference Frames

Two physics students are watching a ladybug on their desk. Each student selects his frame of reference, and each records the position of the bug at 10:00, 10:15, and 10:30.



The first student records the positions $\mathbf{p}_1$ = <−5.0, 2.0> cm, $\mathbf{p}_2$ = <1.0, 4.0> cm, $\mathbf{p}_3$ = <−1.0, −4.0> cm and the other student, $\mathbf{P}_1$ = <0.0, 6.0> cm, $\mathbf{P}_2$ = <6.0, 8.0> cm, $\mathbf{P}_3$ = <4.0, 0.0> cm.

Find the net displacement according to each frame of reference by finding the change in position.

Student 1

$\Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{p}_3 - \mathbf{p}_1 = \mathbf{p}_3 + (-\mathbf{p}_1)$

$\mathbf{p}_1$ = <−5.0, 2.0> cm and
$-\mathbf{p}_1$ = <5.0, −2.0> cm

$\mathbf{p}_3$ = <−1.0, −4.0> cm

$\Delta\mathbf{p}$ = <−1.0, −4.0> cm + <5.0, −2.0> cm
= <4.0, −6.0> cm

Student 2

$\Delta\mathbf{P} = \mathbf{P}_f - \mathbf{P}_i = \mathbf{P}_3 - \mathbf{P}_1 = \mathbf{P}_3 + (-\mathbf{P}_1)$

$\mathbf{P}_1$ = <0.0, 6.0> cm and
$-\mathbf{P}_1$ = <0.0, −6.0> cm

$\mathbf{P}_3$ = <4.0, 0.0> cm

$\Delta\mathbf{P}$ = <4.0, 0.0> cm + <0.0, −6.0> cm
= <4.0, −6.0> cm

Now find the displacements between Position 1 and 2 and between Positions 2 and 3. Then find the displacement by adding the two displacements.

Student 1

$\Delta\mathbf{p}_1 = \mathbf{p}_2 - \mathbf{p}_1 = \mathbf{p}_2 + (-\mathbf{p}_1)$

$\mathbf{p}_1$ = <−5.0, 2.0> cm and
$-\mathbf{p}_1$ = <5.0, −2.0> cm

$\mathbf{p}_2$ = <1.0, 4.0> cm

$\Delta\mathbf{p}_1$ = <1.0, 4.0> cm + <5.0, −2.0> cm
= <6.0, 2.0> cm

(continued)

DIFFERENTIATED INSTRUCTION

Motion according to Different Frames of Reference

The final point of the Two-Dimensional Displacement teacher note is often doubted by students. If you have students who struggle with that conclusion from the images, mathematically prove this point with the example in the background teacher note that begins on the left.

$\Delta \mathbf{p}_2 = \mathbf{p}_3 - \mathbf{p}_2 = \mathbf{p}_3 + (-\mathbf{p}_2)$

$\mathbf{p}_2 = <1.0, 4.0>$ cm and
$-\mathbf{p}_2 = <-1.0, -4.0>$ cm

$\mathbf{p}_3 = <-1.0, -4.0>$ cm

$\Delta \mathbf{p}_2 = <-1.0, -4.0>$ cm $+ <-1.0, -4.0>$ cm

$= <-2.0, -8.0>$ cm

$\Delta \mathbf{p} = \Sigma(\Delta \mathbf{p}) = \Delta \mathbf{p}_1 + \Delta \mathbf{p}_2$

$\Delta \mathbf{p} = <6.0, 2.0>$ cm $+ <-2.0, -8.0>$ cm

$= <4.0, -6.0>$ cm

Student 2

$\Delta \mathbf{P}_1 = \mathbf{P}_2 - \mathbf{P}_1 = \mathbf{P}_2 + (-\mathbf{P}_1)$

$\mathbf{P}_1 = <0.0, 6.0>$ cm and
$-\mathbf{P}_1 = <0.0, -6.0>$ cm

$\mathbf{P}_2 = <6.0, 8.0>$ cm

$\Delta \mathbf{P}_1 = <6.0, 8.0>$ cm $+ <0.0, -6.0>$ cm

$= <6.0, 2.0>$ cm

$\Delta \mathbf{P}_2 = \mathbf{P}_3 - \mathbf{P}_2 = \mathbf{P}_3 + (-\mathbf{P}_2)$

$\mathbf{P}_2 = <6.0, 8.0>$ cm and
$-\mathbf{P}_2 = <-6.0, -8.0>$ cm

$\mathbf{P}_3 = <4.0, 0.0>$ cm

$\Delta \mathbf{P}_2 = <4.0, 0.0>$ cm $+ <-6.0, -8.0>$ cm

$= <-2.0, -8.0>$ cm

$\Delta \mathbf{P} = \Sigma(\Delta \mathbf{P}) = \Delta \mathbf{P}_1 + \Delta \mathbf{P}_2$

$\Delta \mathbf{P} = <6.0, 2.0>$ cm $+ <-2.0, -8.0>$ cm

$= <4.0, -6.0>$ cm

The displacement is the same no matter which reference frame is used. Recall from Chapter 3 that we can also calculate a resultant vector by doing vector addition with the individual vectors. So a total displacement may be found by finding the change in position or adding the displacement vectors. The approach we take will depend on the problem.

Example 4-1 Determining Two-Dimensional Displacement

A jogger starts from home and jogs 400 m west to corner A, turns and jogs 300 m south to corner B, and then turns east and jogs for another 100 m before stopping to rest at point R. Using the jogger's home as the origin of the coordinate system, we draw the jogger's complete path and the position vectors to A, B, and R.

a. What is the jogger's displacement from corner A to corner B?

b. What is his displacement from home to the resting spot?

c. What is the total distance that he jogged?

Write what you know.

Refer to the figure above.

Write the formula and solve for the unknown.

a. $\Delta \mathbf{r}_1 = \mathbf{r}_B - \mathbf{r}_A$

$= <-400, -300>$ m $- <-400, 0>$ m $= <0, -300>$ m

$= 300$ m south

b. $\Delta \mathbf{r}_2 = \mathbf{r}_R - \mathbf{0} = \mathbf{r}_R$ (A *zero vector* ($\mathbf{0}$) is any vector with zero magnitude. The position vector of the origin, which is required when determining any displacement from the origin, is $\mathbf{0}$.)

Determine the components of $\mathbf{r}_R$.

$r_{R_x} = -300$ m (west)

$r_{R_y} = -300$ m (south)

$|\mathbf{r}_R| = \sqrt{(-300\text{ m})^2 + (-300\text{ m})^2} = \sqrt{180\,000\text{ m}^2}$

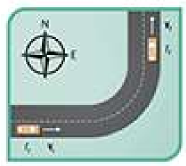
$= 424$ m

$\alpha = \tan^{-1}\frac{|r_{R_y}|}{|r_{R_x}|} = \tan^{-1}\left(\frac{300\text{ m}}{300\text{ m}}\right) = 45°$

The displacement is 424 m at 45° south of west.

c. $d_{total} = (400\text{ m} + 300\text{ m} + 100\text{ m})$

$= 800$ m



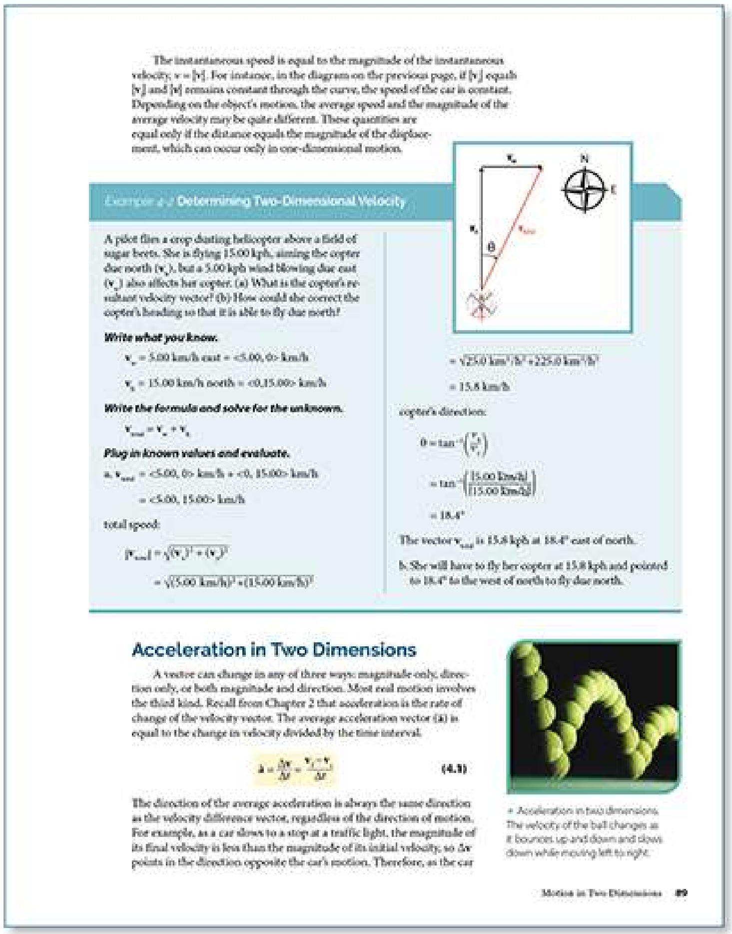
Velocity and Speed in Two Dimensions

Average velocity and average speed between two positions in a Cartesian plane are determined as they are in one dimension. Use vector notation when describing average velocity in two dimensions.

average velocity: $\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}$

average speed: $speed = \frac{distance}{\Delta t}$

The instantaneous velocity vector, which is the velocity of an object at any given moment, has a magnitude equal to the object's speed at that instant and points in the direction that the object is moving at that time. The diagram at left shows the instantaneous velocity of a car traveling around a curve. At time t_i the vector points east, but at time t_f the vector points north. In two- or three-dimensional motion we must include information about the direction of the motion.



The instantaneous speed is equal to the magnitude of the instantaneous velocity, $v = |\mathbf{v}|$. For instance, in the diagram on the previous page, if $|\mathbf{v}_i|$ equals $|\mathbf{v}_f|$ and $|\mathbf{v}|$ remains constant through the curve, the speed of the car is constant. Depending on the object's motion, the average speed and the magnitude of the average velocity may be quite different. These quantities are equal only if the distance equals the magnitude of the displacement, which can occur only in one-dimensional motion.

Example 4-2 Determining Two-Dimensional Velocity

A pilot flies a crop dusting helicopter above a field of sugar beets. She is flying 15.00 kph, aiming the copter due north ($\mathbf{v}_c$), but a 5.00 kph wind blowing due east ($\mathbf{v}_w$) also affects her copter. (a) What is the copter's resultant velocity vector? (b) How could she correct the copter's heading so that it is able to fly due north?

Write what you know.

$\mathbf{v}_w = 5.00$ km/h east $= \langle 5.00, 0 \rangle$ km/h

$\mathbf{v}_c = 15.00$ km/h north $= \langle 0, 15.00 \rangle$ km/h

Write the formula and solve for the unknown.

$\mathbf{v}_{total} = \mathbf{v}_w + \mathbf{v}_c$

Plug in known values and evaluate.

a. $\mathbf{v}_{total} = \langle 5.00, 0 \rangle \text{ km/h} + \langle 0, 15.00 \rangle \text{ km/h}$

$= \langle 5.00, 15.00 \rangle$ km/h

total speed:

$|\mathbf{v}_{total}| = \sqrt{(\mathbf{v}_x)^2 + (\mathbf{v}_y)^2}$

$= \sqrt{(5.00 \text{ km/h})^2 + (15.00 \text{ km/h})^2}$

$= \sqrt{25.0 \text{ km}^2/\text{h}^2 + 225.0 \text{ km}^2/\text{h}^2}$

$= 15.8$ km/h

copter's direction:

$\theta = \tan^{-1}\left(\frac{v_x}{v_y}\right)$

$= \tan^{-1}\left(\frac{5.00 \text{ km/h}}{15.00 \text{ km/h}}\right)$

$= 18.4°$

The vector $\mathbf{v}_{total}$ is 15.8 kph at 18.4° east of north.

b. She will have to fly her copter at 15.8 kph and pointed to 18.4° to the west of north to fly due north.

Acceleration in Two Dimensions

A vector can change in any of three ways: magnitude only, direction only, or both magnitude and direction. Most real motion involves the third kind. Recall from Chapter 2 that acceleration is the rate of change of the velocity vector. The average acceleration vector ($\bar{\mathbf{a}}$) is equal to the change in velocity divided by the time interval.

$$\bar{\mathbf{a}} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_f - \mathbf{v}_i}{\Delta t} \quad (4.1)$$

The direction of the average acceleration is always the same direction as the velocity difference vector, regardless of the direction of motion. For example, as a car slows to a stop at a traffic light, the magnitude of its final velocity is less than the magnitude of its initial velocity, so $\Delta \mathbf{v}$ points in the direction opposite the car's motion. Therefore, as the car

+ Acceleration in two dimensions. The velocity of the ball changes as it bounces up and down and slows down while moving left to right.

Motion in Two Dimensions 89

Velocities in 2D

Use Example 4-2 as a **peer teaching** activity to practice using vectors in a real-world velocity calculation. Write the problem on the board and have students work in pairs to solve it. It is a straightforward vector addition problem, but it will help them understand the vector addition of real-world quantities.

The end of the example problem demonstrates how pilots counteract the effect of wind to fly a particular path along the ground. Pilots call this "killing the drift." It is very important when pilots are trying to land in winds blowing across a runway.

Orientation of Axes

Use the following example to **demonstrate** the freedom we have in orienting our axes and the potential advantage of taking the time to think about how we arrange the axes.

Example: A pilot is flying his plane at 492 kph on a 120° heading. A 131 kph wind blows directly from his left, pushing him to the right at a heading of 210°.

Approach with Standard Axes

Many people would approach the problem using standard axes with the *x*-direction oriented with the east-west direction and the *y*-direction oriented with the north-south direction. We would need to find the components of both vectors and then determine the magnitude and direction of the resultant.

Approach with Adjusted Axes

However, we should always determine whether there is a way to reduce the number of vectors that we must resolve into components. In this example we should align one of the axes (let's choose the *x*-axis) with the plane's direction so that we don't have to resolve the airplane vector into components; all the airplane motion is in the *x*-direction. This approach is especially helpful for this example because, by aligning the *x*-axis with the airplane vector, the wind vector is all in the *y*-direction. All we have left to do is determine the magnitude and direction of the resultant.

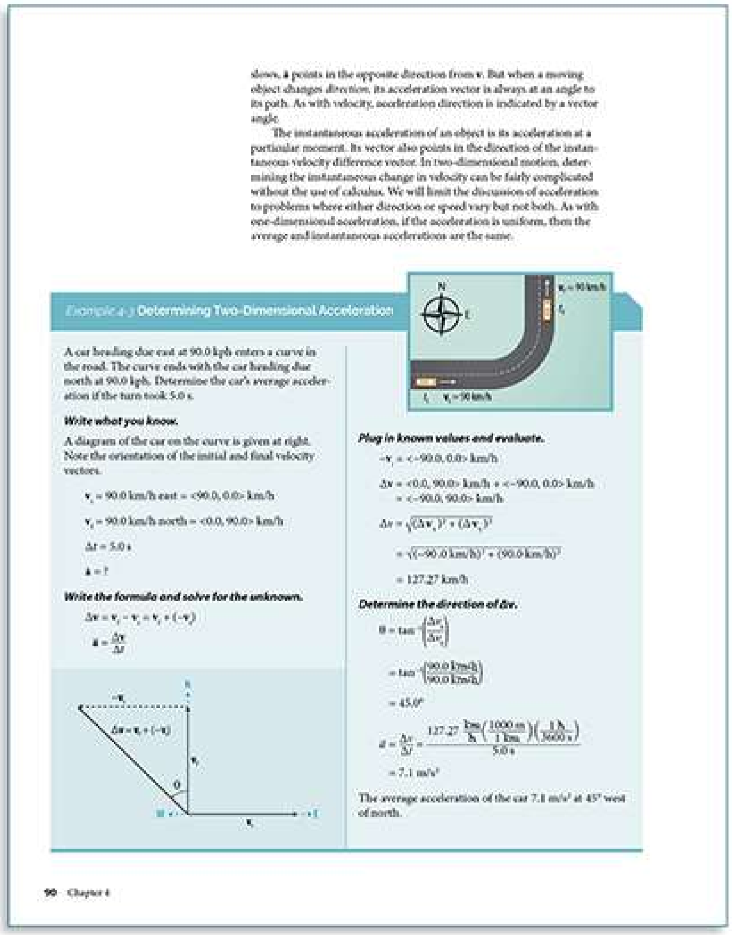
A fairly common approach is to align the *x*-axis with the initial direction of motion of the object.

Accelerations That Cause a Change of Direction

Example 4-3 is a great opportunity to **model** problem solving. The example provides students an opportunity to practice using vectors in a real-world velocity calculation: the change in velocity as a car travels a curve at a constant velocity.

Make sure that you point out the use of positive and negative signs on the vector components. While positive and negative signs are sufficient to indicate the direction of one-dimensional motion, vector angles must be used to indicate the directions of two-dimensional displacements, velocities, and accelerations. However, signs do indicate the directions of the components of two-dimensional motion because the components are one-dimensional vectors.

Essential Question Answer: A pilot corrects for the wind by doing a two-dimensional vector analysis to adjust the aircraft's velocity vector to account for the effect that the wind vector has on his aircraft. If he does it correctly, his path along the ground will be as he has planned.



slows, $\mathbf{a}$ points in the opposite direction from $\mathbf{v}$. But when a moving object changes direction, its acceleration vector is always at an *angle* to its path. As with velocity, acceleration direction is indicated by a vector angle.

The instantaneous acceleration of an object is its acceleration at a particular moment. Its vector also points in the direction of the instantaneous velocity difference vector. In two-dimensional motion, determining the instantaneous change in velocity can be fairly complicated without the use of calculus. We will limit the discussion of acceleration to problems where either direction or speed vary but not both. As with one-dimensional acceleration, if the acceleration is uniform, then the average and instantaneous accelerations are the same.

Example 4-3 Determining Two-Dimensional Acceleration

A car heading due east at 90.0 kph enters a curve in the road. The curve ends with the car heading due north at 90.0 kph. Determine the car's average acceleration if the turn took 5.0 s.

Write what you know.

A diagram of the car on the curve is given at right. Note the orientation of the initial and final velocity vectors.

$\mathbf{v}_i$ = 90.0 km/h east = <90.0, 0.0> km/h

$\mathbf{v}_f$ = 90.0 km/h north = <0.0, 90.0> km/h

$\Delta t = 5.0\text{ s}$

$\bar{\mathbf{a}} = ?$

Write the formula and solve for the unknown.

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i = \mathbf{v}_f + (-\mathbf{v}_i)$$

$$\bar{\mathbf{a}} = \frac{\Delta\mathbf{v}}{\Delta t}$$

Plug in known values and evaluate.

$-\mathbf{v}_i$ = <−90.0, 0.0> km/h

$\Delta\mathbf{v}$ = <0.0, 90.0> km/h + <−90.0, 0.0> km/h
= <−90.0, 90.0> km/h

$$\Delta v = \sqrt{(\Delta \mathbf{v}_x)^2 + (\Delta \mathbf{v}_y)^2}$$

$$= \sqrt{(-90.0\text{ km/h})^2 + (90.0\text{ km/h})^2}$$

$$= 127.27\text{ km/h}$$

Determine the direction of $\Delta\mathbf{v}$.

$$\theta = \tan^{-1}\left(\frac{\Delta v_x}{\Delta v_y}\right)$$

$$= \tan^{-1}\left(\frac{90.0\text{ km/h}}{90.0\text{ km/h}}\right)$$

$$= 45.0°$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{127.27\ \frac{\text{km}}{\text{h}}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)}{5.0\text{ s}}$$

$$= 7.1\text{ m/s}^2$$

The average acceleration of the car 7.1 m/s² at 45° west of north.

90 Chapter 4

4.1 SECTION REVIEW

1. In physics, a location in two dimensions is most efficiently represented by what quantity?
2. Describe how displacement is calculated in two-dimensional motion. In your answer include the formula that you would use.
3. How is the direction of the average acceleration vector determined if you are given the initial and final velocities?
4. A visitor walks directly from the White House 1.83 km in the direction 20° south of east. Then he turns left and walks due east for an additional 0.67 km. He turns and walks 0.25 km due south to the US Capitol Building. What is his displacement from the White House?
5. A quarterback is attempting to run toward his goal line at 4.6 m/s. A linebacker hits him and their combined final velocity is 3.7 m/s at an angle of 120° back and to the left of the quarterback's original direction. The time to change direction was 0.20 s. Calculate the average acceleration of the quarterback during the tackle.
6. An athlete sprints a circular 400.0 m track in 50.0 s. Assume that he runs at a constant speed.
   a. What is his speed over the entire course?
   b. What is the magnitude of his instantaneous velocity at any point along the circuit?
   c. If he starts facing west, what is his instantaneous velocity halfway around the circle?
   d. What is the average acceleration for one-half lap of the track?

Use the careers box below to answer Questions 7–8.

7. Describe a challenge that a humanitarian engineer working with a remote people group might face that would not be an issue for an engineer working in an urban environment.
8. Why might a Christian consider a career as a humanitarian engineer?

careers

SERVING AS A HUMANITARIAN ENGINEER: HELPING WHERE IT IS MOST NEEDED

In 2000 engineering professor Dr. Bernard Amadei visited a Mayan village in Belize that lacked a clean water supply. In fact, the village children could not attend school because their time was devoted to collecting water from sources located miles away. After consulting with his colleagues on possible solutions, Dr. Amadei and a team of students from the University of Colorado Boulder returned to the village. The team built a new water supply system powered by energy from a nearby waterfall. Soon afterward Dr. Amadei founded Engineers Without Borders, an aid organization dedicated to providing engineered solutions to problems faced by people in need around the world.

Helping people in need can provide a great deal of personal satisfaction. Today you can get a college degree in humanitarian engineering. Engineers like the one shown at right, specialize in applying their knowledge and training to meeting the needs of people whose lives have been adversely affected by war, poverty, or disaster. As a humanitarian engineer, you might work on a drinking water project like the one in Belize, or perhaps you'll find ways to use diminishing natural resources more efficiently and sustainably. One key to providing workable long-term solutions to such problems is to find answers that are affordable, self-sustaining, and designed to make good use of local resources—including the skills and labor of local people. If you enjoy physics, like to find answers to real-life problems, and have a heart to serve others, humanitarian engineering may be the right career path for you.

APPLY

Humanitarian Engineer

Use the career box as part of a **discussion** to demonstrate how engineers may be in full-time ministry. A humanitarian engineer uses his engineering skills to complete projects that directly help those in need. Consider visiting the website for Engineers Without Borders. Go to BJU Press Online Resources, select Chapter 4 Web Links, and click on *Engineers Without Borders*.

ASSESS

Section 4.1 Review

Assign the section review as a **formative assessment** to help students solidify their understanding of Section 1.

Section 4.1 Quiz

Use the Section 1.1 Quiz as a **formative** or **summative assessment** to check students' understanding of Section 1.

Section 4.1 Review

* An asterisk before a question indicates that a worked-out solution can be found on pages 105a–105b.

1. a position vector (p. 87)
2. Displacement is calculated by finding the vector difference of two position vectors: $\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$. Students may also mention the sum of a series of displacement vectors. (pp. 87–88)
3. The direction of $\bar{\mathbf{a}}$ is the same as the direction of the velocity difference vector or change in velocity vector: $\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$. (pp. 89–90)

*4. $\Delta \mathbf{R}$ = 2.54 km at 20.1° south of east. (pp. 88–90)

*5. $\bar{\mathbf{a}}$ = 36 m/s² at 153.6° back and to the left from the quarterback's original direction (pp. 88–90)

6. *a. speed = 8.00 m/s
   b. $|\mathbf{v}|$ = 8.00 m/s
   c. $\mathbf{v}_{halfway}$ = 8.00 m/s east
   *d. $\bar{\mathbf{a}} = -0.640\ \text{m/s}^2$ (all pp. 88–90)
7. Answers will vary. Such challenges might include limits on the kinds and amounts of materials that could be used for a project, limits on the kinds of equipment available to build a project, and the need to build a project that can be operated and maintained by unskilled and possibly undereducated people groups.
8. Answers will vary. Humanitarian engineers fulfill the Creation Mandate by helping others. They demonstrate love for God and others by meeting the needs of people in the most difficult times. Such actions glorify God. This is a great example of full-time ministry.

SECTION 4.2 *OVERVIEW*

At what angle should a batter hit a baseball to get the most distance?

OBJECTIVES

» **4.2.1 Describe horizontal projections and the kinematic assumptions they involve.**

» **4.2.2 Solve projectile motion problems.**

» **4.2.3 Evaluate the effectiveness of humanitarian airdrops.** BWS

BIBLICAL WORLDVIEW SHAPING

Ethics (evaluate): The effectiveness of humanitarian aid must be considered in light of risks posed by how and where the aid is delivered. (4.2.3)

PRINTED RESOURCES

Mini Lab: *Catapulting to Fame*

Case Study: *Shot Put Release Angles*

Section 4.2 Review

Ethics: *Humanitarian Airdrops* (p. 105)

Section 4.2 Quiz

DIGITAL RESOURCES

Video: *Projectile Motion*

Web Link: *Animation of Projectile Motion*

MATERIALS

tennis ball

rollerblades

OVERVIEW

Section 4.2 introduces the important topic of projectile motion. While students understand projectile motion intuitively, the mathematics of projectile motion can be challenging for many. Emphasis is on the concept that the horizontal and vertical motions can be analyzed separately.

4.2 Projections

Projectiles

At what angle should a batter hit a baseball to get the most distance?

QUESTIONS
- How are horizontal projections described in physics?
- How do I solve problems involving projectile motion?

TERMS
projectile · projectile motion · trajectory · range

When some volcanoes erupt, they eject fiery bombs of molten rock in high arcs. Each of these blobs of molten rock is an example of a **projectile**—any object near the earth's surface that is launched with an initial velocity and then moves under the influence of gravity. Projectiles include objects such as bullets and bombs, but also baseballs, footballs, and long jumpers, as well as any other object that is launched through the air. The movement exemplified by these objects is, plainly enough, called **projectile motion**.

A projectile's path is called its **trajectory**. A projectile is often assumed to have only gravitational force acting on it. This means that we disregard the effects of air resistance, which is a reasonable simplification for relatively slow-moving, small objects.

» The lava bombs of a volcano are projectiles.

Horizontal Launch

In Chapter 2 we studied the free fall of a dropped object. The path of motion was linear and there was only one component of motion, parallel to the y-axis. A one-dimensional coordinate system was adequate for this kind of motion. But if we were to roll a ball off the top of a stairway, for example, we would have another dimension to consider. As the ball leaves the stairway, it is moving *horizontally*. In the total absence of gravity (and ignoring air resistance), the ball would continue at the same velocity indefinitely. But due to the presence of gravity, the ball also begins to move downward as it leaves the top stair.

The kinematics of the vertical and horizontal components of motion are completely independent, but they occur simultaneously, producing a smooth, continuous path. The illustration at the top of the next page compares the motion of a dropped object with a second object launched horizontally at the same instant. You can see that at each instant, the vertical positions of both objects are identical. This shows that the vertical component of motion does not depend on the horizontal component. The total velocity of a projectile at any time after launch is the vector sum of its horizontal and vertical velocity components.

92 Chapter 4

when she releases it (more like she is pushing it to you). Before she tosses it, have her show the height that she will throw the ball from and tell the class that you will position your hand at the same height to catch the ball. Some students may protest that it won't work. Have your volunteer toss the ball anyway. The ball will miss your hand to the low side.

> **Why did I miss the ball?** *The ball followed a curved path and I didn't account for the ball's vertical motion.*

> **What would I need to do to correct my error and be able to catch the ball?** *I would need to position my hand below the release point to account for the ball's vertical motion.*

Have your volunteer toss the ball again with a horizontal release to demonstrate how you can account for the vertical motion.

> **How could our volunteer correct for my error when she throws the ball?** *She could throw the ball with an upward angle so that the ball would rise and then "fall" into my hand.*

Have your volunteer toss the ball again but with an upward angle to try to get the ball in your hand.

Finish up by tossing the ball back and forth using the terms *projectile*, *trajectory*, and *projectile motion*. Explain that we constantly analyze projectile motion when we play catch. As children play catch, they unconsciously develop a conceptual understanding of projectile motion.

ENGAGE

Students Already Understand Projectile Motion

Use the following **demonstration** to show students that they already have a conceptual understanding of projectile motion.

Ask for a student volunteer to play catch with you. Have her stand in the front of the room to one side of the room while you position yourself approximately 15 feet away. Tell her that you want her to toss the ball from about eye level so that it is moving horizontally

The horizontal and vertical components of projectile kinematics (displacement, velocity, and acceleration) can be treated as two separate one-dimensional problems to which we can apply the equations of motion learned in Chapter 2. Two quantities often of interest in horizontal projections are the time of flight and the magnitude of the horizontal displacement, called the **range**. Because the horizontal and vertical motions occur simultaneously, Δt is the same for motion in both directions.

Projectile Motion: Horizontal Component

Ideally, a projectile experiences no horizontal acceleration, which is a good approximation for small, slow-moving projectiles. Zero acceleration in the horizontal direction means that the horizontal velocity is constant. The first equation of motion,

$$\mathbf{v}_x = \mathbf{v}_{ix} + \mathbf{a}\Delta t,$$

becomes

$$\mathbf{v}_x = \mathbf{v}_{ix},$$

where $\mathbf{v}_x$ and $\mathbf{v}_{ix}$ are the x-components of the projectile's velocity. Since the velocity's x-component is constant, it will simply be called $\mathbf{v}_x$.

The second equation of motion,

$$\mathbf{x}_f = \mathbf{x}_i + \mathbf{v}_{ix}\Delta t + \frac{1}{2}\mathbf{a}\Delta t^2,$$

becomes the equation

$$\mathbf{x}_f = \mathbf{x}_i + \mathbf{v}_x\Delta t,$$

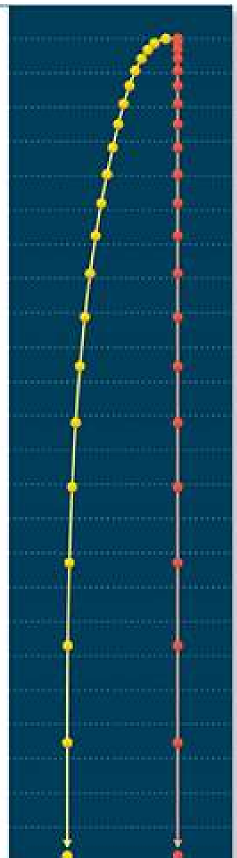
where $\mathbf{x}_i$ and $\mathbf{x}_f$ are the x-components of the projectile's position vectors $\mathbf{r}_i$ and $\mathbf{r}_f$, respectively. This equation is often rearranged into its displacement form.

$$\Delta \mathbf{x} = \mathbf{x}_f - \mathbf{x}_i = \mathbf{v}_x\Delta t \qquad \textbf{(4.2)}$$

The third equation of motion is meaningless when the acceleration is zero since you would be dividing by zero. Just as we noticed with free fall problems, we do not need to learn new motion equations for projectile motion. These are the same equations that you learned in Chapter 2.

Projectile Motion: Vertical Component

You learned in Chapter 2 that all objects experience a uniform downward acceleration near the earth's surface due to gravity. Recall that $\mathbf{g}$ = 9.81 m/s² downward. The initial vertical velocity of a horizontal projection is zero. The final vertical velocity of the projectile is due solely to the amount of time it has to fall. For horizontal projections, the equations of motion in the y-direction take the forms listed below.



What do you notice about the vertical position of the 5th ball, 10th ball, 15th ball, and 20th ball for each type of motion? *They are the same.*

Is the vertical motion dependent on the horizontal motion or independent of it? *independent*

More on Directional Independence

Use the **video** below to solidify students' understanding of the independence of the y-direction (free fall) and x-direction motion (constant velocity). Go to BJU Press Online Resources, select Chapter 4 Videos, and click on *Projectile Motion*.

Identify some people who you think would be experts at conceptually understanding and analyzing projectile motion? *Answers will vary. Students may suggest, for example, quarterbacks, baseball pitchers, or basketball players.*

When you are discussing horizontal projections, you can point out the first attempted toss and catch. When you discuss projections at an angle, you can have the students recall the subsequent tosses and catches.

INSTRUCT

Directional Independence

Use the image on this page for a **visual analysis** to help students understand that the motion in the y-direction (free fall) is independent of the horizontal motion (constant velocity). The two motions are connected by the time interval, which is the same for the motion in each direction.

What do you notice about the motion of the two balls? *Answers will vary. Students should recognize that one ball is moving in just the vertical direction and the other is moving both vertically and horizontally. Others may identify the types of motion. Others may pick up on the fact that the displacements in the vertical are the same for both balls.*

What type of motion is the red ball undergoing? *free fall*

What type of motion is the yellow ball undergoing? *projectile motion; more specifically, a horizontal projection*

DIFFERENTIATED INSTRUCTION

Seeing the Independence of Each Direction's Motion

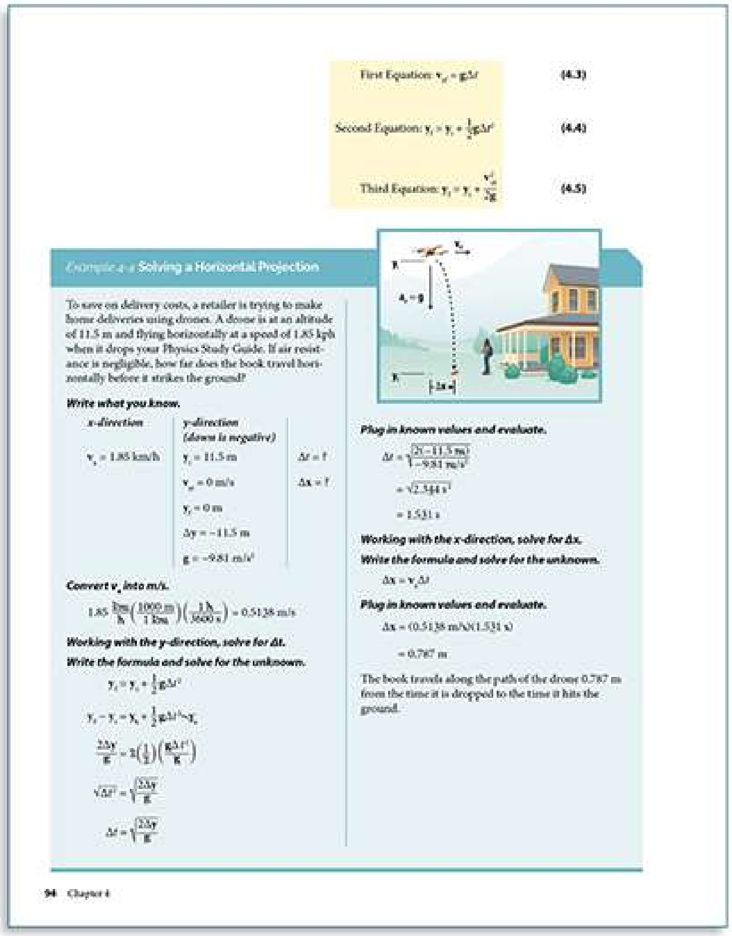
For students who still struggle with the concept that motion in two directions is independent, use the web link below for an animation that will track the vertical motion within two-dimensional projectile motion. The animation allows you to alter the launch angle, speed, and height. Go to BJU Press Online Resources, select Chapter 4 Web Links, and click on *Animation of Projectile Motion*. This source also provides many other physics animations to enhance your teaching of projectile motion.

Solving a Horizontal Projection

Use Example 4-4 to **model** splitting projectile motion problems into two dimensions to make solving these problems easier. Emphasize that the time variable is the same in the motion for both directions.

More Equations?

Make sure that students understand that the equations of motion on pages 93 and 94 are not new equations of motions. They are the equations of motion from Chapter 2 that have been modified specifically for problems involving projectile motion.



First Equation: $v_{y} = g\Delta t$ **(4.3)**

Second Equation: $y_f = y_i + \frac{1}{2}g\Delta t^2$ **(4.4)**

Third Equation: $y_f = y_i + \frac{v_{y}^2}{2g}$ **(4.5)**

Example 4-4 **Solving a Horizontal Projection**

To save on delivery costs, a retailer is trying to make home deliveries using drones. A drone is at an altitude of 11.5 m and flying horizontally at a speed of 1.85 kph when it drops your Physics Study Guide. If air resistance is negligible, how far does the book travel horizontally before it strikes the ground?

Write what you know.

x-direction	*y-direction (down is negative)*	
$v_x = 1.85$ km/h	$y_i = 11.5$ m	$\Delta t = ?$
	$v_{yi} = 0$ m/s	$\Delta x = ?$
	$y_f = 0$ m	
	$\Delta y = -11.5$ m	
	$g = -9.81$ m/s^2	

Convert v_x into m/s.

$$1.85\ \frac{\text{km}}{\text{h}}\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 0.5138\ \text{m/s}$$

Working with the y-direction, solve for Δt.

Write the formula and solve for the unknown.

$$y_f = y_i + \frac{1}{2}g\Delta t^2$$

$$y_f - y_i = y_i + \frac{1}{2}g\Delta t^2 - y_i$$

$$\frac{2\Delta y}{g} = 2\left(\frac{1}{2}\right)\left(\frac{g\Delta t^2}{g}\right)$$

$$\sqrt{\Delta t^2} = \sqrt{\frac{2\Delta y}{g}}$$

$$\Delta t = \sqrt{\frac{2\Delta y}{g}}$$

Plug in known values and evaluate.

$$\Delta t = \sqrt{\frac{2(-11.5\ \text{m})}{-9.81\ \text{m/s}^2}} = \sqrt{2.344\ \text{s}^2} = 1.531\ \text{s}$$

Working with the x-direction, solve for Δx.

Write the formula and solve for the unknown.

$$\Delta x = v_x\Delta t$$

Plug in known values and evaluate.

$$\Delta x = (0.5138\ \text{m/s})(1.531\ \text{s}) = 0.787\ \text{m}$$

The book travels along the path of the drone 0.787 m from the time it is dropped to the time it hits the ground.

94 Chapter 4