

Welcome to *PRE-ALGEBRA*!



Why *pre-algebra*? Just as sports teams use preseason training to ensure readiness for the regular season by improving fitness, strength, and technical ability, *pre-algebra* helps produce optimum performance in algebra courses. As the NFL all-time leading rusher and Hall of Fame running back Emmitt Smith stated, “All men are created equal. Some work harder in preseason.”

More specifically, *PRE-ALGEBRA* will challenge you to improve in the following essential mathematical practices:

1. Persevere in problem solving.
2. Use abstract reasoning.
3. Construct logical arguments.
4. Use mathematical models.
5. Use appropriate tools.
6. Use precise language.
7. See and apply structure.
8. Generalize patterns.

That’s a challenging list, but the text integrates instruction and exercises emphasizing these skills and provides ample review.

As you travel through *PRE-ALGEBRA*, you will join an animated family on their own challenging journey. Enjoy their comic adventures, but also look for the underlying worldview theme in each one. While the Bible is not a mathematics textbook, it reveals the foundational truths that explain why math works, how it should be used, and why it is such an effective tool. You will encounter five themes to help you form a biblical view of mathematics: knowledge, modeling, reasoning, design, and ethics. Each theme is presented within the overarching worldview narrative of Creation, Fall, and Redemption.

Equipped with clearly stated objectives, step-by-step examples, skill checks, tips, and cumulative reviews, this text is designed to optimize your success. Will you make the *Pre-Algebra Hall of Fame* this year?



PROBLEM SOLVING

Draw a Picture

Pictures are often helpful when solving problems. Representing the information with a sketch or diagram allows you to visualize what is happening and see relationships between various parts of the problem. Draw a picture whenever possible.

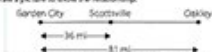
Example 1

Sketching a Picture

Garden City, Scottsville, and Oakley are located along a straight highway. The distance between Garden City and Oakley is 81 mi. Scottsville is located between Garden City and Oakley and is 36 mi from Garden City. How far is it from Scottsville to Oakley?

Answer

Think: What is the relationship of these towns to each other? Draw a picture to show the relationship.



Think: What is being asked? Can I write an equation that models the situation?

Define a variable for the unknown.
 d = the distance from Scottsville to Oakley.
The sum of the two shorter distances is 81.
 $d + 36 = 81$.

Think: How is the equation solved?

Subtract 36 from both sides of the equation:
 $d + 36 = 81 - 36$
 $d = 45$ mi from Scottsville to Oakley

Check: Is your answer reasonable?

Referring to the picture, $45 + 36 = 81$, so the answer is correct.

Example 2

Making a Diagram

There are 8 officials at a council meeting. If each person shakes hands with everyone else at the meeting exactly once, how many handshakes occur?

Answer

Think: How can I visualize the handshakes taking place at the meeting?

If the first person shakes hands with everyone else in the room, 7 handshakes occur.

The second person shakes hands with the other 6 officials.

The third person shakes hands with the 5 remaining officials, and so on until the seventh official shakes hands with the last official.



APPLICATION PROBLEMS

Coding Information

Competitive dating features on the internet reuse valuable transactions and protect their clients' personal information. Modular arithmetic plays a significant role in validating ISBNs (International Standard Book Numbers) and credit card numbers. Keeping personal information secure requires mathematical encryption methods that are also based on modular arithmetic.

Modular arithmetic can be thought of as arithmetic on a clock or a circle rather than on a number line. If you check the time 8 five hours after 9:00^{AM} the sum is 13. What time is it 8 five hours after 9:00^{AM} the sum is 20. Consider the sum $10 + 3$ on a clock. Because the numbers are in a circular form, adding 3 to 10 will bring us to 1.

1. Add the following numbers on the clock face.
 - a. $8 + 6$
 - b. $11 + 7$
 - c. $7 + 10$
 - d. $4 + 8$



Replacing the 12 on the clock with a 0 creates a layout that represents modulus 12 (abbreviated mod 12). If we start at 0 and add 14, we land on 2. This is written $14 \text{ mod } 12 = 2$, read "14 mod 12 is 2".

2. Evaluate each expression.
 - a. $13 \text{ mod } 12$
 - b. $15 \text{ mod } 12$
 - c. $35 \text{ mod } 12$
 - d. $27 \text{ mod } 12$



Modular arithmetic is used in validating ISBNs. Books cataloged in the Library of Congress since 2007 are required to have a 13-digit ISBN. This number provides identifying information such as the language the book was written in, the publisher, and the publisher's specific title number of the book. The last digit in the ISBN acts as a check digit, ensuring that the modulus 10 equivalent of a weighted sum of the ISBN's digits is 0.

To check the validity of a 13-digit ISBN, multiply the first digit by 1, the second by 3, the third by 1, the fourth by 3, and continue multiplying in that pattern for all 13 digits. Then check whether the total is equal to a multiple of 10. The process of validating ISBNs 978-1-6062-889-3 is illustrated in the following table.

254 Chapter 6

210 Chapter 6

Application Problems

Apply learned math skills in each chapter's Application Problems.

Problem Solving

Learn to apply problem-solving strategies.

STEM PROJECT

POTATO POWER

What can you do with potatoes? Well, of course you can bake them or mash them or create cut and fry them, but what about lighting a light bulb with them? In this STEM project, found in Activities, you will generate enough electricity to power an LED light with your very own potato batteries! You will also learn how batteries work and use some simple, but important, equations that model basic properties of electricity.



STEM Project 107

STEM Project

Preview the quarterly STEM projects located in Activities.

Essential Question and Objectives
Start each section with a question about the key idea and a list of the skills you are expected to learn.

4.1 Prime Factorization

How are composite numbers related to prime numbers?

After completing this section, you will be able to:

- list the factors of a number.
- classify numbers as prime, composite, or neither.
- find the prime factorizations of a number.
- explain why some mathematical statements are subjective truth.



Some bicycle frames, phone cases, and car parts are made of a composite material called carbon fiber. Composite materials are made by combining several basic materials to form a product that is often stronger, lighter, and more useful to some way.

In Section 1.4 you learned that multiplying positive or negative integers forms a product that is also an integer. In this lesson you will find positive factors of natural numbers.

Definition

A factor of an integer is any integer that divides the given integer with no remainder.

Since $24 = 3 \times 8$, both 3 and 8 are factors of 24, and 24 is still to be divisible by 3 and 8. Note that 1, 2, 4, 6, 12, and 24 are also factors of 24.

Divisibility tests can help you quickly determine whether 2, 3, 4, 5, 6, 8, 9, or 10 are factors of a given integer.

Divisibility Tests

Test	Rule
Div 2	The integer ends in an even digit: 0, 2, 4, 6, or 8.
Div 3	The sum of the integer's digits is divisible by 3.
Div 4	The last two digits of the integer are divisible by 4.
Div 5	The integer ends in 0 or 5.
Div 6	The integer is divisible by 2 and 3.
Div 8	The last three digits of the integer are divisible by 8.
Div 9	The sum of the integer's digits is divisible by 9.
Div 10	The integer ends in 0.

Factors



Example 1 Listing All the Factors of a Number

List all the factors of 18.
Answer: 1, 2, 3, 6, 9, and 18.

Find all the factors of 18 by listing the pairs of natural numbers that have a product of 18.

Use the square root of a number to limit the search for its factors. For example, examine the factors of 36.

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36. Any factor greater than 6, or $\sqrt{36}$, is paired with a factor less than $\sqrt{36}$ that we need to test numbers greater than the number's square root.

Example 2 Listing All the Factors of a Number

List all the factors of 54 in ascending order.
Answer: 1, 2, 3, 6, 9, 18, 27, and 54.

Since $7 = \sqrt{49} < 8$, do not check for factors greater than 7. 54 is divisible by 1 and 2 but not divisible by 3, 4, 5, 6, or 7.

Skill Check A

1. 15 is a factor of 7.
2. 6 is a factor of 48.
3. 4 is a factor of 712.

Copy the table and determine whether each product is divisible (D) or not divisible (ND) by each possible factor.

Product	2	3	4	5	6	7	8	9	10
1. 1112									
2. 3460									
3. 7224									

List all the factors of each number in ascending order.

4. 7, 12
5. 8, 25
6. 9, 30

Key Concepts

Read through explanations of key concepts and remember important ideas highlighted within colored boxes.

A. Exercises

Evaluate each quotient.

1. $\frac{1}{2} \div \frac{3}{4}$
2. $\frac{3}{4} \div \frac{1}{2}$
3. $5 \div (-\frac{1}{2})$
4. $6 \frac{1}{2} \div 2$
5. $\frac{3}{4} \div \frac{1}{2}$
6. $10 \div -\frac{1}{2} = -1 \frac{1}{2}$
7. $3 \div -\frac{1}{2} = -\frac{3}{2}$
8. $2 \frac{1}{2} \div 25$
9. $\frac{1}{2} \div (-\frac{1}{2})$
10. $4 \frac{1}{2} \div (-\frac{1}{2})$
11. $8 \div -2 = (-2) \frac{1}{2}$
12. $4 \frac{1}{2} \div \frac{1}{2}$

B. Exercises

Evaluate each quotient.

13. $\sqrt[3]{27}$
14. $\sqrt[3]{\frac{8}{27}}$
15. $\sqrt[3]{\frac{1}{8}}$
16. $\sqrt{649}$
17. $-\sqrt{649}$
18. $\sqrt[3]{0.027}$
19. $47.2 \div 8$
20. $-1728 \div 4$
21. $16.32 \div 17$
22. $36.75 \div 33$
23. $76.2 \div 0.06$
24. $3.78 \div (-0.7)$
25. $-21.42 \div (-0.63)$
26. $0.225 \div 7.5$
27. $0.032 \div 0.07$
28. $-2.49 \div 0.9$
29. $13.9 \div (-0.8)$
30. $0.675 \div 0.25$

Use the given and check method to estimate each root to the nearest tenth.

31. $\sqrt{13}$
32. $\sqrt{33}$
33. $\sqrt[3]{27}$
34. $\sqrt[3]{125}$
35. $\sqrt[3]{216}$
36. $\sqrt[3]{729}$
37. $(\frac{1}{2} \div \frac{1}{3}) \div \frac{1}{4}$
38. $(\frac{1}{10} \div 3) \div \frac{1}{2}$
39. $-\frac{1}{2} \div (\frac{1}{3} \div \frac{1}{4})$
40. $10 \div [\frac{1}{2} \div (-\frac{1}{3})]$

41. A real estate developer purchased a 52-acre plot of land. How many $\frac{1}{2}$ -acre residential lots can be developed on the property?

42. Inez is walking on a trail $\frac{1}{2}$ km ahead of Heather, who is running the same trail. How long will it take Heather to catch up to Inez if Heather runs $\frac{1}{3}$ km on Inez's trail each minute?

43. **WRITING** Essential Question: Show that $\frac{1}{a} \div \frac{1}{b} = \frac{b}{a}$ by rewriting the quotient as a fraction and multiplying the numerator and denominator by the reciprocal of the divisor.



C. Exercises

Evaluate each expression.

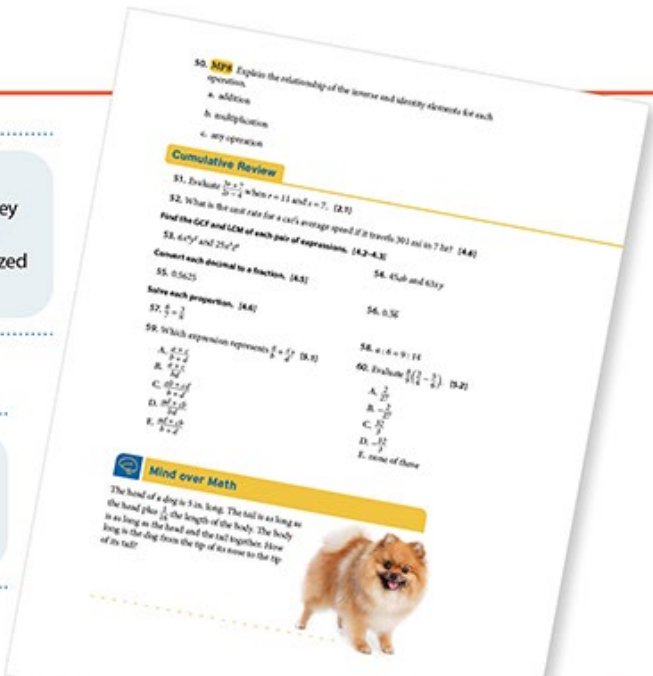
44. $(\frac{1}{2} \div \frac{1}{3}) \div (-\frac{1}{4})$
45. $\frac{1}{2} \div (\frac{1}{3} \div \frac{1}{4})$
46. $\frac{1}{2} \div (-\frac{1}{3} \div \frac{1}{4})$
47. Find the perimeter of a rectangular building if it has a width of $17 \frac{1}{2}$ ft and an area of 1800 ft².
48. $\sqrt{517}$ to the nearest tenth.
49. $\sqrt[3]{67}$ to the nearest hundredth.

Examples and Skill Checks
Study the step-by-step reasoning to solve example problems, and check your understanding by completing the Skill Check exercises.

Exercises
Build and maintain your skills with carefully sequenced exercises that emphasize the essential question, biblical worldview shaping, and essential mathematical practices.

Cumulative Review
Systematically review key concepts and practice strategies for standardized testing.

Mind over Math
Challenge your critical thinking with this unique feature found in each chapter.



Chapter 6 Review

Key Concepts

- Forms of Percents (6.1)**
- A percent is a ratio of a number to 100.
 - To convert a percent to a decimal, remove the percent sign and divide by 100.
 - To convert a decimal to a percent, multiply by 100 and add the percent sign.
 - To convert a percent to a fraction, write the ratio of the number to 100 and simplify.
 - To convert a fraction to a percent, rewrite the percent as a fraction with a denominator of 100, find the decimal equivalent, and convert to a percent. Alternatively, use a proportion.

- Solving Percent Equations (6.2)**
- Use the percent formula: percent \times whole = part or a proportion: $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.
 - Convert percents to decimals or fractions before substituting into an equation.
 - If the percent is greater than 100%, the part is greater than the whole.

- Using Percents (6.3)**
- The percent equation can be used to find the part, the whole, or the percent.
 - An increase of $n\%$ implies the new amount is $(100 + n)\%$ of the original.
 - A decrease by $n\%$ implies the new amount is $(100 - n)\%$ of the original.

Applications of the Percent Formula	
sales tax (6.2)	tax rate = $\frac{\text{tax}}{\text{sales tax}}$
discount (6.4)	discount rate = $\frac{\text{discount}}{\text{original price}}$
markup (6.4)	markup rate = $\frac{\text{markup}}{\text{cost}}$
tip (6.5)	percent = $\frac{\text{tip}}{\text{bill}}$
commission (6.5)	commission = sales \times earnings

- Interest (6.4)**
- simple interest: $I = Prt$ and $A = P + I$
 - annually compounded interest: $A = P(1 + r)^t$
- Percent Change (6.2)**
- percent change = $\frac{\text{amount of change}}{\text{original amount}} \times 100\%$

- Scales (6.6)**
- A scale is a ratio of the length of the drawing or model of an object to the actual length of the object. Scales are used in drawings, maps, blueprints, and scale models.
 - copying settings: percent setting = $\frac{\text{original dimension}}{\text{new dimension}}$

Biblical Worldview Shaping

- Numbers and Reasoning**
- Chapter 5: Creation**—Why is mathematical reasoning a powerful tool? The deductive method used in mathematical reasoning is an effective tool for demonstrating consistency and provides insight into God's creation, God's work, and God's character.
- Chapter 6: Fall and Redemption**—Why can't evil mathematical reasoning lead to certainty? Mathematically accurate statements depend on input that may or may not be true and are subject to misuse or misinterpretation by fallen people.

Vocabulary	6.3 Quizzes and Tests 233
balance (6.4)	commission (6.5)
commission (6.5)	interest (6.4)
compound interest (6.4)	markup (6.4)
cost (6.4)	markup rate (6.4)
discount (6.4)	percent (6.1)
discount rate (6.4)	principal (6.4)
	scale (6.6)
	scale drawing (6.6)
	simple interest (6.4)
	tip (6.5)

Exercises

- Express each percent as a decimal. (6.1)
1. 75% 2. 55.5% 3. 0.4%
- Express each decimal as a percent. (6.1)
4. 5.9 5. 0.795 6. 0.0002
- Express each percent as a fraction. (6.1)
7. 54% 8. 160% 9. $4\frac{1}{2}\%$
- Express each fraction as a percent. (6.1)
10. $\frac{11}{12}$ 11. $\frac{1}{4}$ 12. $\frac{3}{16}$
- Find each amount or percent. (6.2)
13. What is 32% of 90? 14. 14 is 89% of what number? 15. What percent of 18 is 72?
16. Find the sales tax and total cost of a \$200 microwave if the tax rate is $7\frac{1}{2}\%$. (6.2)
17. Find the sales tax rate if Joey pays \$2.60 tax on a video with a price of \$45. (6.2)
18. If 20% of the annual budget of the church is designated for missions and the church's annual income is \$500,000, how much is given to missions each year? (6.3)
19. Jake made 33 out of 60 free throws this season. What was his free throw percentage for the season? (6.3)
20. Ellen sold 12% of the candy bars her class sold to raise money for a new water cooler in the school. If she sold 16 candy bars, what was the total number of candy bars sold by her class? (6.3)
21. The total cost of a dress, including tax, was \$125.37. If the sales tax rate is 6%, what was the pre-tax price of the dress? (6.3)
22. From 1996 to 2016, the average price for a new car rose 68.8%. If the average price in 2000 was \$29,215, what was the average price in 1996? Round to the nearest dollar. (6.3)
23. The warehouse was able to immediately ship 27,072 orders. If this represents 94% of their orders, how many orders were they not able to immediately ship? (6.3)



Chapter Review

Prepare for assessments with a summary of key concepts and the biblical worldview theme, new vocabulary terms, and review exercises.

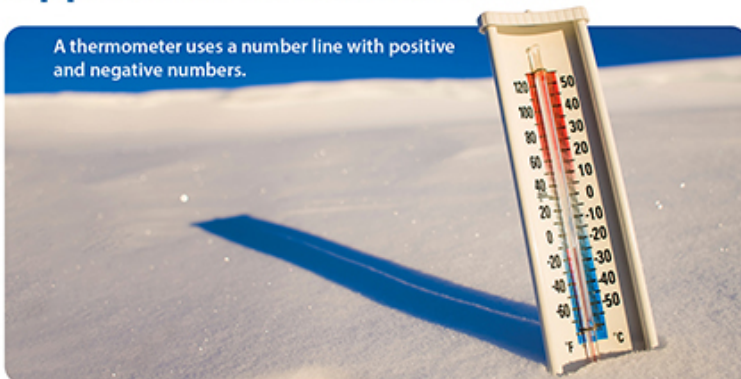
1.1 Opposites and Absolute Value

How do the absolute values of opposite integers compare?

After completing this section, you will be able to

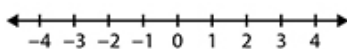
- order integers, using number lines.
- explain the assumption that makes number lines useful.
- state the opposite of an integer.
- find the absolute value of an integer.

A thermometer uses a number line with positive and negative numbers.



Negative numbers are frequently used in everyday life. Temperatures below 0, an overdrawn bank account, and losing points in a game can all be represented by negative numbers.

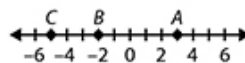
Recall that the set of whole numbers, $W = \{0, 1, 2, 3, \dots\}$, contains 0 and the natural (or counting) numbers, $N = \{1, 2, 3, \dots\}$. The set of integers, $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$, is frequently used to label a number line.



Negative integers are indicated with a negative sign, such as -4 , but positive integers do not require a sign. Zero is neither positive nor negative. The number associated with a point on the number line is called its **coordinate**. The point, which is usually named with a capital letter, is called the **graph** of the number.

Example 1 Stating the Coordinate

State the coordinate of points A , B , and C .



Answers

- A: 3 1. Point A is 3 units to the right of 0.
B: -2 2. Point B is 2 units to the left of 0.
C: -5 3. Point C is 5 units to the left of 0.

Tip

The small end of the symbol points toward the smaller number.

$$5 > 2$$

Skill Check A

Draw a number line and graph each point on the line.

1. $X: -1$ 2. $Y: 4$ 3. $Z: -4$

Number lines can be used to compare integers. Consider a number line with 2 integers graphed. The number to the *left* on the number line is *less than* ($<$) the number on the right. The number to the *right* is *greater than* ($>$) the number on the left.

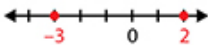
Example 2 Comparing Integers

Compare the following integers, using $<$ or $>$.

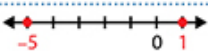
- a. 2 and -3 b. -5 and 1

Answers

- a. $2 > -3$ 2 is to the right of -3 on the number line, so 2 is greater than -3 .



- b. $-5 < 1$ -5 is to the left of 1 on the number line, so -5 is less than 1.



You can also use inequality symbols to order a set of integers from least to greatest or from greatest to least.

Example 3 Ordering Integers

Write the integers -1 , 2 , and -3 in order from least to greatest using $<$.

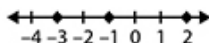
Answer

-3 is farthest to the left on the number line. It is the least number.

2 is farthest to the right on the number line. It is the greatest number.

-1 is between -3 and 2 on the number line. It is greater than -3 but less than 2 .

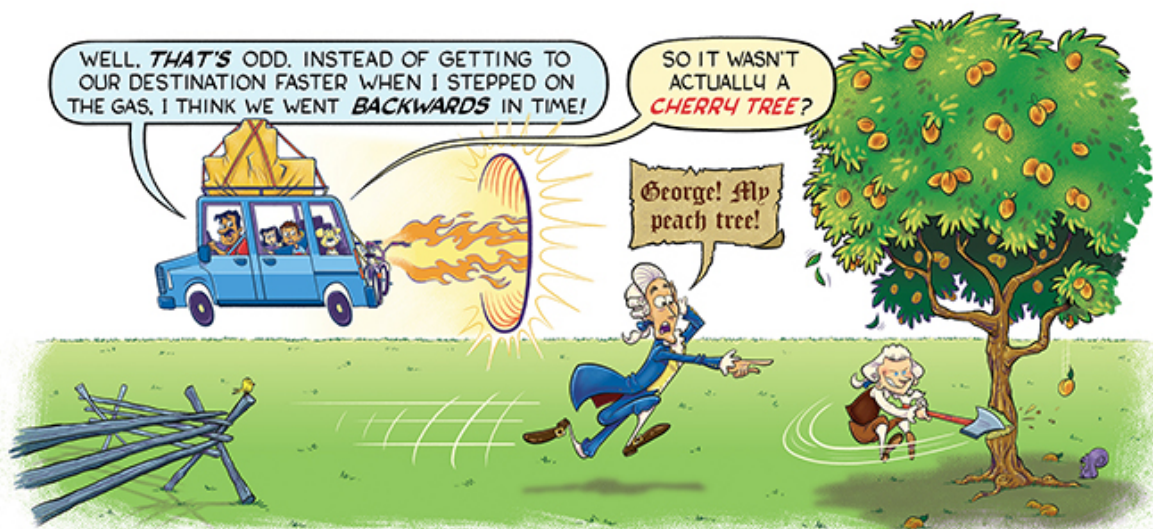
$$-3 < -1 < 2$$



Skill Check B

Compare the integers, using $<$ or $>$.

- -4 $\underline{\hspace{1cm}}$ 0
- -6 $\underline{\hspace{1cm}}$ -11
- 3 $\underline{\hspace{1cm}}$ -9
- Order -3 , 5 , and -7 from least to greatest using $<$.
- Order 8 , -6 , 0 , and -1 from greatest to least using $>$.



A. Exercises

Fill in the blank.

1. The natural numbers consist of the ____ integers.
2. The integer ____ is neither positive nor negative.
3. The set of integers is indicated by the symbol ____.
4. The opposite of a positive integer is a ____ integer.
5. The absolute value of every nonzero integer is a ____ integer.

Draw a number line and graph each point on the line.

6. A: 3
7. B: -3
8. C: 2
9. D: -4

State the opposite of each number.

10. 8 11. -16 12. -4.5 13. $\frac{1}{2}$

Find each absolute value.

14. $|-7|$ 15. $|3|$ 16. $|0|$ 17. $|-152|$

Compare using $<$ or $>$.

18. 9 ____ -3 19. -2 ____ -13 20. -4 ____ 2
21. -15 ____ -10 22. $|-5|$ ____ 1 23. $|-13|$ ____ $|-18|$

B. Exercises

Order the integers from least to greatest using $<$.

24. 1, -3, 0 25. -4, -9, -7 26. -3, -9, -5, -17 27. -4, $|-2|$, 5, 3

Order the integers from greatest to least using $>$.

28. -2, -5, 3 29. -11, 0, -19 30. -2, 6, -7, 2 31. -18, $|11|$, -4, $|-23|$

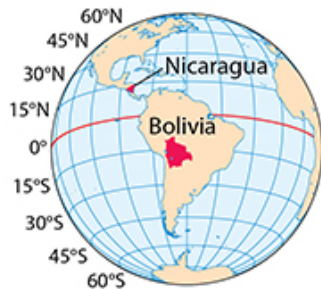
Evaluate each expression.

32. $-(-13)$ 33. $-|5|$ 34. $-|-6|$
35. $-(7 - 4)$ 36. $|17 - 9|$ 37. $-|5 - 3|$

38. Define opposite numbers.
39. Define the absolute value of a number.

Classify each statement as *always*, *sometimes*, or *never* true. Explain your reasoning.

40. The absolute value of a number is negative.
41. The absolute value of an integer is positive.
42. The opposite of the absolute value of an integer is not negative.
43. **Essential Question:** How do the absolute values of opposite integers compare?
44. **BWS Essential Question:** What assumption is necessary for number lines to be useful?
45. The low temperatures for 5 days in northern Minnesota are -12°F , -32°F , 2°F , -10°F , and 8°F .
 - a. Which temperature is the farthest from 0°F ?
 - b. Which temperature is the closest to 0°F ?
46. A team exploring a shipwreck 210 ft below the ocean's surface uses a submarine 100 ft below the surface, a diver at the shipwreck, and a support helicopter 260 ft above the ocean's surface.
 - a. Write integers that represent the elevation of the diver, submarine, and helicopter.
 - b. Which of these objects is closest to the surface?
47. The latitude at the equator is 0° . Positive latitudes are north of the equator, while negative latitudes are south of the equator. Carlos lives in Bolivia at a latitude 16° south of the equator, and Pedro lives in Nicaragua at a latitude 12° north of the equator.
 - a. State an integer that represents the latitude of each boy.
 - b. Which boy lives the farthest from the equator?



C. Exercises

Evaluate each expression and then order the values from least to greatest using $<$.

48. $|2|$, $-|-2|$, $|2 - 2|$, $|2 \cdot 2|$
49. $|-8|$, $|-8| - |-8|$, $-|8|$, $|8| \cdot |-8|$
50. $|6 + 3|$, $|6 - 3|$, $-|6 \cdot 3|$, $-|6 \div 3|$
51. $|-10| \cdot |8|$, $|10| - |-8|$, $|10| + |-8|$, $-|10 \div 5|$
52. **MP2** Classify the following as *always*, *sometimes*, or *never* true:
If $a < b$, then $|a| > |b|$. Explain your reasoning.
53. **MP2** Can $|x| = -x$? Explain why or why not.