CONTENTS

LET'S GO EXPLORING!

Captain Frank Worsley's ship, the *Endurance*, was trapped and crushed by pack ice while carrying Ernest Shackleton's 1914 trans-Antarctic expedition. All 28 men escaped but were then stranded on Elephant Island. Their only hope of rescue was a whaling village on South Georgia Island, 800 nautical miles across some of the stormiest waters on earth.

Worsley, Shackleton, and four other men had no choice but to navigate gale-force winds and icy waves in a modified lifeboat in an attempt to reach this tiny speck in the southern ocean. Clinging to the mast of the *James Caird*, Worsley gripped his sextant in a cold-stiffened hand and attempted to get precise readings while the sun played hide-and-seek in an angry, overcast sky. The sun peeked through for a few seconds, and Worsley was able to obtain the critical readings.

Worsley's calculations using formulas such as

$$\cos(\textit{zenith distance}) = \sin L \sin D + \cos L \cos D \cos A \text{ and}$$

$$\tan(\textit{azimuth}) = \frac{\sin A \tan D}{\cos L} - \sin L \cos A,$$

(where L is the latitude, D is the declination, and A is the local hour angle), showed that they were on course! After reaching South Georgia Island and trekking over dangerous mountains to reach the whaling village, Shackleton's group was able to return and rescue the rest of the expedition.

A hundred years later, algebra still serves to help us explore the world and solve real-world problems. Look for the Biblical Worldview Connection box at the beginning of each chapter to see how algebra will be used to analyze data and solve problems within a Christian worldview. You will explore business and debt management; model motion in crash tests; and analyze sound, music, and pendulums. You will see how algebra can help us improve and save people's lives—people who bear God's image. In this way, mathematics can be used to better fulfill God's commands to wisely care for and utilize His creation (the dominion mandate), love others, and glorify Him (Gen. 1:26–28; Matt. 22:35–40).

Mathematics can reflect the consistency and orderliness of God when it helps us to discover the regularity in His created universe. Apart from the regular, precise orbits of heavenly bodies, Worsley's navigational formulas would have been worthless. Our understanding of the complexities and magnitude of our universe is greatly enhanced by quantitative descriptions. The reflections of God's knowledge and power found in mathematics compel us to worship Him (Ps. 104:24).

Chapter Openers

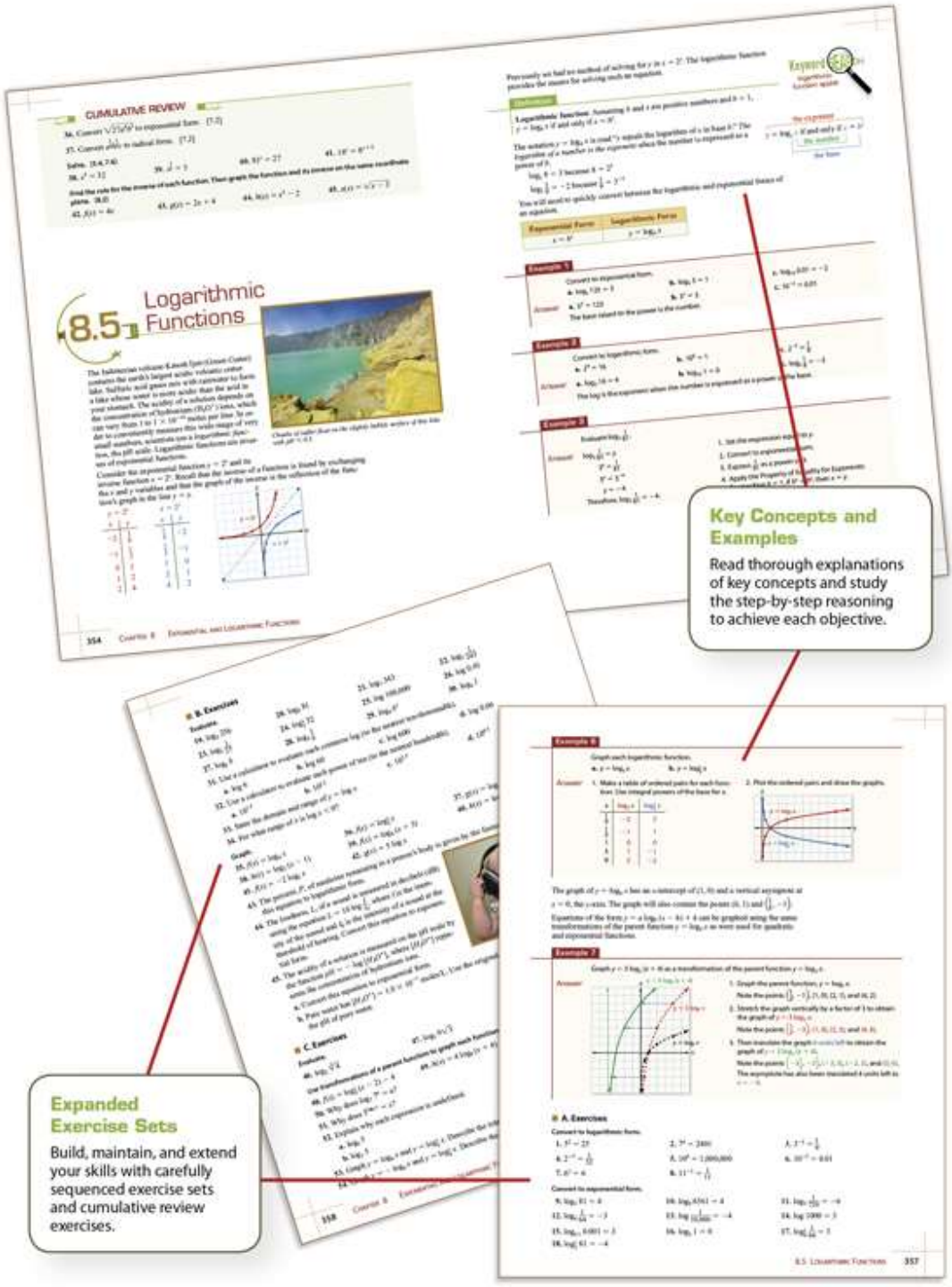
Read the opener to discover the chapter's Biblical Worldview Connection and learning objectives.

Dominion Modeling

Apply mathematical concepts to solve real-life problems within a Christian worldview.

Biographies and History

Discover key mathematical contributions of individuals and cultures in the Math in History and Algebra Around the World features.


CUMULATIVE REVIEW
8.5 Logarithmic Functions
Key Concepts and Examples
Read thorough explanations of key concepts and study the step-by-step reasoning to achieve each objective.
Expanded Exercise Sets
Build, maintain, and extend your skills with carefully sequenced exercise sets and cumulative review exercises.

Technology Corners

Discover how to apply spreadsheets, graphing calculators, and interactive geometry software to create mathematical models and solve problems.

Keyword Searches

Quickly locate additional information, interactive activities, and supplementary resources online.

Chapter Reviews

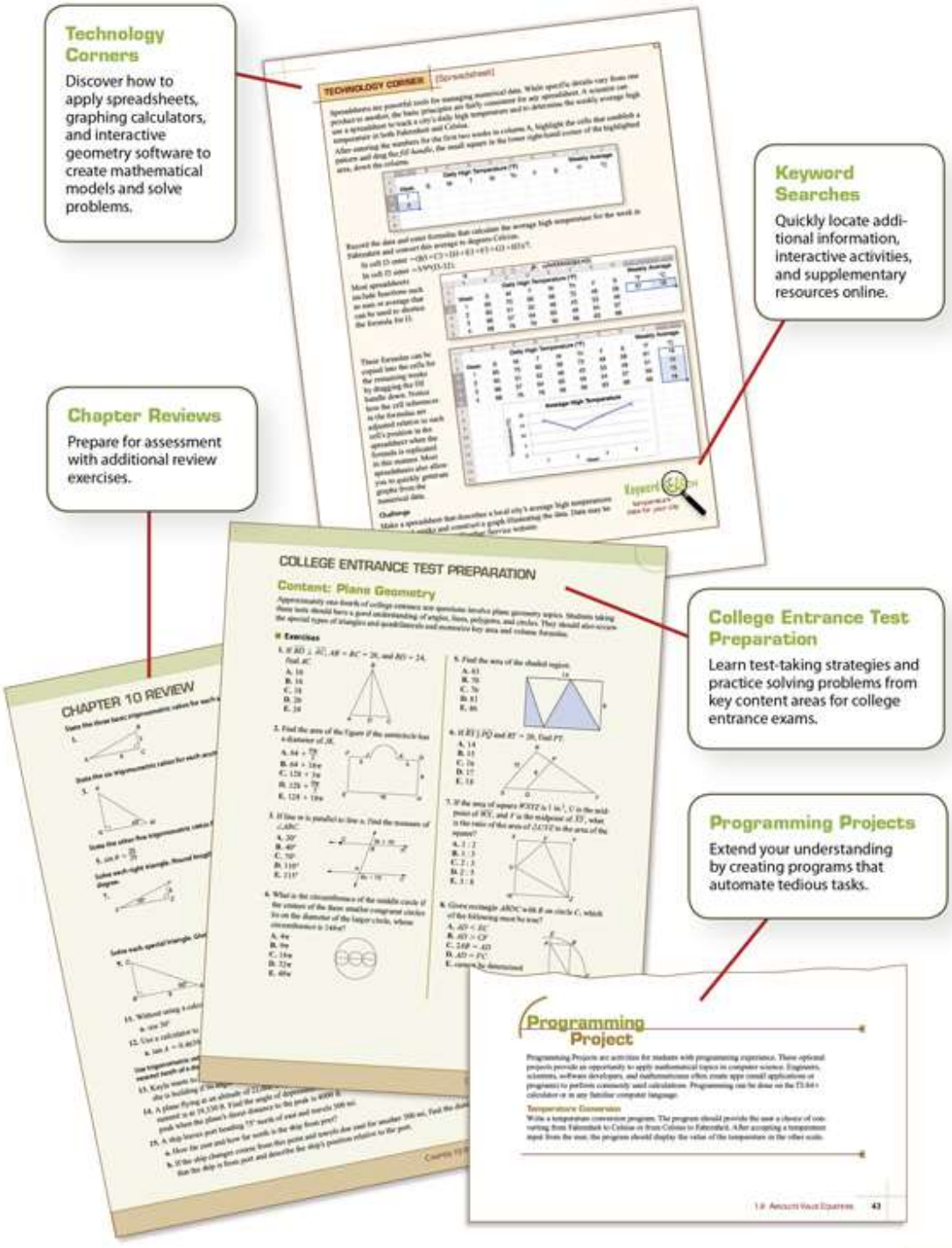
Prepare for assessment with additional review exercises.

College Entrance Test Preparation

Learn test-taking strategies and practice solving problems from key content areas for college entrance exams.

Programming Projects

Extend your understanding by creating programs that automate tedious tasks.



4 MATRICES

"We never landed on the moon." That's what some people really think! They say that the *Apollo* missions to the moon were a conspiracy—a great deception. They analyze images produced by these missions, looking for evidence of a grand hoax. These analyses have been debunked by many professionals, including living astronauts.

That's not the only way people can misuse images. Viral photos circulate on the Internet, passing on urban legends. Many of these images have been heavily edited. But you've probably edited your own photos to get rid of red eye or to make the colors more vivid. What's the difference? Editing can either enhance an image or cause it to deceive or mislead. We value truth because God is a God of truth (Deut. 32:4; John 4:23–24).

Digital technology and image editing have opened new doors for art, graphic design, and photography. It may surprise you to learn that many transformations of images are based on some elegant algebra. These tools can be used to transform images so that they better communicate truth (Eph. 4:25).

Biblical Worldview Connection	
What?	Transforming images
How?	Using matrix operations
Why?	Matrix algebra is a powerful tool for making images that can better communicate truth (Deut. 32:4; Eph. 4:25).

After this chapter you should able to

1. identify matrices and their elements.
2. organize data into matrices.
3. add and subtract matrices.
4. find scalar and matrix products.
5. represent a system of equations using a matrix equation.
6. find the determinant of a matrix.
7. find the inverse of a matrix.
8. solve matrix equations of the form $AX = C$ using Cramer's rule and inverse matrices.
9. use matrix operations to transform images.

4.1 Organizing Data with Matrices

It is unethical for photojournalists to manipulate photos to achieve a more dramatic effect.

A *matrix* is a mathematical tool used in many areas of business and science. In this chapter, you will see how matrices are used to quickly solve systems of equations. They also help graphic artists and computer programmers edit, render, and manipulate images. Matrices are the mathematics behind many amazing animations.

Definitions

A **matrix** is a rectangular array of numbers. Each number in the matrix is called an *element* (or *entry*).

Matrices are named with capital letters. The elements in a matrix are arranged in *rows* (horizontal) and *columns* (vertical) and are enclosed in brackets.

$$M = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 7 & 5 \end{bmatrix}$$

The *dimensions* of a matrix are given in the form $m \times n$, where m is the number of rows and n is the number of columns. The fact that matrix M is a 2×3 (read "two by three") matrix is often notated as $M_{2 \times 3}$.

Example 1

State the dimensions of each matrix.

a. $A = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ **b.** $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$ **c.** $C = [1 \quad 4 \quad 9 \quad 2]$ **d.** $D = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Answer **a.** 2×2 **b.** 4×3 **c.** 1×4 **d.** 3×1

Matrix C is a *row matrix* and matrix D is a *column matrix*. Matrix A is a *square matrix* because it has an equal number of rows and columns. Matrix $A_{2 \times 2}$ is said to be of second order.

Subscripts are used to identify elements in a matrix. The element in row i and column j is expressed as a_{ij}.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Keyword SEARCH
matrix notation