



CHAPTER OVERVIEW

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Chapter 1 Review

1.1

ADDING & SUBTRACTING RATIONAL NUMBERS

How is subtraction the same as addition?

After completing this section, I will be able to

- add rational numbers.
- identify the properties of addition.
- subtract rational numbers.
- evaluate sums and differences to solve real-world problems.
- explain the unity and diversity of rational numbers.

RECALL

$$\begin{array}{c} 8 + (-3) \\ \swarrow \searrow \\ \text{addends} \end{array}$$



The ability to add both positive and negative rational numbers is essential to success in algebra. Using the absolute value of the addends simplifies the addition process.

The value of stocks on a given day can be modeled by the addition and subtraction of rational numbers.

DEFINITION

The **absolute value** of a number, denoted $|x|$, is the number of units (distance) between 0 and the location of the number on a number line.

ADDING POSITIVE & NEGATIVE NUMBERS

Addends with the Same Sign	Addends with Different Signs
1. Add the absolute values of the addends.	1. Subtract the smaller absolute value from the larger absolute value.
2. Use the same sign as the 2 addends.	2. Use the sign of the addend with the larger absolute value.

EXAMPLE 1: Adding Integers

Find each sum.

a. $-1 + (-3)$ b. $8 + (-3)$ c. $4 + (-10)$

Answers

- a. $|-1| + |-3| = 1 + 3 = 4$ 1. When addends have the same sign, add their absolute values.
 $-1 + (-3) = -4$ 2. The sum of 2 negatives must be negative.
-
- b. $|8| - |-3| = 8 - 3 = 5$ 1. When addends have different signs, subtract the smaller absolute value from the larger.
 $8 + (-3) = 5$ 2. The sum is positive because $|8| > |-3|$.
-
- c. $|-10| - |4| = 10 - 4 = 6$ 1. When addends have different signs, subtract the smaller absolute value from the larger.
 $4 + (-10) = -6$ 2. The sum is negative because $|-10| > |4|$.

SKILL CHECK EXERCISE 5

Mathematicians have created special names for the rules governing operations on numbers.

DEFINITION

A **mathematical property** or *identity* is an equation or statement that is true for any value of the variable.

PROPERTIES OF ADDITION	
Commutative Property of Addition $a + b = b + a$	When adding 2 numbers together in different orders, the sum is the same. $3 + 5 = 5 + 3$
Associative Property of Addition $(a + b) + c = a + (b + c)$	When grouping numbers together differently while adding, the sum is the same. $(2 + 9) + 4 = 2 + (9 + 4)$
Additive Identity Property $a + 0 = 0 + a = a$	The sum of any number and 0 is that number. $3 + 0 = 0 + 3 = 3$
Additive Inverse Property $a + (-a) = 0$	The sum of any number and its additive inverse (opposite) is 0. $3 + (-3) = 0$

Applying these properties can often simplify sums. The Commutative and Associative Properties establish that numbers can be added in any order. Rearranging the order of the addends produces an equivalent expression that can simplify the adding process.

EXAMPLE 2: Adding Multiple Integers

Find each sum.

a. $-1 + 5 + (-3) + (-7) + 4 + (-6)$

b. $8 + (-3) + 15 + (-8)$

Answers

a. $-1 + 5 + (-3) + (-7) + 4 + (-6)$
 $= 5 + 4 + (-1) + (-3) + (-7) + (-6)$
 $= 9 + (-17)$
 $= -8$

1. Apply the Commutative Property of Addition.
2. Add all the positive numbers and add all the negative numbers.
3. Add the results.

b. $8 + (-3) + 15 + (-8)$
 $= 8 + (-8) + (-3) + 15$
 $= 0 + 12$
 $= 12$

1. Apply the Commutative Property of Addition.
2. Apply the Additive Inverse Property.
3. Apply the Additive Identity Property.

SKILL CHECK EXERCISE 17

The process of subtracting signed numbers can often be simplified by rewriting a difference as an equivalent sum.

DEFINITION

Subtraction is defined as adding the opposite. For any real numbers a and b , $a - b = a + (-b)$.

This equation is read “ a minus b equals a plus the opposite of b .” When subtracting a smaller positive number from a larger positive number ($8 - 3$), it is unnecessary to use the definition to rewrite the expression as addition [$8 + (-3)$]. However, more complicated subtractions such as $4 - (-2)$ and $-3 - 2$ are more easily understood by using the definition to add the opposite.

EXAMPLE 3: Subtracting Integers

Find each difference.

a. $4 - (-2)$ b. $-3 - 2$ c. $2 - 5$

Answers

a. $4 - (-2) = 4 + 2 = 6$

Subtract -2 by adding its opposite, 2 .

b. $-3 - 2 = -3 + (-2) = -5$

Subtract 2 by adding its opposite, -2 .

c. $2 - 5 = 2 + (-5) = -3$

Subtract 5 by adding its opposite, -5 .

SKILL CHECK EXERCISE 11

The unity and diversity we find in mathematics reflects the unity and diversity found in our triune God. Without the revelation of Scripture, we would have no adequate explanation for why our world functions this way. Rational numbers, for example, may be very different, yet they can all be expressed as a quotient of integers.

Our number system reflects diversity in the many different forms of numbers (integers, fractions, decimal numbers, radicals, etc.). At the same time, our number system represents unity since the principles of adding and subtracting integers also apply to all real numbers.



RECALL

Align decimals when adding or subtracting.

AFTERSCHOOLHELP



Adding & Subtracting Rational Numbers

EXAMPLE 4: Adding or Subtracting Rational Numbers

Find each sum or difference.

a. $-112.6 + 39.27$ b. $2.39 - (-14.682)$ c. $(-\frac{1}{12}) + \frac{3}{4}$ d. $-\frac{5}{12} - \frac{5}{8}$

Answers

a.
$$\begin{array}{r} 112.6 \\ -39.27 \\ \hline 73.33 \end{array}$$
$$-112.6 + 39.27 = -73.33$$

1. When addends have different signs, find the difference of their absolute values.

2. The sum is negative because $|-112.6| > |39.27|$.

b.
$$2.39 - (-14.682) = 2.39 + 14.682 = 17.072$$

1. To subtract, add the opposite.

2. The sum is positive since both addends are positive.

$$\begin{aligned} \text{c. } \left(-\frac{1}{12}\right) + \frac{3}{4}\left(\frac{3}{9}\right) &= \left(-\frac{1}{12}\right) + \frac{9}{12} \\ \frac{9}{12} - \frac{1}{12} &= \frac{8}{12} = \frac{2}{3} \\ \left(-\frac{1}{12}\right) + \frac{3}{4} &= \frac{2}{3} \end{aligned}$$

1. Rename so the fractions have a common denominator.
2. When addends have different signs, find the difference of their absolute values.
3. The sum is positive because $\left|\frac{3}{4}\right| > \left|-\frac{1}{12}\right|$.

$$\begin{aligned} \text{d. } -\frac{5}{12} - \frac{5}{8} &= -\frac{5}{12} + \left(-\frac{5}{8}\right) \\ &= -\frac{5}{12}\left(\frac{2}{2}\right) + \left(-\frac{5}{8}\right)\left(\frac{3}{3}\right) \\ &= \left|-\frac{10}{24}\right| + \left|-\frac{15}{24}\right| = \frac{25}{24} \\ -\frac{5}{12} + \left(-\frac{5}{8}\right) &= -\frac{25}{24} \end{aligned}$$

1. To subtract, add the opposite.
2. Add the absolute values.
3. The sum is negative since both addends are negative.

SKILL CHECK EXERCISES 23, 33

EXAMPLE 5: Solving a Real-World Problem



On Monday the price of stock in an electronics company opened at \$15.75, fell \$0.19 to its daily low before climbing \$0.56 to its daily high, and then fell \$0.20 to its closing price. What was the value of the stock at the end of the day on Monday?

Answer

$$\begin{aligned} &15.75 - 0.19 + 0.56 - 0.20 \\ &= 15.75 + (-0.19) + 0.56 + (-0.20) \\ &= 15.75 + 0.56 + (-0.19) + (-0.20) \\ &= 16.31 + (-0.39) \\ &= |16.31| - |-0.39| \\ &= 16.31 - 0.39 = 15.92 \end{aligned}$$

1. Write an expression modeling the price of the stock.
2. To subtract, add the opposite.
3. Apply the Commutative Property to rearrange the order of the numbers.
4. Find the sum of the positives and the sum of the negatives.
5. Subtract the smaller absolute value from the larger. The answer is positive since the larger absolute value is positive.

The value of the stock at the end of the day was \$15.92.

SKILL CHECK EXERCISE 41

VOCABULARY

- absolute value
- mathematical property
- subtraction

A. EXERCISES

Fill in the blank.

1. The _____ of a number, n , is the distance between n and 0 on the number line.
2. A _____ is a statement that is true for any value of the variable in the statement.
3. **Essential Question:** Subtraction is defined as _____ the opposite.

Find each sum.

- | | | |
|-----------------|----------------|-----------------|
| 4. $3 + 21$ | 5. $-13 + 32$ | 6. $26 + (-28)$ |
| 7. $-14 + (-7)$ | 8. $-126 + 35$ | 9. $7 + (-3)$ |

Find each difference.

- | | | |
|-----------------|-------------|-------------------|
| 10. $3 - (-2)$ | 11. $4 - 5$ | 12. $-12 - 5$ |
| 13. $-8 - (-2)$ | 14. $6 - 8$ | 15. $-16 - (-31)$ |

B. EXERCISES

Simplify each expression.

- | | | |
|--|---|--|
| 16. $-7 + (-8) + 18 + (-5)$ | 17. $14 + (-9) + 6 + (-28)$ | 18. $-1 + 2 - (-4) - 3$ |
| 19. $5 - 3 - [-(-9)] + 13$ | 20. $[-5] + (-4) - 3 + (-6) - (-8)$ | 21. $-8 + -10 - (-9) - 17 $ |
| 22. $-527.69 + (-89.41)$ | 23. $0.23 + (-624.4)$ | 24. $-25.6 + 11.23$ |
| 25. $621.4 - 0.23$ | 26. $3.896 - (-11.42)$ | 27. $-13.716 - (-7.07)$ |
| 28. $\frac{1}{5} + \frac{8}{15}$ | 29. $\frac{4}{7} + \left(-\frac{11}{7}\right)$ | 30. $-\frac{2}{15} + \frac{3}{5}$ |
| 31. $-\frac{12}{21} + \left(-\frac{10}{3}\right)$ | 32. $-\frac{4}{9} + \frac{5}{2}$ | 33. $\frac{2}{9} - \left(-\frac{5}{9}\right)$ |
| 34. $5 - \frac{3}{4} - \left(-\frac{8}{9}\right)$ | 35. $-\frac{7}{10} + \frac{3}{5} - \frac{7}{2}$ | 36. $\frac{8}{9} - \frac{14}{9} + \left(-\frac{5}{3}\right)$ |
| 37. $5\frac{3}{7} - 9\frac{1}{5} + \left(-3\frac{13}{35}\right)$ | | |

Write and evaluate an expression for each exercise.

38. The temperature on a September day is 68°F at 7:00 AM, increases 14° by noon, and then decreases 20° from noon to 10:00 PM. What is the temperature at 10:00 PM?
39. A motorboat travels at an average speed of 17 knots (nautical miles per hour) in still water. Determine the actual speed of the boat when it is traveling against a current of 5 knots.
40. If the temperature on January 5 increased from -3°F to 14°F , how much did it increase?
41. Find the difference in the height of Mt. Whitney (14,494 ft elevation) and the depth of Death Valley (282 ft below sea level).
42. Mathilda is sewing a top treatment for her bedroom window that requires a special self-enclosed French seam. The first part of the seam measures $\frac{1}{4}$ in., and the second part enclosing the first seam measures $\frac{3}{8}$ in. What is the total amount of fabric that Mathilda should allow for the top treatment seam?

43. Bob is putting gutters on the last home in a new subdivision. The individual measurements (in feet) for each length of gutter needed are 20 , $3\frac{3}{4}$, $30\frac{1}{2}$, $7\frac{1}{6}$, $20\frac{5}{6}$, $7\frac{1}{6}$, and $25\frac{1}{4}$. Find the total length of gutters that Bob needs to order if he has 3 pieces measuring $5\frac{1}{2}$ ft, $7\frac{7}{12}$ ft, and $8\frac{2}{3}$ ft left over from another job site.

State the property of addition that justifies each numbered step in the following simplification.

$$(-3 + 10) + (-7)$$

44. $= [10 + (-3)] + (-7)$ _____

45. $= 10 + [-3 + (-7)]$ _____
 $= 10 + (-10)$ addition

46. $= 0$ _____

47. **BWS Essential Question:** What is one way that rational numbers exhibit unity and diversity?

C. EXERCISES

Simplify each expression.

48. $-|3 - 15| + (-9)$

49. $2.9 + 13.05 + (-26.472)$

50. $|-8| - |7| - |-8 - 7|$

51. $4 - (-5) + |2 - 19| - (-6) - 4$

52. $\frac{3}{7} + \frac{5}{14} - \frac{1}{3} + \frac{9}{28}$

53. **MP3** Simplify the expression $-\frac{3}{4} - (-\frac{2}{3})$ and compare your work with Emily's work.

- a. Did Emily get the correct answer if she put $-\frac{1}{12}$ on her answer sheet?
- b. Determine which steps in Emily's work contain errors and describe each error that was made.

Step	Emily's Work
	$-\frac{3}{4} - (-\frac{2}{3})$
1.	$= -\frac{3}{4} + (-\frac{2}{3})$
2.	$= -\frac{9}{12} + (-\frac{8}{12})$
3.	$= -\frac{1}{12}$

1.2 MULTIPLYING & DIVIDING RATIONAL NUMBERS

How can I use a property of multiplication to solve a division problem?

After completing this section, I will be able to

- multiply rational numbers.
- divide rational numbers.
- identify the properties of multiplication.
- evaluate products and quotients to solve real-world problems.

RECALL

$$25 \cdot 10 = 250$$

$\begin{array}{c} \diagdown \quad \diagup \\ \text{factors} \quad \text{product} \end{array}$



Lola manages a food truck in the park during the summer. After the first 5 days, she notices that her expenses exceeded her sales, and she has lost \$7 each day. What is her net income so far?

$$-7 + (-7) + (-7) + (-7) + (-7) = -\$35$$

Lola can save time if the repeated addition is replaced with multiplication.

$$5(-7) = -\$35$$

Recall the following rules for multiplying signed numbers.

MULTIPLICATION RULES FOR THE PRODUCT OF 2 NUMBERS

1. If the factors are both positive or both negative, the product is positive.
2. If 1 factor is positive and the other is negative, the product is negative.

Note how these rules are applied to find each of the following products.

$$2 \cdot 3 = 6 \quad -2 \cdot 3 = -6 \quad 2 \cdot (-3) = -6 \quad -2 \cdot (-3) = 6$$

The following generalizations can be made when finding the product of more than 2 factors since the product of every pair of negative factors is positive.

MULTIPLICATION RULES FOR THE PRODUCT OF MULTIPLE NUMBERS

1. If there is an even number of negative factors, the product is positive.
2. If there is an odd number of negative factors, the product is negative.

EXAMPLE 1: Multiplying Integers

Calculate each product.

a. $-4(-6)$ b. $-2(-3)(-5)$

Answers

- a. $-4(-6) = 24$ A product of 2 factors with the same sign is positive.
- b. $2 \cdot 3 \cdot 5 = 30$ 1. Multiply the absolute values.
- $-2(-3)(-5) = -30$ 2. A product of 3 negative factors is negative.

SKILL CHECK EXERCISE 7

