LIFEPAC®

Math
Teacher's Guide

Grade 12

Alpha Omega Publications®

Mathematics 1200
Teacher's Guide

CONTENTS

Revision Editor: Alan Christopherson, M.S.

Alpha Omega Publications®

LIFEPAC

MANAGEMENT

STRUCTURE OF THE LIFEPAC CURRICULUM

The LIFEPAC curriculum is conveniently structured to provide one teacher handbook containing teacher support material with answer keys and ten student worktexts for each subject at grade levels two through twelve. The worktext format of the LIFEPACs allows the student to read the textual information and complete workbook activities all in the same booklet. The easy to follow LIFEPAC numbering system lists the grade as the first number(s) and the last two digits as the number of the series. For example, the Language Arts LIFEPAC at the 6th grade level, 5th book in the series would be LAN0605.

Each LIFEPAC is divided into 3 to 5 sections and begins with an introduction or overview of the booklet as well as a series of specific learning objectives to give a purpose to the study of the LIFEPAC. The introduction and objectives are followed by a vocabulary section which may be found at the beginning of each section at the lower levels, at the beginning of the LIFEPAC in the middle grades, or in the glossary at the high school level. Vocabulary words are used to develop word recognition and should not be confused with the spelling words introduced later in the LIFEPAC. The student should learn all vocabulary words before working the LIFEPAC sections to improve comprehension, retention, and reading skills.

Each activity or written assignment has a number for easy identification, such as 1.1. The first number corresponds to the LIFEPAC section and the number to the right of the decimal is the number of the activity.

Teacher checkpoints, which are essential to maintain quality learning, are found at various locations throughout the LIFEPAC. The teacher should check 1) neatness of work and penmanship, 2) quality of understanding (tested with a short oral quiz), 3) thoroughness of answers (complete sentences and paragraphs, correct spelling, etc.), 4) completion of activities (no blank spaces), and 5) accuracy of answers as compared to the answer key (all answers correct).

The self test questions are also number coded for easy reference. For example, 2.015 means that this is the 15th question in the self test of Section II. The first number corresponds to the LIFEPAC section, the zero indicates that it is a self test question, and the number to the right of the zero the question number.

The LIFEPAC test is packaged at the centerfold of each LIFEPAC. It should be removed and put aside before giving the booklet to the student for study.

Answer and test keys have the same numbering system as the LIFEPACs and appear at the back of this handbook. The student may be given access to the answer keys (not the test keys) under teacher supervision so that he can score his own work.
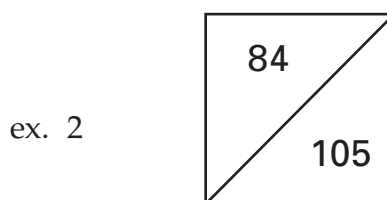
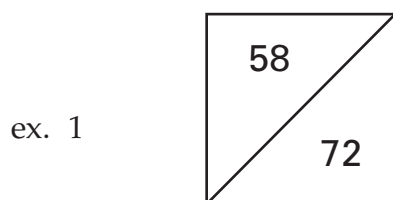A thorough study of the Curriculum Overview by the teacher before instruction begins is essential to the success of the student. The teacher should become familiar with expected skill mastery and understand how these grade level skills fit into the overall skill development of the curriculum. The teacher should also preview the objectives that appear at the beginning of each LIFEPAC for additional preparation and planning.

TEST SCORING and GRADING

Answer keys and test keys give examples of correct answers. They convey the idea, but the student may use many ways to express a correct answer. The teacher should check for the essence of the answer, not for the exact wording. Many questions are high level and require thinking and creativity on the part of the student. Each answer should be scored based on whether or not the main idea written by the student matches the model example. "Any Order" or "Either Order" in a key indicates that no particular order is necessary to be correct.

Most self tests and LIFEPAC tests at the lower elementary levels are scored at 1 point per answer; however, the upper levels may have a point system awarding 2 to 5 points for various answers or questions. Further, the total test points will vary; they may not always equal 100 points. They may be 78, 85, 100, 105, etc.

ex. 1
```
 ┌──────────┐
 │ 58   ╱   │
 │    ╱     │
 │  ╱    72 │
 └──────────┘
```

ex. 2
```
 ┌──────────┐
 │ 84   ╱   │
 │    ╱     │
 │  ╱   105 │
 └──────────┘
```

A score box similar to ex.1 above is located at the end of each self test and on the front of the LIFEPAC test. The bottom score, 72, represents the total number of points possible on the test. The upper score, 58, represents the number of points your student will need to receive an 80% or passing grade. If you wish to establish the exact percentage that your student has achieved, find the total points of his correct answers and divide it by the bottom number (in this case 72.) For example, if your student has a point total of 65, divide 65 by 72 for a grade of 90%. Referring to ex. 2, on a test with a total of 105 possible points, the student would have to receive a minimum of 84 correct points for an 80% or passing grade. If your student has received 93 points, simply divide the 93 by 105 for a percentage grade of 89%. Students who receive a score below 80% should review the LIFEPAC and retest using the appropriate Alternate Test found in the Teacher's Guide.

The following is a guideline to assign letter grades for completed LIFEPACs based on a maximum total score of 100 points.

LIFEPAC Test   =   60% of the Total Score (or percent grade)
Self Test   =   25% of the Total Score (average percent of self tests)
Reports   =   10% or 10* points per LIFEPAC
Oral Work   =   5% or 5* points per LIFEPAC
*Determined by the teacher's subjective evaluation of the student's daily work.

Example:

LIFEPAC Test Score	=	92%	92 x .60	=	55 points
Self Test Average	=	90%	90 x .25	=	23 points
Reports				=	8 points
Oral Work				=	4 points

_____

TOTAL POINTS	=	90 points

Grade Scale based on point system:	100	–	94	=	A
	93	–	86	=	B
	85	–	77	=	C
	76	–	70	=	D
	Below		70	=	F

TEACHER HINTS and STUDYING TECHNIQUES

LIFEPAC Activities are written to check the level of understanding of the preceding text. The student may look back to the text as necessary to complete these activities; however, a student should never attempt to do the activities without reading (studying) the text first. Self tests and LIFEPAC tests are never open book tests.

Language arts activities (skill integration) often appear within other subject curriculum. The purpose is to give the student an opportunity to test his skill mastery outside of the context in which it was presented.

Writing complete answers (paragraphs) to some questions is an integral part of the LIFEPAC Curriculum in all subjects. This builds communication and organization skills, increases understanding and retention of ideas, and helps enforce good penmanship. Complete sentences should be encouraged for this type of activity. Obviously, single words or phrases do not meet the intent of the activity, since multiple lines are given for the response.

Review is essential to student success. Time invested in review where review is suggested will be time saved in correcting errors later. Self tests, unlike the section activities, are closed book. This procedure helps to identify weaknesses before they become too great to overcome. Certain objectives from self tests are cumulative and test previous sections; therefore, good preparation for a self test must include all material studied up to that testing point.

The following procedure checklist has been found to be successful in developing good study habits in the LIFEPAC curriculum.

1. Read the introduction and Table of Contents.
2. Read the objectives.
3. Recite and study the entire vocabulary (glossary) list.
4. Study each section as follows:
    a. Read the introduction and study the section objectives.
    b. Read all the text for the entire section, but answer none of the activities.
    c. Return to the beginning of the section and memorize each vocabulary word and definition.
    d. Reread the section, complete the activities, check the answers with the answer key, correct all errors, and have the teacher check.
    e. Read the self test but do not answer the questions.
    f. Go to the beginning of the first section and reread the text and answers to the activities up to the self test you have not yet done.
    g. Answer the questions to the self test without looking back.
    h. Have the self test checked by the teacher.
    i. Correct the self test and have the teacher check the corrections.
    j. Repeat steps a–i for each section.

5. Use the SQ3R* method to prepare for the LIFEPAC test.
6. Take the LIFEPAC test as a closed book test.
7. LIFEPAC tests are administered and scored under direct teacher supervision. Students who receive scores below 80% should review the LIFEPAC using the SQ3R* study method and take the Alternate Test located in the Teacher Handbook. The final test grade may be the grade on the Alternate Test or an average of the grades from the original LIFEPAC test and the Alternate Test.

    *SQ3R: **S**can the whole LIFEPAC.
               Question yourself on the objectives.
               Read the whole LIFEPAC again.
               Recite through an oral examination.
               Review weak areas.

GOAL SETTING and SCHEDULES

Each school must develop its own schedule, because no single set of procedures will fit every situation. The following is an example of a daily schedule that includes the five LIFEPAC subjects as well as time slotted for special activities.

Possible Daily Schedule

8:15	–	8:25	Pledges, prayer, songs, devotions, etc.
8:25	–	9:10	Bible
9:10	–	9:55	Language Arts
9:55	–	10:15	Recess (juice break)
10:15	–	11:00	Mathematics
11:00	–	11:45	Social Studies
11:45	–	12:30	Lunch, recess, quiet time
12:30	–	1:15	Science
1:15	–		Drill, remedial work, enrichment*

*Enrichment: Computer time, physical education, field trips, fun reading, games and puzzles, family business, hobbies, resource persons, guests, crafts, creative work, electives, music appreciation, projects.

Basically, two factors need to be considered when assigning work to a student in the LIFEPAC curriculum.

The first is time. An average of 45 minutes should be devoted to each subject, each day. Remember, this is only an average. Because of extenuating circumstances a student may spend only 15 minutes on a subject one day and the next day spend 90 minutes on the same subject.

The second factor is the number of pages to be worked in each subject. A single LIFEPAC is designed to take 3 to 4 weeks to complete. Allowing about 3-4 days for LIFEPAC introduction, review, and tests, the student has approximately 15 days to complete the LIFEPAC pages. Simply take the number of pages in the LIFEPAC, divide it by 15 and you will have the number of pages that must be completed on a daily basis to keep the student on schedule. For example, a LIFEPAC containing 45 pages will require 3 completed pages per day. Again, this is only an average. While working a 45 page LIFEPAC, the student may complete only 1 page the first day if the text has a lot of activities or reports, but go on to complete 5 pages the next day.

Long range planning requires some organization. Because the traditional school year originates in the early fall of one year and continues to late spring of the following year, a calendar should be devised that covers this period of time. Approximate beginning and completion dates can be

noted on the calendar as well as special occasions such as holidays, vacations and birthdays. Since each LIFEPAC takes 3-4 weeks or eighteen days to complete, it should take about 180 school days to finish a set of ten LIFEPACs. Starting at the beginning school date, mark off eighteen school days on the calendar and that will become the targeted completion date for the first LIFEPAC. Continue marking the calendar until you have established dates for the remaining nine LIFEPACs making adjustments for previously noted holidays and vacations. If all five subjects are being used, the ten established target dates should be the same for the LIFEPACs in each subject.

FORMS

The sample weekly lesson plan and student grading sheet forms are included in this section as teacher support materials and may be duplicated at the convenience of the teacher.

The student grading sheet is provided for those who desire to follow the suggested guidelines for assignment of letter grades found on page 3 of this section. The student's self test scores should be posted as percentage grades. When the LIFEPAC is completed the teacher should average the self test grades, multiply the average by .25 and post the points in the box marked self test points. The LIFEPAC percentage grade should be multiplied by .60 and posted. Next, the teacher should award and post points for written reports and oral work. A report may be any type of written work assigned to the student whether it is a LIFEPAC or additional learning activity. Oral work includes the student's ability to respond orally to questions which may or may not be related to LIFEPAC activities or any type of oral report assigned by the teacher. The points may then be totaled and a final grade entered along with the date that the LIFEPAC was completed.

The Student Record Book which was specifically designed for use with the Alpha Omega curriculum provides space to record weekly progress for one student over a nine week period as well as a place to post self test and LIFEPAC scores. The Student Record Books are available through the current Alpha Omega catalog; however, unlike the enclosed forms these books are not for duplication and should be purchased in sets of four to cover a full academic year.

NOTES

INSTRUCTIONS FOR TWELFTH GRADE MATHEMATICS

The LIFEPAC curriculum from grades two through twelve is structured so that the daily instructional material is written directly into the LIFEPACs. The student is encouraged to read and follow this instructional material in order to develop independent study habits. The teacher should introduce the LIFEPAC to the student, set a required completion schedule, complete teacher checks, be available for questions regarding both content and procedures, administer and grade tests, and develop additional learning activities as desired. Teachers working with several students may schedule their time so that students are assigned to a quiet work activity when it is necessary to spend instructional time with one particular student.

Mathematics is a subject that requires skill mastery. But skill mastery needs to be applied toward active student involvement. Measurements require measuring cups, rulers, empty containers. Boxes and other similar items help the study of solid shapes.   Construction paper, beads, buttons, beans are readily available and can be used for counting, base ten, fractions, sets, grouping, and sequencing. Students should be presented with problem situtations and be given the opportunity to find their solutions.

Any workbook assignment that can be supported by a real world experience will enhance the student's ability for problem solving. There is an infinite challenge for the teacher to provide a meaningful environment for the study of mathematics.   It is a subject that requires constant assessment of student progress.   Do not leave the study of mathematics in the classroom.

The Teacher Notes section of the Teacher's Guide lists the required or suggested materials for the LIFEPACs and provides additional learning activities for the students. Additional learning activities provide opportunities for problem solving, encourage the student's interest in learning and may be used as a reward for good study habits.

Materials Needed for LIFEPAC
    Required: Suggested:
                                                          reference materials i.e. precalculus textbook

Additional Learning Activities
  Section I Ordered-Pair Numbers
    1. Discuss symmetry with respect to the axes, with respect to a point, and with respect to the origin.
    2. Have students plot the following sets of ordered pairs, one set to each graph. Then ask them to name the corresponding geometric figures.
        a. (2, 5), (5, -2), (-1, -2)
        b. (6, 4), (3, -1), (-5, -1), (-2, 4)
        c. (0, 3), (3, -1), (6, -4), (3, 2)
        d. (1, 4), (-2, 0), (0, -4), (5, -3), (6, 2)

  Section II The Algebra of Functions
    1. The distance from a person to a strike of lightning is a function of the time between the flash and the thunder according to the following table:

Time	1	2	3	4	5
Distance (ft.)	1,100	2,200	3,300	4,400	5,500

        a. Have the students draw a graph of the preceding relationship.
        b. Have the students write a formula for this function.
        c. How far away would a person be if the time between the flash and the thunder to the person were 11 seconds?

Additional Activity
The following activity may be reproduced as a student worksheet.

PLOTTING SYMMETRIC POINTS

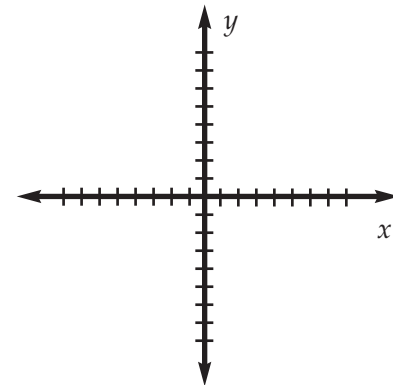Model: Plot (2, 3) and the point symmetric to it with respect to the *x*-axis. List its coordinates.
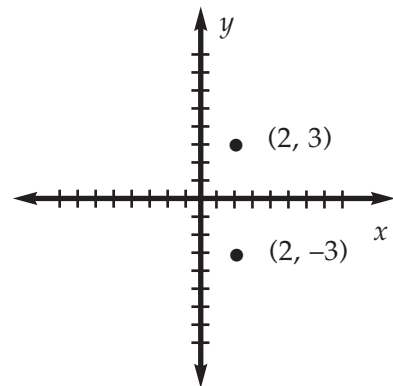
The point symmetric to (2, 3) with respect to the *x*-axis is (2, -3).
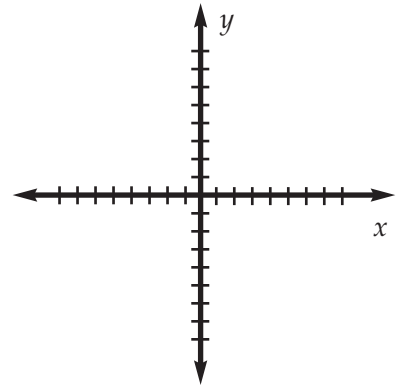
Plot the required points.

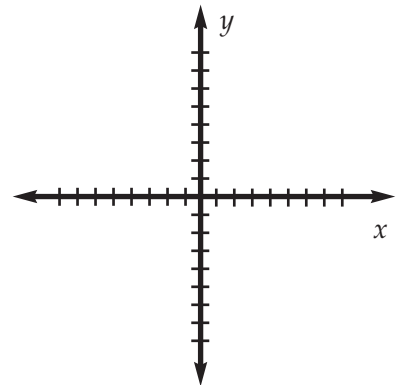1. Plot (3, -7) and the point symmetric to it with respect to the *x*-axis. List its coordinates.
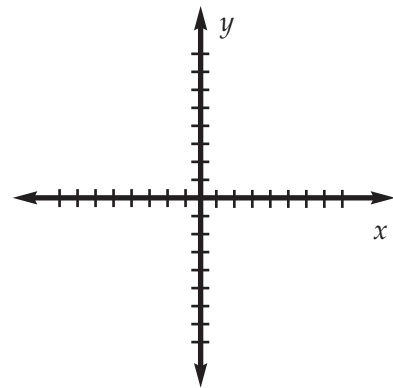
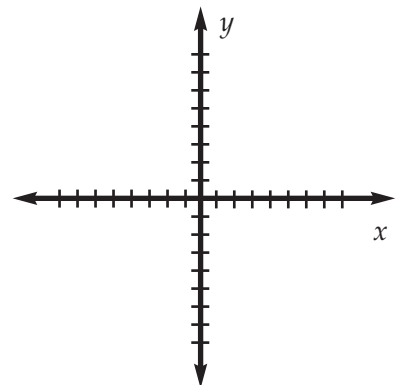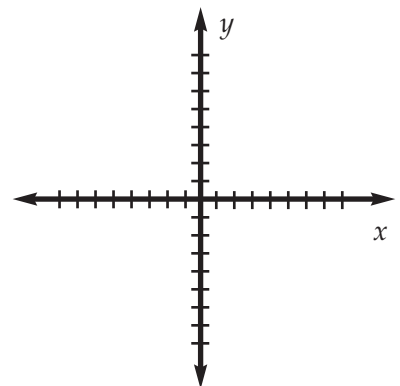2. Plot (-4, 3) and the point symmetric to it with respect to the *y*-axis. List its coordinates.
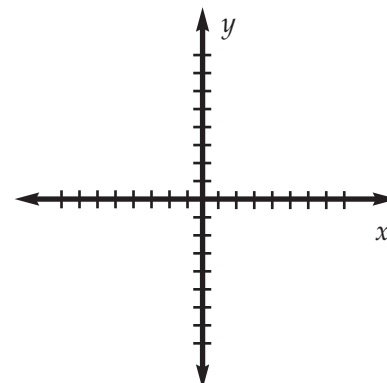
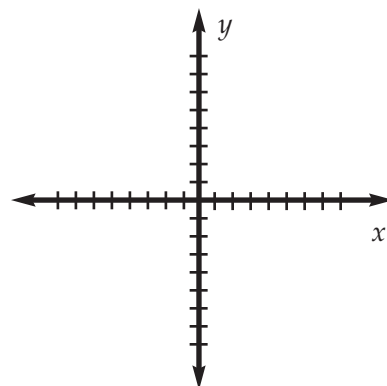3. Plot (-3, -6) and the point symmetric to it with respect to the origin. List its coordinates.

4. Plot (2, 5) and the point symmetric to it with respect to the *y*-axis. List its coordinates.

5. Plot (-5, 1) and the point symmetric to it with respect to the *x*-axis. List its coordinates.

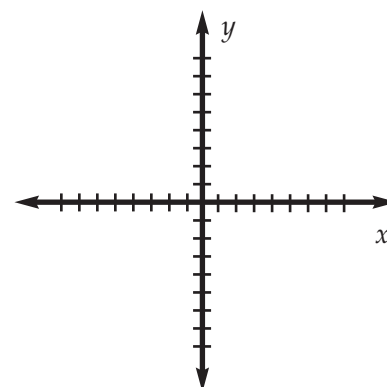6. Plot (1, 7) and the point symmetric to it with respect to the origin. List its coordinates.

7. Plot (-2, -5) and the point symmetric to it with respect to the *x*-axis. List its coordinates.

8. Plot (4, -2) and the point symmetric to it with respect to the origin. List its coordinates.

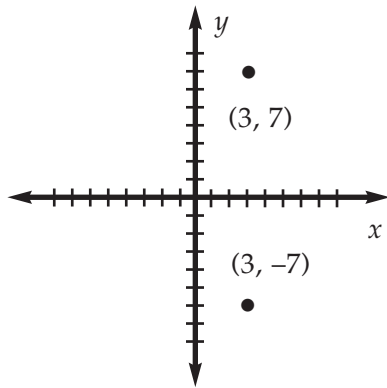9. Plot (-1, -1) and the point symmetric to it with respect to the *y*-axis. List its coordinates.

10. Plot (3, 6) and the point symmetric to it with respect to the *x*-axis. List its coordinates.
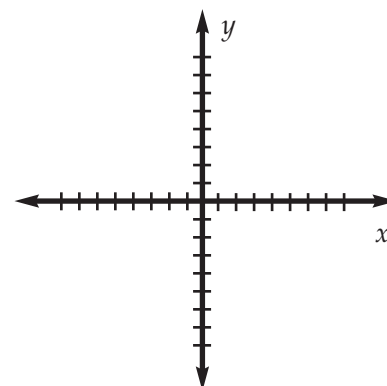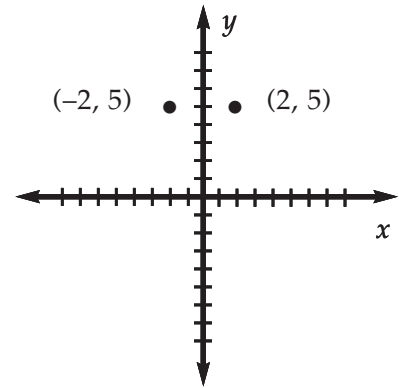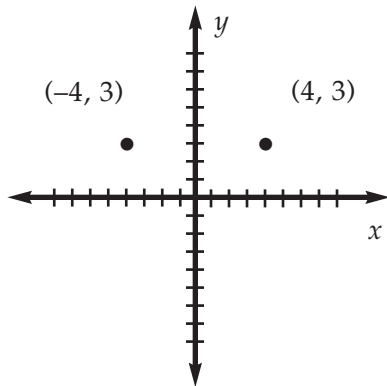
Additional Activity, Solution Key

1.

Point (3, 7) is symmetric to (3, -7) with respect to the *x*-axis.

4.
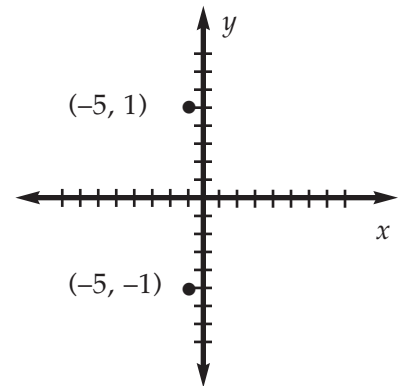

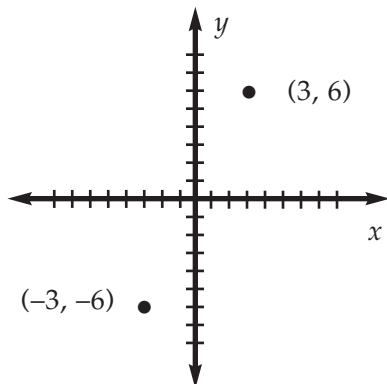Point (-2, 5) is symmetric to (2, 5) with respect to the *y*-axis.

2.

Point (4, 3) is symmetric to (-4, 3) with respect to the *y*-axis.

5.

Point (-5, -1) is symmetric to (-5, 1) with respect to the *x*-axis.

3.

Point (3, 6) is symmetric to (-3, -6) with respect to the origin.

6.
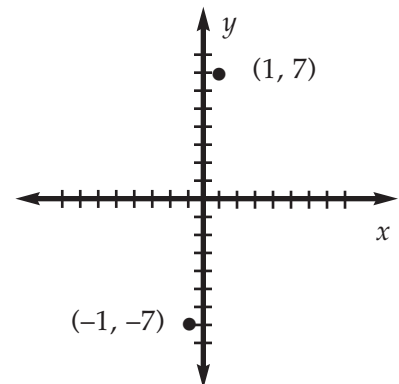
Point (-1, -7) is symmetric to (1, 7) with respect to the origin.

7.

(−2, 5)

(−2, −5)
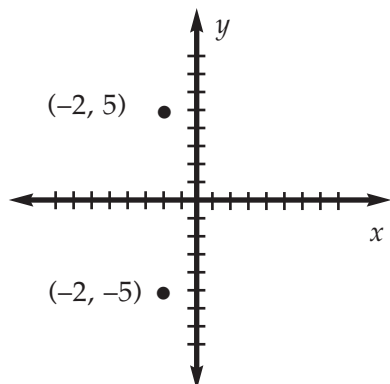
Point (2, 5) is symmetric to (-2, -5)
with respect to the *x*-axis.

10.

(3, 6)

(3, −6)
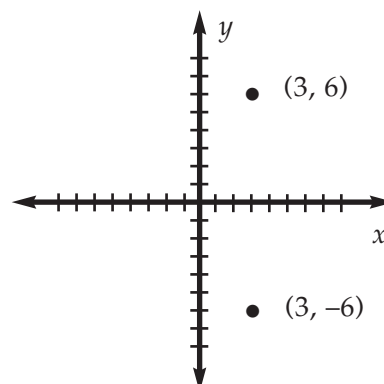
Point (3, -6) is symmetric to (3, 6)
with respect to the *x*-axis.

8.

(−4, 2)

(4, −2)
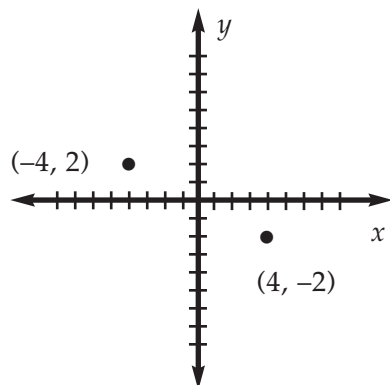
Point (-4, 2) is symmetric to (4, -2)
with respect to the origin.

9.

(−2, −1)   (1, −1)
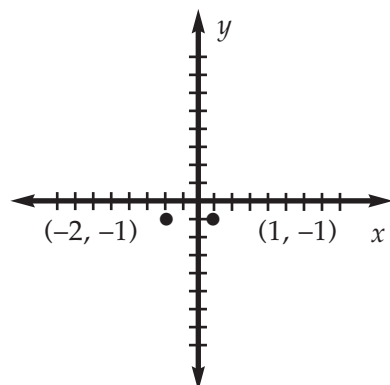
Point (1, -1) is symmetric to (-1, -1)
with respect to the *y*-axis.

A L T E R N A T E

T E S T S

Reproducible Tests
for use with the Mathematics
1200 Teacher's Guide

Name _____

Write the required information (each answer, 3 points).

1. State the range of the relation $G = \{(4, \sqrt[3]{7}), (6, \sqrt[3]{9}, (8, \sqrt[3]{11})\}$.

    _____

2. Let $x = \{1, 2, 3, 4\}$ and $y = \{a, b, c\}$. Let H be the set of ordered pairs $\{(1, b), (2, a), (3, b), (1, a), (4, c)\}$. Is H a function?

    _____

3. Write the set of ordered pairs for the relation $\{(x, y): x = |4y + 1| \text{ and } y \in \{-2, -1, 0, 1, 2\}\}$.

    _____

4. The following figures are graphs of relations between real numbers. Which graphs are functions?

    _____

a.

b.

c.
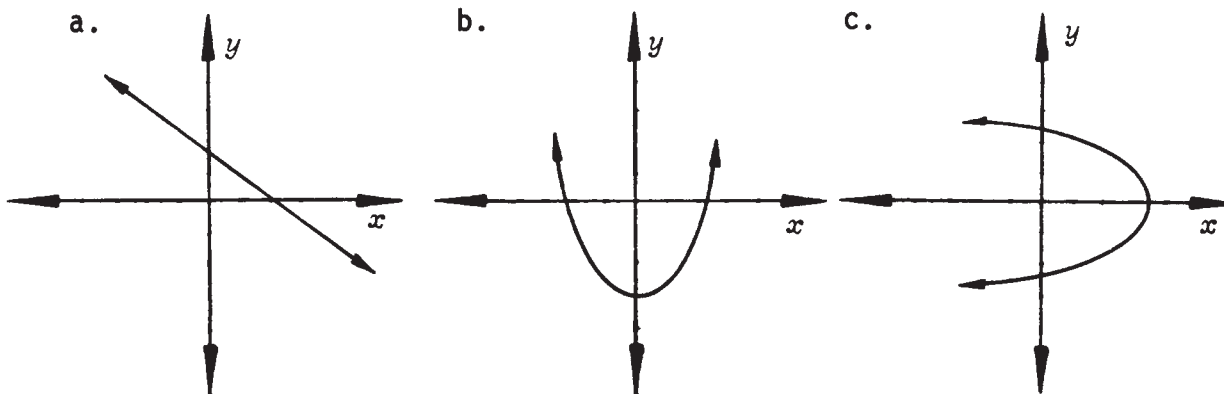
5. State the domain of $\{(x, y): y = 3x^2 - 2\}$.

    _____

Write the required rules (each answer, 5 points).

6. $A = \{(x, f(x)): (4, 2), (9, 3), (16, 4), (25, 5)\}$.

    _____

7. $B = \{(x, g(x)): (0, -1), (-1, -4), (2, 5), (-2, -7)\}$.

    _____

Perform the required operations on the following functions (**each answer, 5 points**).

Given: $F(x) = 2x^2 - 2$, $G(x) = x + 1$, $H(x) = x$

8. $F(3)$ _____

9. $F(x) - G(x)$ _____

10. $F \circ G$ _____

11. $G^{-1}(x)$ _____

12. $F(1) + G(2) - 3H(-1)$ _____

13. $(F \cdot G)(x)$ _____

14. $(\frac{F}{G})(x)$ _____

15. $G \circ H$ _____

16. $G^{-1} \circ H^{-1}$ _____

17. $(F + G)(x)$ _____

18. $(G + H)(x)$ _____

19. $(G \cdot H)(x)$ _____

20. Is $F^{-1}(x)$ a function? _____

72 / 90
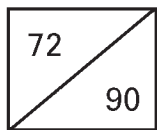
Date _____

Score _____

ANSWER KEYS

MATHEMATICS 1201
SOLUTION KEY

I. SECTION ONE

1.1   a. {5, 6, 7}
      b. {0, 1, 2}

1.2   a. {6, 7, 8, 9}
      b. {$\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$}

1.3   a. {$\frac{1}{2}$}
      b. {$\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$}

1.4   a. {6.2, 7.3, 8.4, 9.5}
      b. {0.3}

1.5   {(10, \$2), (15, \$3), (20, \$4),
      (30, \$6), (60, \$8), (90, \$10)}

1.6 F = {(1, 16), (2, 64),
        (3, 144), (4, 256),
        (5, 400)}

1.7   {x: $x \in R$, $x \geq 0$}

1.8   {r: $r \in R$, $r \neq 0$}

1.9   {a: $a \in R$, $a \neq 0$}

1.10 a. {x: x is an even integer}
      b. {y: y is an odd integer}

1.11 a. {x: $x \in R$}
      b. {y: $y \geq 0$}

1.12 a. yes
      b. ---

1.13 a. no
   b. 1 is not paired with a
         unique second element
         of the range.

1.14 a. yes
      b. ---

1.15 a. yes
      b. ---

1.16 a. no
      b. The empty set is not a set
         of ordered-pair numbers.

1.17 a. yes
      b. No two points lie on the
         same vertical line.

1.18 a. no
      b. Two or more points lie
         on the same vertical
         line.

1.19 a. yes
      b. No two points lie on
         the same vertical line.

1.20 a. no
      b. Two or more points lie
         on the same vertical
         line.

1.21 a. no
      b. Two or more pointe lie
         on the same vertical
         line.

1.22 Multiply x by 3 and subtract
      1; hence, $y = 3x - 1$.

1.23 Divide x by 2 and add 2;
      hence, $y = \frac{x}{2} + 2$.

1.24 Multiply x by 3 and subtract
      3; hence, $y = 3x - 3$.

1.25 y is the square of x; hence,
      $y = x^2$.

1.26 Multiply x by 5 and add 5;
      hence, $y = 5x + 5$.

1.27 Subtract 3 from n, multiply
      by n, and divide by 2;
      hence, $D = \frac{n(n - 3)}{2}$.

1.28 D is 50 times t; hence,
      $D = 50t$.

1.29 S is 16 times the square
      of t; hence, $S = 16t^2$.

1.30 a. $S_1 = \frac{0}{2}(0 + 1) = 0$

  b. $S_2 = \frac{1}{2}(1 + 1) = 1$

  c. $S_3 = \frac{2}{2}(2 + 1) = 3$

  d. $6 = \frac{n}{2}(n + 1)$

  $12 = n(n + 1)$

  $n = 3$ or $n = -4$

  e. $S_5 = \frac{100}{2}(100 + 1)$

  $= 5,050$

1.31 a.
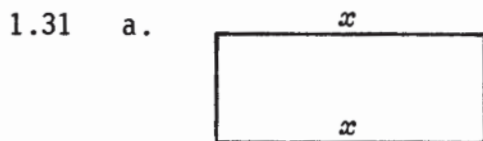
  $A = 2,000$ sq. ft.

  $x l = 2,000$

  $l = \dfrac{2,000}{x}$

  $c(x) = 0.50x + 0.30x$

  $+ 2 \cdot (0.30) \cdot \dfrac{2,000}{x}$

  $c(x) = 0.80x + 0.60(\dfrac{2,000}{x})$

  $c(x) = 0.80x + \dfrac{1,200}{x}$

  b. $x > 0$

II. SECTION TWO

2.1 $F(1) = 1^2 + 3(1) - 2$

  $= 1 + 3 - 2$

  $= 2$

2.2 $F(3) = 3^2 + 3(3) - 2$

  $= 9 + 9 - 2$

  $= 16$

2.3 $F(-1) = (-1)^2 + 3(-1) - 2$

  $= 1 - 3 - 2$

  $= -4$

2.4 $F(a) = a^2 + 3a - 2$

2.5 $F(x - 1) = (x - 1)^2 + 3(x - 1)$

  $- 2$

  $= x^2 - 2x + 1 + 3x$

  $- 3 - 2$

  $= x^2 + x - 4$

2.6 $G(6) = 3(6)^2 - 2(6) - 1$

  $= 3(36) - 12 - 1$

  $= 108 - 12 - 1$

  $= 95$

2.7 $G(a + b) = 3(a + b)^2$

  $- 2(a + b) - 1$

  $= 3(a^2 + 2ab + b^2)$

  $- 2a - 2b - 1$

  $= 3a^2 + 6ab + 3b^2$

  $- 2a - 2b - 1$

2.8 $F(x + h) = (x + h)^2 + 2$

  $= x^2 + 2xh + h^2 + 2$

2.9 $F(x - h) = (x - h)^2 + 2$

  $= x^2 - 2xh + h^2 + 2$

2.10 $F(x^2) = (x^2)^2 + 2 = x^4 + 2$

2.11 $\{2, 4, 6\}$

2.12 $(F + G)(x) = \{(2, 9), (4, 13),$
  $(6, 17)\}$

2.13 $(F \cdot G)(x) = \{(2, 20), (4, 42),$
  $(6, 72)\}$

2.14 $(\frac{F}{G})(x) = \{(2, \frac{4}{5}), (4, \frac{6}{7}),$
  $(6, \frac{8}{9})\}$

2.15 $(F - G)(x) = \{(2, -1),$
  $(4, -1),$
  $(6, -1)\}$

2.16 $(f + g)(x) = (x + 2)$
  $+ (3x + 5)$
  $= 4x + 7$

2.17 $(g \cdot f)(x) = (3x + 5)(x + 2)$
  $= 3x^2 + 6x + 5x + 10$
  $= 3x^2 + 11x + 10$

2.18 $(f - g)(x) = (x + 2)$
  $- (3x + 5)$
  $= x + 2 - 3x - 5$
  $= -2x - 3$

2.19 $\left(\frac{f}{g}\right)(x) = \frac{x + 2}{3x + 5}$; $3x + 5 \neq 0$;

$x \neq -\frac{5}{3}$

2.20 $[f(x)]^2 = (x + 2)^2 = x^2 + 4x + 4$

2.21 $2(x + 2) - \frac{1}{x - 1}$

$= 2x + 4 - \frac{1}{x - 1}$; $x \neq 1$

2.22 $\dfrac{\left[\dfrac{1}{x - 1}\right]^2}{x + 2} = \frac{1}{(x - 1)^2} \cdot \frac{1}{x + 2}$

$= \frac{1}{(x - 1)^2(x + 2)}$

2.23 $(x + h)^2 + 2(x + h) + 1$
$= x^2 + 2xh + h^2 + 2x + 2h + 1$

2.24 $\dfrac{2(x + h) + 3 - (2x + 3)}{h}$

$= \dfrac{2x + 2h + 3 - 2x - 3}{h}$

$= \dfrac{2h}{h} = 2$

2.25 $\dfrac{(x + h)^2 - x^2}{h}$

$= \dfrac{x^2 + 2xh + h^2 - x^2}{h}$

$= \dfrac{2xh + h^2}{h} = \dfrac{h(2x + h)}{h} = 2x + h$

2.26 $\dfrac{(x + h)^3 - x^3}{h}$

$= \dfrac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$

$= \dfrac{3x^2h + 3xh^2 + h^3}{h} = \dfrac{h(3x^2 + 3xh + h^2)}{h}$

$= 3x^2 + 3xh + h^2$

2.27 $\dfrac{\sqrt{x + h} - \sqrt{x}}{h} = \dfrac{\sqrt{x + h} - \sqrt{x}}{h}$

$\cdot \dfrac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}} = \dfrac{x + h - x}{h(\sqrt{x + h} + \sqrt{x})}$

$= \dfrac{h}{h(\sqrt{x + h} + \sqrt{x})} = \dfrac{1}{\sqrt{x + h} + \sqrt{x}}$

2.27 cont.

$-\dfrac{1}{\sqrt{x + h} + \sqrt{x}} \cdot \dfrac{\sqrt{x + h} - \sqrt{x}}{\sqrt{x + h} - \sqrt{x}}$

$= \dfrac{\sqrt{x + h} - \sqrt{x}}{x + h - x} = \dfrac{\sqrt{x + h} - \sqrt{x}}{h}$

2.28 $\dfrac{[(x + h)^2 + 6(x + h) + 9]}{h}$

$- \dfrac{(x^2 + 6x + 9)}{h}$

$= \dfrac{x^2 + 2xh + h^2 + 6x + 6h}{h}$

$+ \dfrac{9 - x^2 - 6x - 9}{h}$

$= \dfrac{2xh + h^2 + 6h}{h} = \dfrac{h(2x + h + 6)}{h}$

$= 2x + h + 6$

2.29 a. $3(x + 5) + 2$
$= 3x + 15 + 2 = 3x + 17$
b. $(3x + 2) + 5 = 3x + 7$
c. all real numbers
d. all real numbers

2.30 a. $(2x - 1)^2 + 6$
$= 4x^2 - 4x + 1 + 6$
$= 4x^2 - 4x + 7$
b. $2(x^2 + 6) - 1$
$= 2x^2 + 12 - 1$
$= 2x^2 + 11$
c. all real numbers
d. all real numbers

2.31 a. $(x^2 + 1) + 7 = x^2 + 8$
b. $(x + 7)^2 + 1$
$= x^2 + 14x + 49 + 1$
$= x^2 + 14x + 50$
c. all real numbers
d. all real numbers

2.32 a. $\dfrac{1}{(3x - 4) + 2} = \dfrac{1}{3x - 2}$
b. $3\left(\dfrac{1}{x + 2}\right) - 4 = \dfrac{3}{x + 2} - 4$
c. $x \neq \dfrac{2}{3}$
d. $x \neq -2$

2.33 a. $\dfrac{1}{\left(\dfrac{1}{2x+4}\right)^2} = \dfrac{1}{\dfrac{1}{(2x+4)^2}}$

$= 1 \cdot \dfrac{(2x+4)^2}{1}$

$= 4x^2 + 16x + 16$

b. $\dfrac{1}{2\left(\dfrac{1}{x^2}\right)+4} = \dfrac{1}{\dfrac{2}{x^2}+4}$

$= \dfrac{1}{\dfrac{2+4x^2}{x^2}} = 1 \cdot \dfrac{x^2}{2+4x^2}$

$= \dfrac{x^2}{4x^2+2}$

c. all real numbers, $x \neq -2$
d. all real numbers, $x \neq 0$

2.34 $g[I(x)] = x^2 + 2$ and $I[g(x)]$
$= x^2 + 2$ and $g(x) = x^2 + 2$

2.35 $f\{g[h(x)]\} = f[g(7)]$
$= f(7+6) = f(13) = 13^2$
$= 169$

2.36 $g\{f[h(x)]\} = g[f(7)] = g(7^2)$
$= g(49) = 49 + 6 = 55$

2.37 $h\{g[f(x)]\} = h[g(x^2)]$
$= h(x^2 + 6) = 7$
$h(x)$ is constant at 7; hence,
h of any value x is 7.

2.38 Given that $f[g(x)] = I$ and $I = x$.
Then $f[g(x)] = x$ and $f(ax + b)$
$= x$.
Now let f be some function,
such as $f(x) = Px + c$.
Then $f(ax + b) = P(ax + b) + c$.
Therefore, $P(ax + b) + c = x$
and $Pax + Pb + c = x$.
Hence, $Pa = 1$ and $Pb + c = 0$;

$P = \dfrac{1}{a}$ and $c = -Pb$

$= -\dfrac{b}{a}$.

Then, $f(x) = \dfrac{1}{a}x - \dfrac{b}{a} = \dfrac{x-b}{a}$.

2.39 a. $\{(4, 1), (3, 2), (8, 5), (7, 4)\}$
b. yes

2.40 a. $\{(3, 5), (3, 2), (4, 6)\}$
b. No, the first element 3 has two values.

2.41 a. $\{(1, 0)\}$
b. yes

2.42 a. $\{(1, 2), (2, 3), (1, 4), (3, 5)\}$
b. No, the first element 1 has two values.

2.43 a. Let $y = F(x)$. Then
$y = 3x + 6$. Interchange
the variables: $x = 3y + 6$
and $y = \dfrac{x-6}{3}$.

$\therefore F^{-1}(x) = \dfrac{x-6}{3}$.

b. yes

2.44 a. Let $y = F(x)$. Then
$y = x^2 + 5$. Interchange
the variables:
$x = y^2 + 5$ and $y^2 = x - 5$.
$y = \pm\sqrt{x-5}$ or $F^{-1}(x)$
$= \pm\sqrt{x-5}$
b. No, for each value of x
in the domain of F^{-1}, two
function values occur.

2.45 a. Let $y = G(x)$. Then
$y = \dfrac{1}{x-1}$. Interchange
the variables: $x = \dfrac{1}{y-1}$
and $y - 1 = \dfrac{1}{x}$. $y = \dfrac{1}{x} + 1$
and $F^{-1}(x) = \dfrac{1}{x} + 1$
b. yes

2.46 a. Let $y = H(x)$. Then
$y = \sqrt{x^2-1}$. Interchange
the variables:
$x = \sqrt{y^2+1}$. $x^2 = y^2 + 1$,
$y^2 = x^2 - 1$; $y = \pm\sqrt{x^2-1}$
and $F^{-1}(x) = \pm\sqrt{x^2-1}$
b. No, for each value of x
in the domain of F^{-1}, two
function values occur.

MATHEMATICS 1201
SELF TEST
SOLUTION KEY

SELF TEST 1

1.01   no

1.02   yes

1.03 ϕ; No, range, because x is not a set of ordered-pair numbers.

1.04   No, not all are pairs.

1.05   {2}

1.06   {4}

1.07   yes

1.08   yes

1.09   no

1.010 yes

1.011 $y = 4x - 4$

1.012 $y = x^3$

1.013 $y = \dfrac{x + 4}{2}$

1.014 $y = \dfrac{x}{2} + 3$

SELF TEST 2

2.01   {-3, 0, 2, 5}

2.02   {-2, 1, 3}

2.03   yes

2.04   No, there is no set of ordered-pair numbers.

2.05   all real numbers; $x \in R$

2.06   all real numbers less than or equal to zero; $y \le 0$

2.07 $g[f(x)] = g(3 - x)$
$= -2(3 - x) = -6 + 2x$
$= 2x - 6$

2.08 $f[g(2)] = f(-4) = 3 - (-4) = 7$

2.09 $g[f(-1)] = g(4) = -2(4) = -8$

2.010 $F[G(x)] - F(x)$
$= F(3x + 2) - F(2x - 1)$
$= 2(3x + 2) - 1 - (2x - 1)$
$= 6x + 4 - 1 - 2x + 1$
$= 4x + 4$

2.011 $G[H(1)] = G(1) = 3(1) + 2 = 5$

2.012 $F\{G[H(2^2)]\} = F[G(4)] = F(14)$
$= 2(14) - 1$
$= 28 - 1$
$= 27$

2.013 $H(x + a) - H(x)$
$= (x + a)^2 - x^2$
$= x^2 + 2ax + a^2 - x^2$
$= a^2 + 2ax$

2.014 $\dfrac{H(x + a) - H(x)}{a}$
$= \dfrac{(x + a)^2 - x^2}{a}$
$= \dfrac{x^2 + 2ax + a^2 - x^2}{a}$
$= \dfrac{2ax + a^2}{a} = \dfrac{a(2x + a)}{a} = 2x + a$

2.015 $F(x) + G(x) + H(x)$
$= (2x - 1) + (3x + 2) + (x^2)$
$= x^2 + 5x + 1$

2.016 a. The second element is one-half the first element:
$y = \frac{1}{2}x$ or $y = \dfrac{x}{2}$.
b. yes
c. $y = \frac{1}{2}x$
$2y = x$
$x = 2y$
The inverse is $y = 2x$.

2.017 a. In each case the second
          element is a square root
          of the first element:
          $y = \sqrt{x}$.
       b. no
       c. not a function

2.018 no

2.019 {(2, 1), (0, 1), (1, 1),
        (3, 0), (0, 0), (1, 2)}

MATHEMATICS 1201
LIFEPAC TEST
SOLUTION KEY

1. $\{x: \ x \in R, \ x \neq -4, \ x \neq 7\}$

2. $(\sqrt{3}, -4), (-\sqrt{3}, 4)$

3. $\{(-2, 2), (-1, 0), (0, 2),$
    $(1, 4), (2, 6)\}$

4. no

5. set of all real numbers

6. $f(x) = x^2 + 2$

7. $g(x) = 2x - 1$; then $y = 2x - 1$
    or $x = \dfrac{y + 1}{2}$. Interchanging

    x and y results in $y = \dfrac{x + 1}{2}$;

    $g^{-1}(x) = \dfrac{x + 1}{2}$.

8. $F(-2) = 3(-2)^2 + 1$
    $\qquad = 3(4) + 1$
    $\qquad = 13$

9. $F(x) + G(x) = (3x^2 + 1)$
    $+ (2x - 3) = 3x^2 + 2x - 2$

10. $F \circ G = F[G(x)] = F(2x - 3)$
    $\qquad = 3(2x - 3)^2 + 1$
    $\qquad = 3(4x^2 - 12x + 9) + 1$
    $\qquad = 12x^2 - 36x + 27 + 1$
    $\qquad = 12x^2 - 36x + 28$

11. Let $y = G(x)$; $y = 2x - 3$.

    Solve for x: $x = \dfrac{y + 3}{2}$.

    Interchange the variables:

    $y = \dfrac{x + 3}{2}$. $\therefore G^{-1}(x) = \dfrac{x + 3}{2}$.

12. Let $y = H(x)$; $y = x$ or $x = y$.
    $H^{-1}(x) = x$.

13. $F(3) + G(4) - 2H(5)$
    $= 3(3)^2 + 1 + 2(4) - 3 - 2(5)$
    $= 27 + 1 + 8 - 3 - 10$
    $= 23$

14. Let $y = F(x)$; $y = 3x^2 + 1$.

    Solve for x: $x = \pm \sqrt{\dfrac{y - 1}{3}}$.

    Interchange the variables:

    $y = \pm \sqrt{\dfrac{x - 1}{3}}$. $\therefore F^{-1}(x)$

    $= \pm \sqrt{\dfrac{x - 1}{3}}$.

15. no, for each value of x
    in the domain of F^{-1}, two
    function values occur.

16. $F[G(x)] = F(x - 4)$
    $\qquad = 5(x - 4) - 6$
    $\qquad = 5x - 20 - 6$
    $\qquad = 5x - 26$
    Let $y = 5x - 26$. Then
    $x = \dfrac{y + 26}{5}$. Then
    $(F \circ G)^{-1} = \dfrac{x + 26}{5}$.

17. $G^{-1} = x + 4$ and $F^{-1} = \dfrac{x + 6}{5}$.

    $G^{-1}(F^{-1}) = G^{-1}(\dfrac{x + 6}{5})$

    $= \dfrac{x + 6}{5} + 4 = \dfrac{x + 6 + 20}{5}$

    $= \dfrac{x + 26}{5}$.

18. $(F + G)(x) = (2x) + (x^2 + 2)$
    $\qquad = x^2 + 2x + 2$

19. $(F - G)(x) = (x^2 + 2) - (2x)$
    $\qquad = x^2 - 2x + 2$

20. $(\dfrac{F}{G})(x) = \dfrac{2x}{x^2 + 2}$

1. $\{y: \quad y = \sqrt[3]{7}, \sqrt[3]{9}, \sqrt[3]{11}\}$

2. H is not a function because two of the given x-values are the same.

3. $y = -2$: $\quad x = |4(-2) + 1| =$
   $$|-8 + 1| =$$
   $$|-7| = 7$$
   $$(7, -2)$$

   $y = -1$: $\quad x = |4(-1) + 1| =$
   $$|-4 + 1| =$$
   $$|-3| = 3$$
   $$(3, -1)$$

   $y = 0$: $\quad x = |4 \cdot 0 + 1| =$
   $$|0 + 1| =$$
   $$|1| = 1$$
   $$(1, 0)$$

   $y = 1$: $\quad x = |4 \cdot 1 + 1| =$
   $$|4 + 1| =$$
   $$|5| = 5$$
   $$(5, 1)$$

   $y = 2$: $\quad x = |4 \cdot 2 + 1| =$
   $$|8 + 1| =$$
   $$|9| = 9$$
   $$(9, 2)$$
   $\{(7, -2), (3, -1), (1, 0),$
   $(5, 1), (9, 2)\}$

4. a and b (c is not a function because the values for x do not have unique values).

5. set of all real numbers

6. $f(x) = \sqrt{x}$

7. $g(x) = 3x - 1$

8. $F(3) = 2(3)^2 - 2 =$
   $$2(9) - 2 = 18 - 2 = 16$$

9. $F(x) - G(x) =$
   $(2x^2 - 2) - (x + 1) =$
   $2x^2 - 2 - x - 1 =$
   $2x^2 - x - 3$

10. $F \circ G = 2(x + 1)^2 - 2$

11. Let $y = G(x)$; then $y = x + 1$.
    Interchange x and y: $\quad x = y + 1$.

    Solve for y: $\qquad x = y + 1$
    $$x - 1 = y + 1 - 1$$
    $$x - 1 = y$$
    $$y = x - 1$$
    $$G^{-1}(x) = x - 1$$

12. $F(1) = 2(1)^2 - 2 =$
    $$2 \cdot 1 - 2 = 2 - 2 = 0$$

    $G(2) = 2 + 1 = 3$
    $3H(-1) = 3(-1) = -3$

    $F(1) + G(2) - 3H(-1) =$
    $0 + 3 - (-3) =$
    $0 + 3 + 3 = 6$

13. $(F \cdot G)(x) = F(x) \cdot G(x) =$
    $(2x^2 - 2)(x + 1) =$
    $2x^3 + 2x^2 - 2x - 2$

14. $(\frac{F}{G})(x) = \dfrac{2x^2 - 2}{x + 1} = \dfrac{2(x^2 - 1)}{x + 1} =$

    $\dfrac{2(\cancel{x + 1})(x - 1)}{\cancel{x + 1}} = 2(x - 1)$

15. $G \circ H = x + 1$

16. To find G^{-1}, let $y = G(x)$;
then $y = x + 1$. Interchange
x and y: $x = y + 1$.

Solve for y:
$$x = y + 1$$
$$x - 1 = y + 1 - 1$$
$$x - 1 = y$$
$$y = x - 1$$
$$G^{-1} = x - 1$$

To find H^{-1}, let $y = H(x)$; then
$y = x$. Interchange x and y.
$y = x$
$x = y$

Solve for y: $y = x$
$$H^{-1} = x$$

$G^{-1} \circ H^{-1} = x - 1$

17. $(F + G)(x) = F(x) + G(x) =$
$(2x^2 - 2) + (x + 1) =$
$2x^2 + x - 1$

18. $(G + H)(x) = G(x) + H(x) =$
$(x + 1) + (x) = 2x + 1$

19. $(G \cdot H)(x) = G(x) \cdot H(x) =$
$(x + 1)(x) = x^2 + x$

20. To find $F^{-1}(x)$, let $y = F(x)$;
then $y = 2x^2 - 2$.
Interchange x and y:
$x = 2y^2 - 2$.

Solve for y:
$$x = 2y^2 - 2$$
$$x + 2 = 2y^2 - 2 + 2$$
$$x + 2 = 2y^2$$
$$\frac{x + 2}{2} = \frac{2y^2}{2}$$
$$\frac{x + 2}{2} = y^2$$
$$\pm\sqrt{\frac{x + 2}{2}} = \sqrt{y^2}$$
$$\pm\sqrt{\frac{x + 2}{2}} = y$$
$$y = \pm\sqrt{\frac{x + 2}{2}}$$

Since y is double valued,
$F^{-1}(x)$ is also double valued
and is therefore not a function.