Introduction

As your student journeys further up and further in the mountainous land of mathematics, it's good to remember the beauty and joy to be found along the way. Charlotte tells us that imagination and reason travel alongside intellect and, "By degrees, absolute truth unfolds itself. We are so made that truth, absolute and certain truth, is a perfect joy to us; and that is the joy that mathematics afford" (*Ourselves*, Book 1, p. 63). Irene Stephens, who headed up the studies in Science and Math for the House of Education, reminds us that Miss Mason "wanted the children to get a real sense of number; some vision of its innate power and beauty far beyond the sum of the moment. Miss Mason taught us that 'Education is the science of relations' and that a child should feel from the very beginning that his relations with number are opening up to him yet another realm of beautiful and wonderful things for his enjoyment and delight" (*The Parents' Review*, Vol. XL, "Number: A Figure and a Step Onward," 1929, p. 36).

In addition to education being the science of relations, Charlotte also held the view that education is an atmosphere, a discipline, and a life. Thus, a number of guiding principles and practices are found in these three educational keys. Some to bear in mind:

Atmosphere. Your good attitude toward math can help your student enjoy a healthy relationship with numbers while also maintaining a healthy relationship between you and your student. Avoid anxiety and conflict by never giving her the idea that she is somehow behind, that math is only for the gifted few or for those going into a STEM career, and by allowing your student the time she needs to wonder at and grapple with math.

Be sure to schedule Arithmetic at a time when your child is mentally fresh and then vary the subjects before and after to help keep her from tiring. Charlotte tells us, "If the lessons be judiciously alternated—sums first, say, while the brain is quite fresh; then writing, or reading—some more or less mechanical exercise, by way of a rest; and so on, the programme varying a little from day to day, but the same principle throughout—a 'thinking' lesson first, and a 'painstaking' lesson to follow,—the child gets through his morning lessons without any sign of weariness *(Home Education,* p. 142). In Miss Mason's own schoolrooms, the schedule for this form began with a subject that involved listening and narrating, next came Arithmetic, to be followed by a mechanical lesson such as Dictation or Grammar.

Discipline. Charlotte Mason believed the good habits cultivated during math lessons gave the subject its rightful place in the curriculum and, over time, these habits would serve a person well throughout her life. Keep lessons short and engaging—they should never exceed 30 minutes with at least 5 of these minutes given to mental work in the form of rapid oral review. More can be accomplished when students are able to give their full attention to concentrated lessons rather than becoming accustomed to dawdling over long, drawn out ones. Allow your student the time needed for careful execution of written problems while the challenge of oral work will help develop fixed attention, concentration, and quick thinking.

A gridded math notebook also goes a long way in fostering habits of neatness and order. The size of grid chosen should be based on your student's writing so that one number can go inside each square, helping to keep everything in proper place value order and easy to read. As writing ability matures, a $\frac{1}{2}$ " grid should give way to the standard $\frac{1}{4}$ " grid. In the beginning stages of new notation—such as the fraction work found in this book—we start with simpler equations involving little work in order to first secure neatness and orderly arrangement.

Never allow sloppy work, give excessive explanations, or immediately help your child over each difficulty. Instead, allow her time to investigate, imagine, reason, and do the work herself. If she gets an answer wrong due to carelessness, a do-over will only reinforce the behavior. If a mistake is made because she doesn't understand the concept, then slow down and secure her understanding before giving a new problem.

Life. Charlotte recognized that mathematics, like music, is a living language that rings clearly with undeniable logic, and is able to feed a child's mind without the literary presentation she felt imperative in other subjects (*A Philosophy of Education*, pp. 333, 334).

In the early years, ideas were presented using common everyday objects. While concrete objects are used less in Book 5, they now give way to the presentation of ideas by means of concrete examples. These examples, together with carefully graduated teaching, and engaging questions that spark the imagination and cultivate the child's power of reasoning, often allow the student to arrive at rules on her own—which you will notice is a focus of this book.

Arithmetic Concepts in Book 5

- Addition of decimals
- Subtraction of decimals
- Multiplication of decimals
- Division of decimals
- Dollars and cents
- Approximations
- Measurement by decimals
- Factors
- Measures of numbers
- Greatest Common Measure
- Numbers divisible by 2 through 12
- Prime numbers
- Greatest Common Factor
- Multiples of a number
- Least Common Multiple
- Fractions
- Improper fractions
- Simplifying fractions
- Addition of fractions
- Subtraction of fractions
- Addition & subtraction of mixed numbers
- Turning decimals into fractions
- Turning fractions into decimals
- Canceling

Overview of Lessons

In the previous book, the student had further practice in the four operations—working with numbers within 1,000,000. She learned to draw to scale, delved deeper into long multiplication and long division, and worked with averages and rounding. She also had more exact work with weights and measures in both the US standard system and the metric system—including measures of time, weight, distance, area, and cubic measure with an active hands-on approach. She continued work with fractions, handled more complex compound addition and subtraction, and was formally introduced to decimals.

In Book 5, the student will advance in her work with decimals and fractions, working with factors and both greatest and least common measures. Both US standard system and the metric system will occur using questions of a concrete nature throughout the book. She'll also continue her work with drawing to scale along with measures in decimals as well as fractions.

The arithmetic work for each week contains three important components: New, Review, and Mental Math too. New refers to work in the newest concept, Review is time given to maintain fluency in past concepts, while pencil and paper are put away for a stimulating time of Mental Math that helps build habits such as attention, speed, and accuracy. The daily lesson length is again 30 minutes per lesson, which includes 5 minutes of mental math. Note that many sections in the book may be used for oral work while additional mental math may be found at the back of the book. If your child is unable to give focused attention for this amount of time, work gradually to build her power of concentration. When her attention wanes, try switching to some lively mental math before returning to the main work of the lesson. Put the book away, though, if she is unable to concentrate following this change. This book is recommended for use with Charlotte Mason Practical Geometry, Part I. When used with Practical Geometry, four days should be given to arithmetic and one day to geometry. The recommended schedule is Monday through Thursday given to arithmetic lessons with Friday reserved for the hands-on geometry lesson, which is the traditional schedule followed by Charlotte's students.

Independent Work

While math is not considered an independent subject, you increasingly want your student to be able to do more and more work independently. As an aid in nurturing her ability to work alone, answers to some problems are placed in the back of the book. Here's the target balance to achieve:

- teacher and student work on initial concept and a number of problems together until comprehension and comfort is exhibited
- the student then works independently for a set amount of time
- the student then meets with teacher again to look over the accomplished work

Any Mixed Review and Review sections not specified as oral work may also be taken independently if the teacher ensures the student is working carefully and with understanding. It is not recommended to simply hand the book over to the student to work alone. Care should always be taken that reading and writing never overshadow the ideas of the math lesson, accuracy and understanding are maintained, and undesirable habits are not taking hold.

Assign a given amount of time to the child for independent work, while also allowing time to evaluate the work together in order to immediately address any misunderstandings. A lesson might look like:

- 5–10 minutes in new concept, student working first three exercises with you at her side
- 10–15 minutes of independent work
- 2–5 minutes looking over independent work
- 5 minutes mental math

and another like this:

- 2–5 minutes going over with student what is expected in the Review section, noting the amount of time allotted
- 10 minutes independent work of Review section
- 10 minutes work together in latest concept
- 5 minutes mental math

yet another like this:

- 15 minutes of work together in new section
- 5 minutes mental math
- 10 minutes continued work in new section

Notation and Writing

Understanding our system of notation is foundational to the idea of place value. When regrouping or exchanging occurs, i.e., borrowing and carrying, the student should say she is working with 4 ten thousands, 7 thousands, 5 hundreds, or 9 tens—not simply 4, 7, 5, or 9. The same is true when working with decimal numbers, with the student saying she is working with tenths, hundredths, thousandths, etc.

As your student's ease and ability in reading and writing increases, be sure she continues to maintain physical habits of neatness and order, even while working independently. The orderliness of each number contained in just one box on the grid of graph paper will help her maintain orderliness of thought. Alternating between written and mental work during the lesson will further nurture these habits.

Mental Arithmetic

The student will be expending mental effort throughout the lesson but there should also be a daily time given to work taken strictly orally, known as rapid oral work or mental math. Work noted to be given orally, Table Work, and additional review that is found throughout or in the back of the book are all fit for this activity. This type of work should be lively and engaging and is a good way to regain your child's responsiveness if you notice it is waning. Mental math may occur at the end of the lesson or another time during the day—indoors or out, with the student adding to the questions or multiple students posing mental work to each other. This type of mental arithmetic should always fall within the scope of a child's learning. If necessary, adjust to using lower numbers as she builds proficiency in this area.

If your child has yet to learn her math facts, or speed and fluency need to either be secured or maintained, spend at least 5 minutes

daily in this endeavor. The Number Sentence Cards found at simplycm.com/cmeas5-a are wonderful aids for mental math, solidifying math facts, as well as building speed and accuracy. If your child has never had the opportunity to make connections and investigate the logic of addition, subtraction, and multiplication tables, please refer to Books 2 & 3 in this series.

Review

While a student may have mastered or internalized a concept, review is still an important part of the weekly lessons and should not be skipped. Review acts as an aid in maintaining fluency while making gradual gains in speed and helps concepts and processes become second nature. Review of material covered in Book 4 as well as review of the new material have been built into this book, along with extra review of new concepts found in the back of the book. Review sections not specified as oral work may be taken independently as long as the teacher ensures the student is working carefully and with a good command of the material.

The target balance is:

- teacher and student look over the review section to ensure the student understands what is being asked of her,
- the student then works independently at her own pace for a set time,
- the student immediately meets with teacher to look over the accomplished work and address any concerns.

While a time of refreshing is normal after a long break, if your child has never worked with the concepts found in the initial review sections—or you believe she needs additional review—you will find completely guided lessons in the previous books in this series.

Tip: If you are continuing straight from Book 4 without a break, some of the initial review may be tiresome. If this is the case, explain to your student that you will go at her pace, which may be quite quick.

When to Advance

The number of problems necessary for a student to master or internalize a concept will vary with each individual child as well as the specific concept. You want the child to work with the newest process or idea until she is both comfortable and confident in her work but not so much that she becomes bored. Just as good habits must be reinforced over time, periodic review must also be given to solidify concepts and facts as well as build relative speed in using them. Math concepts often build upon each other so gaining fluency is vital. The importance of a strong foundation will become increasingly evident as the student meets with more complex math.

This book contains the components necessary to attain and maintain fluency through immediate work in the new material, ample review, and mental math—i.e., rapid oral work. Use of bookmarks or page markers can help you adapt and customize the lessons to fit the unique needs of your child. You are at liberty to not use every question at a given moment if they are found to be excessive or to give additional work found in the back if more work in a concept is necessary. Unused questions may be bookmarked and utilized for later review. If more work is found to be needed, the questions are written in such a way as to allow you to easily change numbers, names, and objects that relate to your own child's life and interests.

Familiarize yourself with each lesson by giving it a brief look-through ahead of time. This will allow you to gather any necessary materials in advance as well as help you maintain focus on your student during the lesson.

Practical Geometry

Charlotte Mason Practical Geometry is a weekly 30-minute lesson in which the student learns to handle mathematical tools, such as the compass and protractor, while gaining foundational ideas in geometry. This handson approach—spread over 2 years—develops a student's comfort with the mechanics of the tools, drawings, geometrical terms, and behavior of geometrical forms before formal geometry and proofs are ever introduced. In a Charlotte Mason education, Practical Geometry is traditionally taken in tandem with Arithmetic, with one lesson now replacing one day of arithmetic during the week. The discoveries made during this time lend interest to the branch's formal study, while the physical habits gained in the mechanics of geometry allow the student to concentrate on ideas when formal geometry is begun.

For more information, see *Charlotte Mason Practical Geometry, Part I*. While the work in these books may be taken separately, it is recommended that *Charlotte Mason Practical Geometry, Part I* be taken along with this book and *Charlotte Mason Practical Geometry, Part II* be taken with Book 6 of The Charlotte Mason Elementary Arithmetic Series.

Pacing

While the concepts found in Book 5 generally take place in a child's fifth year of formal education, you should progress at a pace that ensures each step is taken on solid ground. Pace should be adjusted to the progress of the individual child rather than a standardized rate of learning or the pace of siblings and peers.

Aim for consistent and regular lessons and you should find the pace takes care of itself. Help instill a sense of confidence based on your student's own progress and achievements, remembering that communicating the idea that she is somehow behind can injure her confidence. The week's work should include work in the newest concept, review, and mental arithmetic. You may wish to bookmark certain pages to help distribute work in these three areas as best fits the needs of your student.

The charts given here are for your convenience as a loose planning guide. As always, adjust the chart to your child's pace rather than attempting to make your child fit the chart. The following is based on lessons that are 25 minutes in length with 5 minutes of mental arithmetic. If a child is unable to maintain attention for that length of time, simply make it a target goal to be gradually worked toward.

Term 1												
Suggested Weeks	1	2	3	4	5	6	7	8	9	10	11	12
Review of Previous Year's Work												
Addition, Subtraction, Division, Multiplication of Decimals												
Approximation & Rounding												
Measurement by Decimals												
Exam												

Term 2												
Suggested Weeks	13	14	15	16	17	18	19	20	21	22	23	24
Measures of Numbers												
Factors												
Numbers Divisible by 2 through 12												
Prime Numbers & Prime Factorization												
Greatest Common Factor												
Multiples of a Number												
Exam												

Term 3												
Suggested Weeks	25	26	27	28	29	30	31	32	33	34	35	36
Least Common Multiple												
Simplifying Fractions												
Improper Fractions												
Converting Decimals to Fractions & vice versa												
Addition & Subtraction of Fractions												
Addition & Subtraction of Mixed Numbers												
Canceling & Extra Written Review												
Exam												

Exams

Three end-of-term exams are provided in the back of this book. These are customizable to reflect where your child is in the lessons at the end of each term. See page 295 for details.

Supplies Needed

- Gridded math notebook
- Personal chalkboard, dry-erase board, or the like together with its appropriate writing instrument. For brevity, it will be referred to as *the slate* in the lessons.
- Four strips of paper of equal size—an 8.5" by 11" sheet of paper cut lengthwise into four equal strips will work
- Place value chart (visit simplycm.com/cmeas5-supplies for download and instructions)
- 12" ruler with US Standard and metric measures
- 36" length of string or fabric measuring tape
- Cardboard or heavy card stock at least 1" by 12"
- Drawing compass or circular object that can be traced, such as a small plate
- Meter stick
- Low-tack tape such as painter's tape, washi, or paper tape
- Analog clock

Optional

- The handbook, *Mathematics: An Instrument for Living Teaching*, contains a fuller explanation of Charlotte Mason's unique approach in mathematics, including the principles upon which the practices found in this book rest. Available from Simply Charlotte Mason.
- Number Sentence Cards. A collection of number sentence cards with over 1,000 ready-to-use equations at your fingertips. Valuable to have on hand so your student can keep working if your math lesson is unexpectedly interrupted or to use for five minutes of mental math to retain math facts, build speed, and promote accuracy and concentration. Available from Simply Charlotte Mason.

- Concrete objects, commonly called manipulatives, may no longer be necessary for a child this far along. Feel free to allow their use as needed as it will save time in the long run.
- Printed hundred chart from simplycm.com/cmeas5-supplies. Students will be guided to create a Sieve of Eratosthenes chart and a pre-printed hundred chart can be used to reduce writing.