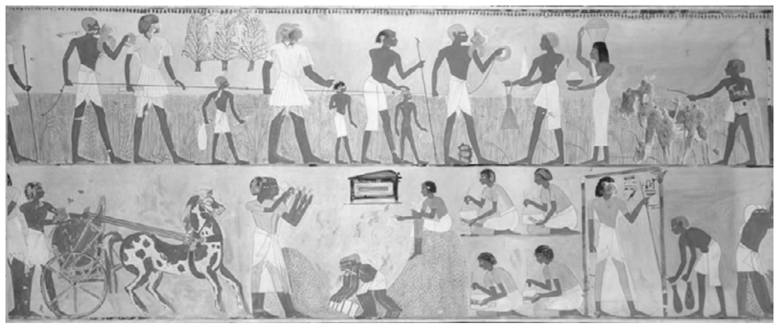
THE STUDY OF GEOMETRY

What do you know of the Ancient Egyptians? Perhaps when you think of this ancient civilization, you envision camels traversing great deserts of sand or powerful pharaohs. Do you picture one of the many great structures they built, such as a pyramid or the Sphinx? They were able to use simple tools to build such grand structures that can still be seen thousands of years later. Think of the size of the Great Pyramid of Giza, the only remaining wonder of the Seven Wonders of the Ancient World. The pyramid was the tallest structure in the world for more than 4,000 years! In addition to the physical power it took to build it, the Egyptians needed to take many measurements and make complex calculations to create a structure that, as the Father of History, Herodotus, said, was "jointed with the greatest exactness."

The Ancient Egyptians were a large and prosperous civilization. While their pyramids are awe-inspiring, perhaps more impressive is their ability to harness the properties of the Nile River in order to grow crops and raise livestock. Ancient Egypt was mostly desert land, but running through the middle of it was the Nile River. Every year, around the month of August, the Nile flooded its banks. This was actually a good thing, because these flood waters carried fertile soil from the hills that were upriver. When the flood waters receded, they left behind an abundance of rich soil in the middle of what was otherwise only desert. Without the annual flood, the Ancient Egyptians would not have been able to grow the abundance of food that allowed such a large civilization to thrive.

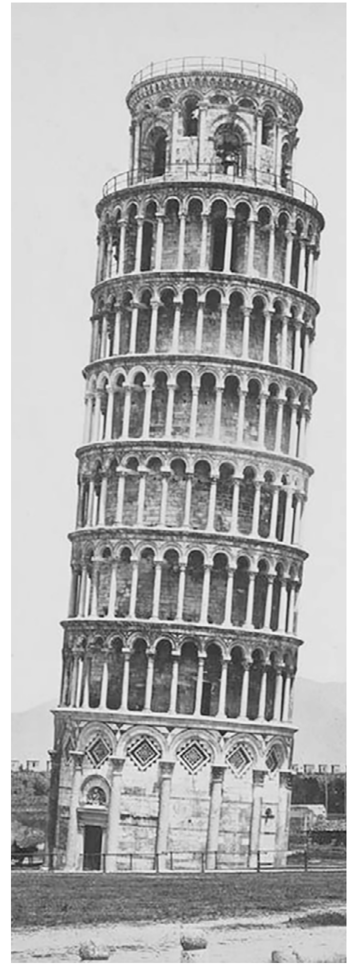
Although the annual flood provided much-needed resources, it also left behind a problem. In the wake of each flood, the boundary markings on the edge of a family's land were washed away. Thus, they would have to take measurements of the land to re-mark each family's field. This measuring of the earth was called *geometry*, which means "earth measure." Ancient Egyptian wall paintings show us that there were groups of men that used simple tools, including knotted ropes and wooden pegs, to measure the vast fields and deserts of Egypt.



It was with such tools these men, called surveyors or rope-stretchers, created a study of the measurement of the earth. Herodotus wrote of this study making its way to Greece. From there, it traveled throughout the Mediterranean region and the rest of the ancient world. It was with these simple tools that, before the birth of Christ, a man named Eratosthenes was able to determine with amazing accuracy the distance around the earth! The Wise Men may have used them to track the Bethlehem star, Galileo used them to track the planets, and Columbus used them to navigate across the Atlantic. Now, thousands of years later, you will also use similar simple tools to join in on this ancient study of geometry.

SOLIDS, SURFACES, AND LINES

LESSON 1—SOLID FIGURES



OBJECTS USED: cylinder (e.g., canned good, oatmeal container with lid), cube (e.g., a die, a wooden block), and sphere (e.g., ball, globe)

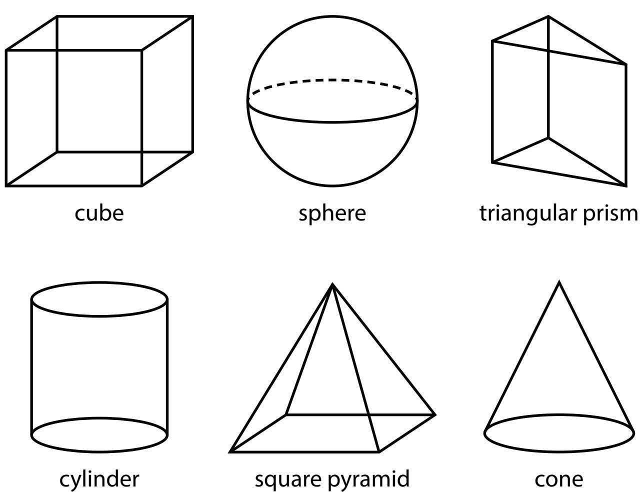
OPTIONAL OBJECTS USED: cone (e.g., ice cream sugar cone, party hat), triangular prism, and square pyramid

The world is full of items you can see, touch, and sometimes hold. Some are rather ordinary, such as the pencil in your hand, the textbook in front of you, or the ice cream cone you enjoy on a hot summer day. Others are extraordinary, such as the Leaning Tower of Pisa or the Great Pyramid of Giza. Ordinary or extraordinary, each of these things consists of basic three-dimensional shapes like those seen below.

Each of these *solids* or *solid figures* has its own name according to its shape. Pictured below you can see a cube, sphere, triangular prism, cylinder, square pyramid, and cone. Take a moment to observe your surroundings. Can you find any of these shapes around you? See if you can identify them. Gather some examples to observe.

Now take a look at your examples and notice the different parts of these solid figures. First, consider the cylinder. What are some items in your home or classroom that have this shape? From the examples you gathered earlier, take a cylinder in your hand. You may have found a canned good, an oatmeal container, a roll of paper towels, a new spool of thread, or one of any number of cylinders around you. How would you describe the cylinder you are holding? There are likely many ways to describe it! It may be smooth, heavy, or hollow. You could describe its contents or its color. However, for the purpose of geometric study, the attribute of interest is the *outside* of the cylinder, the part you can see and touch.

This outside part is called the *surface* of a solid. Sometimes the surface of a solid is all in one piece, as with the sphere. More often, it consists of several parts. As you take a look at your cylinder, you will notice that its surface has three parts, two flat and one rounded. Consider the surface of some of the other solids that you gathered.



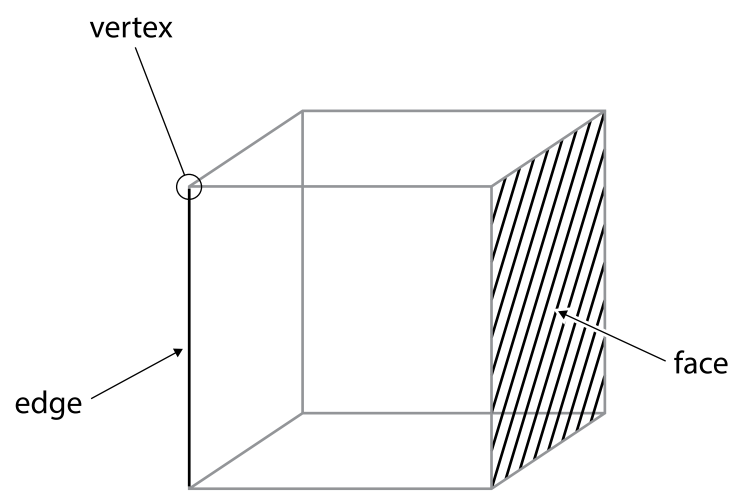
Exercise 1

a. Hold and examine the cube you found. How would you describe its surface? Are the surfaces flat or rounded?

b. Look closely at your cone-shaped object, or consider the drawing above. How would you describe its surface? Is each surface flat or rounded?

Just as each of these solids has its own name, each of the parts of their surfaces do as well. The largest part of a solid's surface is its sides. You have observed two types of sides that a solid figure could have: flat and rounded. A side that is flat has a special name. It is called a *face*. How many faces does your cube have? Of course, if you have ever played a board game, you will probably be familiar with the six sides of a die! Although the surface of a cylinder has three parts, only two are faces.

Now consider how two neighboring faces of a surface meet on your cube. Two faces of a solid figure meet to form a piece of a straight line, called an *edge*. Edges can be found on the cube, prism, and pyramid, because they each have faces that meet another face, or flat side.



Exercise 2

a. How many edges are there on your cube?

b. Were you able to find a triangular prism or square pyramid? If so, how many edges does each have? If not, refer to the drawings on the previous page. How many edges can you count?

c. Does a cone or cylinder have edges? What about a sphere? Why?

Now consider how the edges of a solid meet each other. Follow one of the edges of your cube until it meets another edge. You'll notice a point where the edges meet. The point where two or more edges meet is called a *vertex*. Notice how a cube has more than one vertex. This means that a cube has *vertices*, the plural of vertex.

Exercise 3

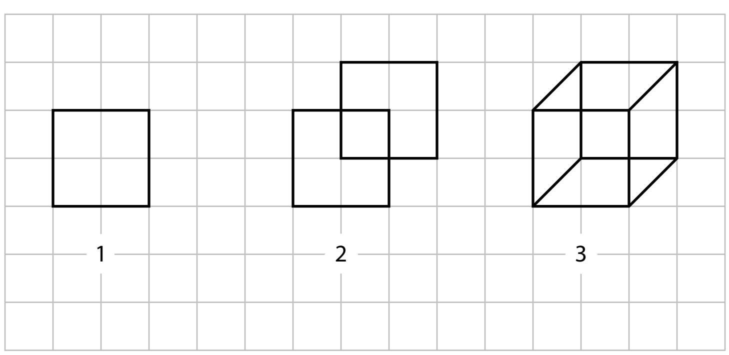
a. How many vertices are there on your cube? How many edges meet at each vertex of your cube?
b. How many vertices are there on a square pyramid? How many edges meet at each vertex of a square pyramid?
c. How many vertices are there on a triangular prism? How many edges meet at each vertex of the triangular prism?

MATH NOTEBOOK ENTRY

Choose one or two solid figures to draw in your notebook. Name the figures, and, if it has any, label an edge and vertex of each. Enter into your notebook what you have learned of *solids, faces, edges,* and *vertices.*

TIPS FOR SKETCHING A CUBE

1. Using your grid paper, draw a two-by-two square.
2. Next, draw another two-by-two square slightly offset from the first, as shown in the picture below.
3. Then, connect the two squares at each of their corresponding vertices with straight lines.



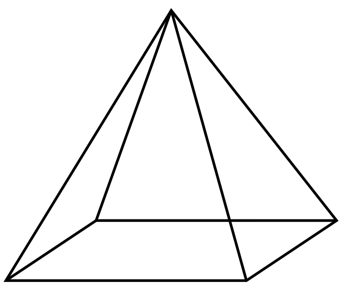
LESSON 2—POINTS AND LINES

OBJECTS USED: light-colored thick-tip marker, medium-colored fine-tip marker, finely sharpened pencil or mechanical pencil, and straightedge (e.g., index card, ruler)

REVIEW

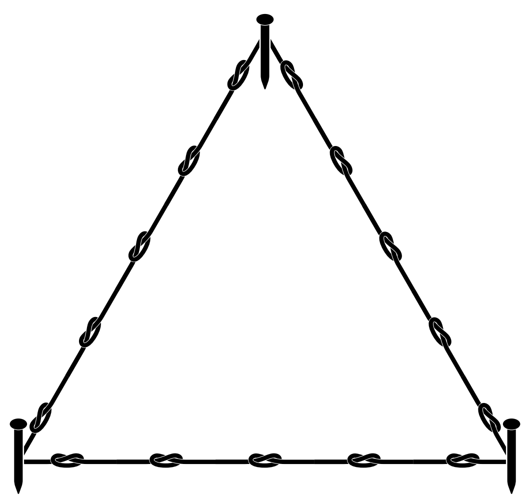
Recall what you learned in the previous lesson about solids and their various parts.

1. Name three solids.
2. How many vertices does a cube have?
3. How many edges does a square pyramid have?
4. How many faces does a cylinder have?


pyramid

Each face of a solid figure is a simple shape. For instance, the faces of a cube are all *squares*. What shapes are the faces of this pyramid?

In Ancient Egypt, surveyors would use ropes to consider the simple shapes of the faces of solid figures. Consider one face of a pyramid: the triangle. The surveyors would use a knotted rope to form the edges of the shape and a stake to mark the location of each vertex. If you were to draw a diagram to show only the locations of each stake, what would it look like? It would simply be three dots on your paper.



Exercise 1

1. Make a small dot on your paper with a thick-tip marker.
2. In the center of that dot, make a smaller dot with a fine-tip marker.
3. Finally, make an even smaller dot with a finely sharpened pencil in the center of that dot.

The smallest dot you can make on your paper with a sharp pencil will give you an idea of what is meant by a geometric point. You could make an even smaller point with a fine sewing needle and an even smaller one with a very fine pin. A *point* is so small that one does not think of its length, breadth, size, or shape. In fact, it *has no size or shape*; all there is to consider is its *position*.

Points are named and distinguished from one another by labeling them with capital letters; thus, you can speak of the "point *A*" or the "point *B*."

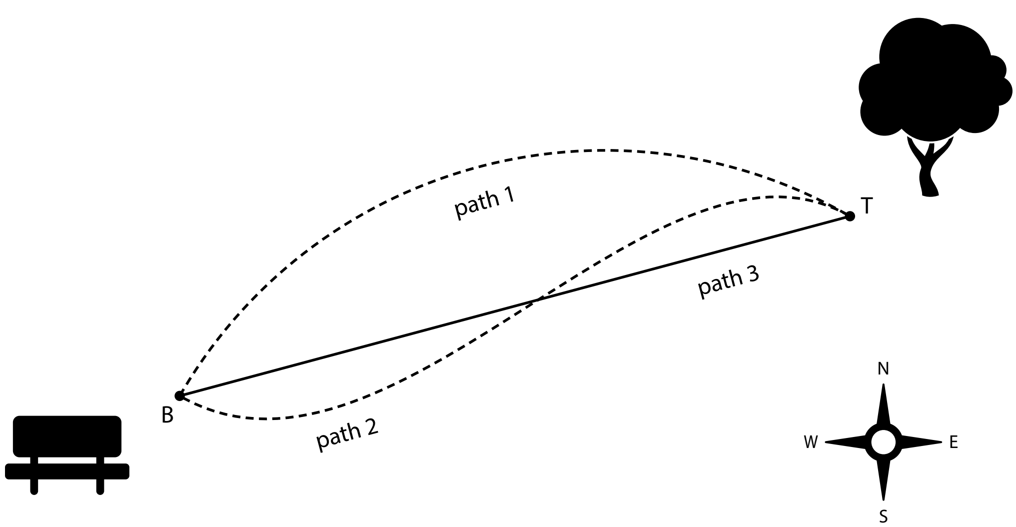
Exercise 2

In your math notebook, draw two points and label them B and T.

Let's say that you are looking at a map of a park, with point B marking the location of a bench and point T marking the location of a tree. Now you see that these points represent two distinct positions.

Suppose you wish to go from the bench to the tree by the shortest way. You can see at once what course you must choose; you must go straight from B to T. There are numberless curved paths along which you could go from the bench to the tree, but the shortest way of all is the straight path. Notice that it is *the only* straight path, for you can see for yourself that there can only be *one* straight path from B to T.

A map of these different paths has been drawn in the picture. Just like a map, it includes a directional compass to indicate which way is north. If you are traveling toward the top of the page, then the compass indicates that you are traveling north. Toward the bottom of the page is south, to the right is east, and to the left is west. With this in mind, consider the different paths on the map. If you travel along path 1, you begin by heading in the direction of north and then gradually adjust your course until you are heading southeast.

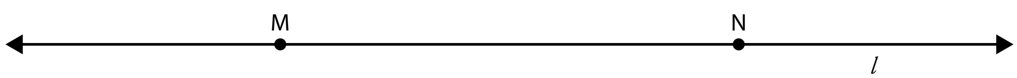


Exercise 3

How would you describe the direction taken along path 2? Along path 3?

Paths 1 and 2, and indeed every other path you could draw to connect B and T besides path 3, are examples of *curves*. A curve changes direction. You will learn much more about curves in the Circles chapter. The straight path connecting B and T, path 3, is called a *line segment*. A line segment is straight and never changes direction. The points B and T are the *endpoints* of the line segment. A line segment is named by its endpoints and with the symbol of a horizontal line placed above the names of the points like this: $\overline{BT}$. Because a line segment has endpoints, it is only a piece of a line.

So, then, what is meant by a *line* in the study of geometry? Intuitively, you may know what a line is, but coming up with a mathematical definition can be challenging! We can, however, describe its attributes. A *line* is straight, has no width, and is infinitely long (that is, it extends forever in either direction). It is drawn with arrows on either end, indicating that it continues on in that direction. A line can be labeled with a single letter, usually lowercase, or by naming any two points that lie on the line. For instance, below is line l or line MN, the symbol ↔ is used to indicate a line. So, this line can be written as $\overleftrightarrow{MN}$.



TIPS FOR DRAWING A LINE

In order to produce a straight line on paper, it is necessary to use a straightedge to assist you. Any straightedge can be used to draw a line. A common tool to use is a ruler, but an index card or bookmark would work as well. All of these have a straight edge that can act as the straightedge geometry tool. When using a straightedge, be sure to press it firmly onto the paper.

1. Using your non-dominant hand, plant your thumb and forefinger on the straightedge. Attempting to secure it with just one finger will result in pivoting of the straightedge and a line that is not truly straight.
2. Taking your pencil in your dominant hand, be sure that the tip of your pencil is angled toward your ruler and the eraser away from it. It is necessary for the pencil trace to be as close to the edge of the ruler as possible.
3. Run your pencil tip along the edge of your straightedge to draw your line.

Exercise 4

Using your straightedge, draw a line in your notebook. Be sure to include the arrows on the end and to label it with a lowercase letter.

When you connect two points with a line segment, you are said to *join* the two points.

Exercise 5

1. Join B and T, the points you drew in exercise 2 (*i.e.,* draw a line segment from B straight to T). Be sure that your straightedge is placed just below the points and that your line goes to the center of each point. If it does not, adjust the position of your straightedge or the angle of your pencil.
2. Draw three meandering paths, or curves, from B to T.

Just as a very fine pencil dot is the best representation of a point, a very fine pencil trace is the best representation of a line.

Exercise 6

1. Repeat the process in exercise 1 to draw a line segment: with your thick-tip marker, draw two points, R and S. Join them with this marker.
2. Next, use your fine-tip marker to repeat the process on top of R, S, and $\overline{RS}$.
3. Finally, use your pencil to repeat the procedure once more.

MATH NOTEBOOK ENTRY

Enter into your notebook what you have learned of *points, line segments,* and *lines.* You could choose to write this in words or to make a geometric drawing to illustrate these concepts.

CIRCLES

LESSON 8—PARTS OF A CIRCLE

OBJECTS USED: drawing compass, metric ruler, US standard ruler, blank sheet of paper, and scissors

REVIEW

1. How many edges does a square pyramid have?
2. If two lines are congruent and one measures 3", what does the other measure?
3. How can you write $2\frac{3}{4}$" as a decimal?
4. For what purpose have you used a drawing compass?
5. How many lines can be drawn through a given point?

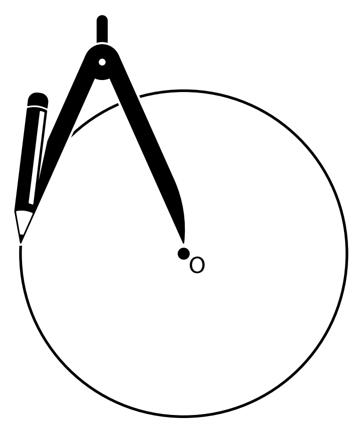
Thus far, you have used a drawing compass to measure distances, but it can be used to draw curved lines as well.

TIPS FOR DRAWING A CURVE USING A DRAWING COMPASS

1. Mark a point *O* on a sheet of paper.
2. Set your drawing compass to any length.
3. Placing the steel point on point *O*, slowly and carefully rotate the compass by turning the grip between your forefinger and thumb so as to draw a curved line with the pencil point. Take care that the steel point does not move from point *O*.

Exercise 1

1. Mark a point *O* in your math notebook. Set a distance of 5 cm between the points of your compass, and use your compass to draw a curve with point *O* as the center.
2. As the curved line is being traced out, notice carefully that the pencil point always keeps the same distance from *O*. What is this distance?
3. Continue the curve all the way around point *O*. Notice also that the pencil returns to its starting point, so as to close the curve. Why is this?

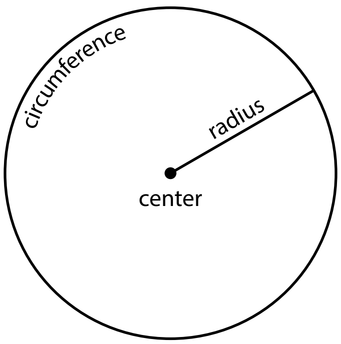


The curve you have thus drawn is called a *circle*, and the point *O* is its *center*. The symbol ⊙ can be used to indicate a circle, so you can write this as ⊙*O*. Sometimes the word *circle* means the space enclosed by the curve, and the curve itself is said to be the *circumference* of the circle.

Exercise 2

a. Mark four points anywhere on the circumference of the circle you have drawn; call them A, B, C, and D.

b. Draw the line segments $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$. Are these line segments all congruent? How do you know? Tell their lengths without measuring them.



A line segment drawn from the center of a circle to its circumference is called a *radius*. The plural of radius is *radii* (ray-dee-ai); all the radii of a circle are equal.

Exercise 3

1. Mark a point E in your notebook.
2. Mark another point F at a distance of 2" from E.
3. Mark four more points that are also 2" from E. Label them G, H, I, and J.
   a. How many points could you mark whose distance from E is 2"?
   b. On what curve do all of these points lie? Draw a curve to pass through all of them.
   c. Draw a point X that is 1.75" from the center of the circle you have just drawn, another point Y that is 2" from the center, and a third point Z that is 2.5" from the center. Which of these points is on the circumference? Which is outside it? Which is within it?

Exercise 4

a. Can you think of another way to draw a circle with a given center and radius without using a compass?

b. How might the Ancient Egyptians have achieved such a task if they desired to mark off a large circle in a field? *(Remember the tools they often used included a rope and pegs.)*

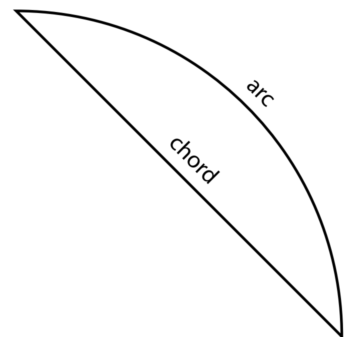
Now consider how you might *define* a circle. Perhaps as "a round shape"? Certainly it is, but would not an oval also be considered a round shape? You must be precise in your definition. Can you explain in your own words what a circle is?

In geometry, a circle is defined as *the collection of all of the points that are an equal distance from a center point*. Imagine a horse that is tethered to a post in a field. He moves round the post, keeping the rope tight. The path that he travels is a circle; at the center of the circle is the post. The length of the rope is the circle's radius.

Exercise 5

1. Taking a point O as center, draw a circle with a radius of 1.5".
2. Draw any line segment that does not pass through the center of the circle and that has endpoints that lie on the circumference of the circle.
3. Label the endpoints G and H, and measure the length of $\overline{GH}$.

A line segment such as $\overline{GH}$, that has its endpoints on a circle, is called a *chord*. Notice that a chord divides the circumference of the circle into two parts. Any part of the circumference of the circle, such as the two parts created by $\overline{GH}$, is called an *arc*. The symbol for an arc is $\frown$, so we can write $\overset{\frown}{GH}$. Does the shape of the chord and arc shown here remind you of anything?

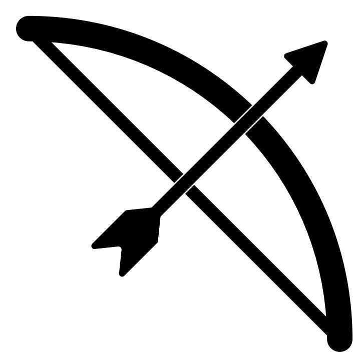


It certainly looks a lot like a bow from a bow and arrow set. In fact, the origin of these terms comes from the Latin *arcus*, "a bow," and *chorda*, "a string."

Exercise 6

Draw a circle with center O and radius 2".

a. Draw, if possible, the following chords inside the circle, using your compass for marking off the lengths of the chords: $\overline{PQ}$ of length 2", $\overline{RS}$ of length 3.5", and $\overline{TU}$ of length 5".
b. Was it possible to draw all three chords? What is the length of the longest chord that you can draw in this circle? Draw such a chord, and name it $\overline{AB}$. Draw any other chord going through the center of the circle, and name it $\overline{CD}$.
c. What is the length of $\overline{CD}$?
d. Point out the two arcs that chord $\overline{PQ}$ divides the circumference into. Are the two arcs congruent?
e. Do any of the chords that you have drawn appear to create two congruent arcs? Which ones?

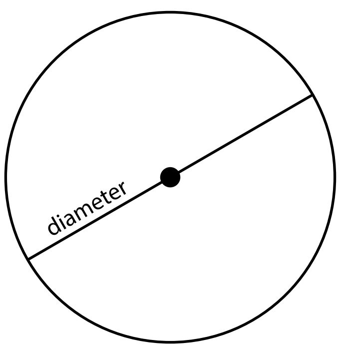


The longest chord that can be drawn in a circle is called a *diameter*. A diameter will always go through the center of a circle and create two equal arcs. Both $\overline{AB}$ and $\overline{CD}$ are diameters in exercise 6.

Exercise 7

1. On a separate piece of paper, so that you can cut it out, draw a circle centered at V with a radius of 4 cm.

2. Draw a chord, $\overline{WX}$, that goes through the center of the circle.
    a. Name a radius of this circle.
    b. What is the length of $\overline{WX}$? Can you answer this without measuring?
    c. Are all diameters of a circle congruent to one another?
3. Now carefully cut out your circle and fold it along the diameter $\overline{WX}$, dividing the circle into two parts.
    d. What do you notice?
4. Flatten out the circle; rule any other one diameter, and fold the circle along it as before.
    e. Do you find the same result?



So a diameter is always twice the radius of a circle. Also, when a circle is folded along its diameter, one part fits exactly over the other. Just as two line segments that are of equal measure are congruent, two figures are *congruent* if they are exactly the same size and shape. Thus, these two parts of the circle are congruent, which can be expressed by saying that a circle is *symmetrical* about any diameter. The two congruent parts into which a circle is divided by a diameter are called *semicircles*.

MATH NOTEBOOK ENTRY

Draw and label a circle with the following parts in your notebook: *center, circumference, radius, diameter, chord, arc.*

LESSON 9—OVERLAPPING AND CONCENTRIC CIRCLES

OBJECTS USED: metric ruler, US standard ruler, and drawing compass

REVIEW

1. How many faces does a cube have?
2. How can you define a circle?
3. If a circle has a radius of 2", what is its diameter?
4. If a circle has a radius of 6 cm, could you draw a chord of length 2 cm in it? 8 cm? 15 cm?

a. What is the measure of $\angle DCA$?

b. $\overline{CE}$ bisects $\angle DCB$. What is the measure of $\angle DCE$?

c. $\overline{CF}$ bisects $\angle DCA$. What is the measure of $\angle DCF$?

d. What is the measure of $\angle ECF$?

Exercise 5

1. Draw a line segment $\overline{AB}$.
2. Anywhere on $\overline{AB}$, place a point O.
3. From O, draw a line segment $\overline{OC}$, making any angle with $\overline{OA}$.
4. Using your straightedge and compass only, bisect $\angle AOC$ and $\angle BOC$, and call the bisectors $\overline{OX}$ and $\overline{OY}$.
5. Measure $\angle XOY$. What do you notice?

You can see that bisecting two adjacent supplementary angles results in the formation of a right angle.

MATH NOTEBOOK ENTRY

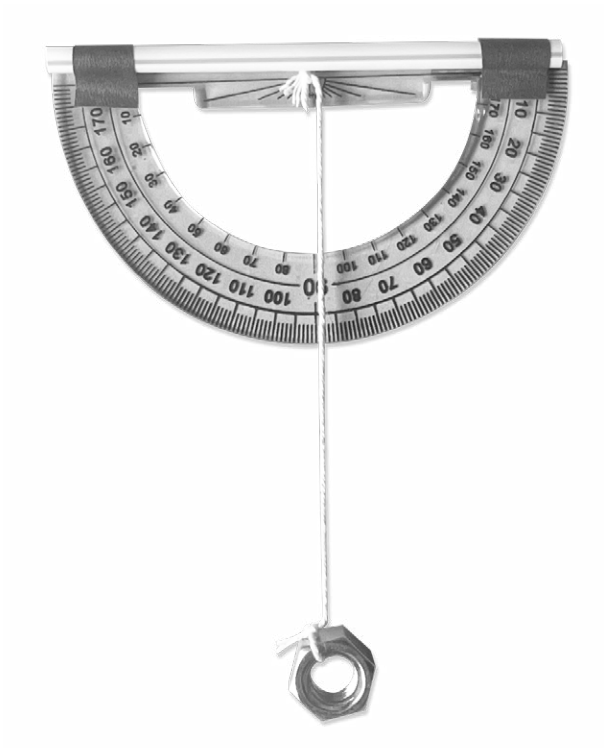
Enter into your notebook *how to divide an angle into four equal parts*. Include what happens when you bisect two vertical angles and what happens when you bisect adjacent supplementary angles.

OPTIONAL LESSON—OUTDOOR GEOGRAPHY

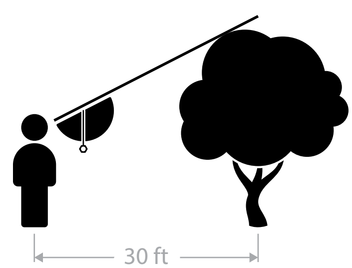
Let us use our knowledge of angles to estimate the height of an object which we cannot easily measure, such as a building, a tree, or a power pole.

OBJECTS USED: drinking straw, tape, protractor, string (about 8 in. long), weight or metal nut, and tape measure

1. First, you must construct a *clinometer*, an instrument used to measure the angle of elevation, or angle from the ground, to an object. Using your supplies from the list above, assemble as shown.

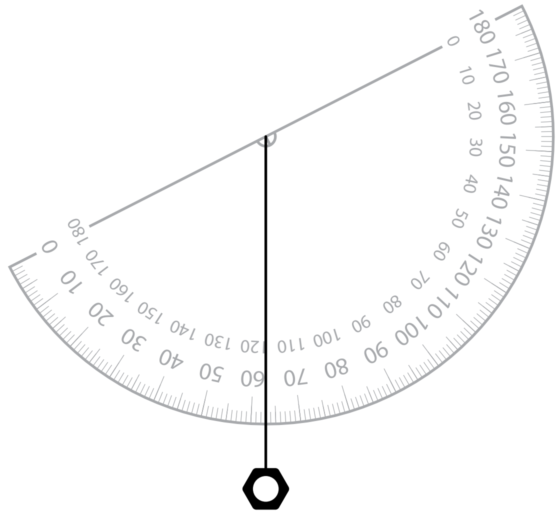


2. Find a tall object of which you wish to determine the height. It should be on level ground.
3. Position yourself a good distance from your object.
4. Using your tape measure, determine your distance to its base. The figure shown illustrates someone standing 30' from the tree.



> TIP: *Standing at some multiple of 10 ft from the tree will keep this activity simple.*

5. Hold the straw up to your eye and position the clinometer so that you can just see the very top of the object through the straw. Be sure that the string is able to hang freely. Before lowering the clinometer, have a helper note at which angle measure the string is hanging. In the scenario illustrated in the figures, the clinometer indicated 63°. Record your data.

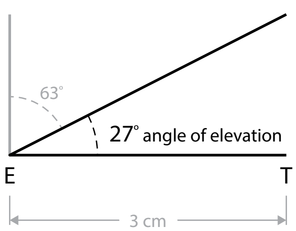


6. Make a plan using your measurements. Use a scale of 10 ft. to 1 cm.

   a. First, indicate the position of your eye with a point, and label it *E*.

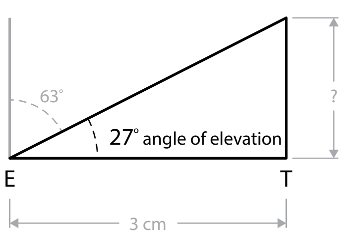
   b. Using your ruler, draw the correct distance to the base of your object. The distance in the example was 30', which is represented by 3 cm according to the scale being used. The end of the line segment was labeled *T* to stand for the tree.

NOTE: *The scale used in this exercise does mix metric and US standard units. This will likely be a new experience for your student.*



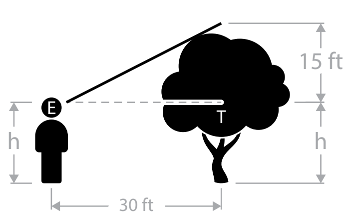
7. Determine your angle of elevation by *subtracting your clinometer reading from 90°*. Since the clinometer indicated 63° in the example, the angle of elevation was 90° – 63° = 27°.

8. Position your protractor with *E* as the vertex, and draw the angle you just found in the previous step. Here, the tree had an angle of elevation of 27°, so a line segment was drawn extending from the vertex *E* using that angle measure.



9. Beginning at the point *T*, draw a line segment at a 90° angle up from $\overline{ET}$, which will represent the tree. If necessary, extend these two line segments until they intersect.

10. Measure the vertical line segment representing the tree, and determine the height according to your scale. The vertical line segment in the example measures approximately 1.5 cm, which represents 15 ft. according to the given scale.



11. Notice in the diagram below that the height of the tree has been determined from the observer's eyeline. Distance *h*, the measurement from the ground to the observer's eye, must be added in order to determine the total height of the tree. The observer's eye is located 5 ft. 8 in. above the ground, so the total height of the tree can be determined by adding the two figures: 15 ft. + 5 ft. 8 in. = 20 ft. 8 in.