Chapter 4

Ratio, Proportion, and Percent

Fractions, decimals, and percents are different ways of representing the same quantities. For some situations, you'll know whether it's better to represent a quantity by a fraction, a decimal, or a percent. Let's review how to change fractions to decimals and terminating decimals to fractions.

Remember from Chapter 1 that any fraction can be changed to a terminating (a decimal that stops) or to a repeating decimal.

Converting Fractions to Decimals and Terminating Decimals to Fractions

Example 1: Change $\frac{3}{8}$ to a decimal.

The fraction bar means "divide." You're dividing 3 by 8. You can think of the 3 sitting on a chair and falling into the box. Since 8 cannot go into 3, put a decimal point after the 3 and add a few zeros. Then start dividing and keep dividing until the decimal terminates or repeats.

$$\begin{array}{r|l} & 0.375 \\ \hline 8 & 3.000 \\ & -24 \\ \hline & 60 \\ & -56 \\ \hline & 40 \\ & -40 \\ \hline & 0 \end{array}$$

Example 2: Change .375 to a fraction.

The 5 sits on the thousandths place, so write 375 as the numerator and 1,000 as the denominator and reduce. Then divide the fraction by 5. If the GCF (which is 125) was found, the fraction could have been reduced in one step.

$$\frac{375}{1{,}000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$$

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Example 3: Change $-5\frac{2}{3}$ to a decimal.

Since −5 is an integer, you just need to find out what $\frac{2}{3}$ is as a decimal. Again, drop the 2 into the box and divide.

$$\begin{array}{r} 0.666... \\ 3\overline{)\,2.000} \\ -\underline{18} \\ 20 \\ -\underline{18} \\ 20 \\ -\underline{18} \\ 0 \end{array}$$

You can see that the 6 will repeat, so the answer is $-5.\overline{6}$. The bar on top of the 6 indicates that the 6 repeats. And yes, you can write your answer as $-5.\overline{66}$.

Common Fractions/Decimals

You can always use the above methods to covert fractions to decimals and terminating decimals to fractions. However, there are 10 common fractions/decimals that appear in many real life situations, as well as in many standardized tests that you might take in the future. It is a great advantage to memorize these. Making flashcards may be useful. Make sure you do not memorize these unless you understand the concept.

$\frac{1}{2} = .5$	$\frac{3}{8} = .375$
$\frac{1}{4} = .25$	$\frac{5}{8} = .625$
$\frac{3}{4} = .75$	$\frac{7}{8} = .875$
$\frac{1}{5} = .2$	$\frac{1}{3} = .\overline{3}$
$\frac{1}{8} = .125$	$\frac{2}{3} = .\overline{6}$

$\frac{1}{5} = .2$. What are the decimals for $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$. Why?

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PLACE VALUE CHART

millions	hundred-thousands	ten-thousands	thousands	hundreds	tens	ones	•	tenths	hundredths	thousandths	ten-thousands	hundred-thousandths	millionths
							•	0	0	0	1	6	

Practice

Change the following decimals to fractions. Make sure to simplify each fraction. If you memorized the "Common Fractions/Decimals," you do not need to show work.

1. .08
2. .004
3. −5.025
4. .375
5. −9.875
6. .00016 (see chart)
7. −6.008
8. 26.75
9. −9.248
10. .00065

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Change the following fractions to decimals. Remember to reduce the fraction first. If you memorized the "Common Fractions/Decimals," then you do not need to show work.

11. $\frac{7}{100}$

12. $\frac{3}{24}$

13. $\frac{2}{5}$

14. $\frac{5}{8}$

15. $-8\frac{3}{10{,}000}$

16. $\frac{3}{16}$

17. $-10\frac{1}{3}$

18. $-9\frac{27}{72}$

19. $\frac{125}{100}$

20. $\frac{18}{24}$

21. $-\frac{5}{32}$

22. $\frac{22}{7}$

23. a. Change $\frac{1}{9}$ to a decimal.

b. Change $\frac{2}{9}$ to a decimal.

Finish the pattern.

c. $\frac{3}{9}$ = ______, $\frac{4}{9}$ = ______, $\frac{5}{9}$ = ______, $\frac{6}{9}$ = ______, $\frac{7}{9}$ = ______, $\frac{8}{9}$ = ______

d. So $\frac{9}{9}$ = ______. But isn't $\frac{9}{9}$ the same as 1 whole?

Explain your thinking. ____

____

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How to Tell if a Fraction Terminates or Repeats

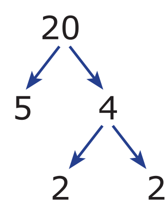
When changing a fraction to a decimal, it helps to know ahead of time if the decimal will terminate or repeat. To figure this out, follow these steps.

Example 1: Does the fraction $\frac{6}{40}$ terminate or repeat when changed to decimal?

Step 1: Reduce the fraction first! It becomes $\frac{3}{20}$.

Step 2: Do a prime factorization of the denominator. Do not worry about the numerator.

The prime factorization of 20 is: $2 \bullet 2 \bullet 5$ or $2^2 \bullet 5$.



Step 3: If you have only factors of 2, or only factors of 5, or both, but no other primes such as 3, 7, 11, etc. in the prime factorization of the denominator, then the fraction will terminate when changed to a decimal.

Example 2: Does the fraction $\frac{3}{70}$ terminate or repeat when changed to a decimal?

The fraction is already reduced. The prime factorization of 70 is $2 \bullet 5 \bullet 7$. Therefore the prime number 7 will make this fraction repeat when changed to a decimal repeat.

Practice

1. Why do you think having only the primes 2 and/or 5 and no other primes will make the fraction terminate when changed to a decimal? Explain your thinking.

____

____

2. Do these fractions terminate (T) or repeat (R) when changed to a decimal? Use an extra sheet of paper if needed. Don't forget to reduce each fraction first.

a. $\frac{1}{20}$ = ______ b. $\frac{3}{30}$ = ______ c. $\frac{22}{14}$ = ______

d. $\frac{1}{64}$ = ______ e. $\frac{9}{54}$ = ______ f. $\frac{8}{41}$ = ______

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Ratio, Rate, and Unit Rate

A ratio is the relationship between two quantities.

Example 1: In a classroom there are 15 girls to 7 boys. What is the ratio of girls to boys?

You can write the answer as 15 to 7, 15:7, or $\frac{15}{7}$.

Since the question reads "girls" to "boys," then the number that matches with girls goes first. If you write the ratio as a fraction, you can reduce.

Example 2: In the problem above, what is the ratio of boys to the total number of students in the classroom?

The answer is 7 to 22, 7:22, or $\frac{7}{22}$.

Example 3: At a dance the principal of a school wants to have the ratio of chaperones to students to be 2 chaperones for every 9 students. If the total number of chaperones plus students was 110, how many chaperones were there?

2 chaperones + 9 students = 11 people. $\frac{2}{11}$ is the ratio of chaperones to total people.

So $\frac{2}{11}$ of 110 must be chaperones. $\frac{2}{\cancel{11}} \bullet \overset{10}{\cancel{110}} = 20$. There are 20 chaperones.

Example 4: Dana drove 200 miles using 5 gallons of gas. Simplify the ratio of miles to gallons.

$\frac{200 \textit{ miles}}{5 \textit{ gallons}} = \frac{40 \textit{ miles}}{1 \textit{ gallon}}$ You can say Dana's car gets 40 miles per gallon or 40 mi/gal or 40 mpg (miles per gallon).

$\frac{200}{5}$ is called a rate (a special type of ratio used to compare measurements with different units) and $\frac{40}{1}$ is called a unit rate.

A unit rate is a rate with a denominator of 1.

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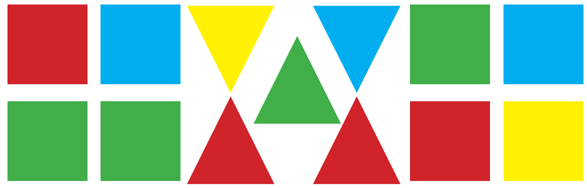
Practice

Find the answer to these problems using these bathroom tiles. Be sure to reduce if possible. Use a separate sheet of paper if needed.



1. What is the ratio of yellow shapes to squares? ____________________
2. What is the ratio of red triangles to total triangles? ____________________
3. What is the ratio of green triangles to green squares? ____________________
4. What is the ratio of triangles to squares? ____________________
5. What is the ratio of triangles to total shapes? ____________________
6. If the design above has to be repeated multiple times on a bathroom wall to make a border and the final design has 104 total shapes (squares plus triangles), how many triangles are there on the banner? Show your work.
7. Dan drove his car for 250 miles and used 10 gallons of gas. Elsie drove her truck for 300 miles and used 20 gallons of gas. DJ drove his motorcycle 120 miles and used up 3 gallons of gas. You can also use the abbreviation "mpg" for miles per gallon. Complete the table below.

Vehicle	Rate	Unit Rate
		
		
		

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8. If gas costs $2.30 per gallon, how much did each trip in problem 7 cost? Show your work.

   a. Dan's trip ________

   b. Elsie's trip ________

   c. DJ's trip ________

9. A banana bread recipe calls for 1 cup of sugar for every 2 cups of flour. Finish the following table.

Flour (Cups)	Sugar(Cups	Flour and Sugar (Cups)
2	1	3
3		
6		
		12
	5	
		$16\frac{1}{2}$

10. The same recipe above calls for $\frac{1}{4}$ tsp of salt for every 2 cups of flour. If Mr. Baker used $1\frac{1}{2}$ tsp salt, how many cups of flour should he use? Show your work.

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11. Find each unit rate.

a. The average human walks 9 miles in 3 hours.

b. Michael Phelps swims 3 miles in a $\frac{1}{2}$ hour.

c. Katie Ledecky swims 800 meters in about 8 minutes.

d. A gray wolf runs 70 miles in 2 hours.

e. Usain Bolt runs 55 miles in 2 hours.

f. A cheetah runs 245 miles in $3\frac{1}{2}$ hours.

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12. Fill in the table below.

Two 8” pizza pies for $21.90.

Number of Pizza Pies	Cost
1	
2	$21.90
3	
16	

13. In problem 12, the dependent variable is the "cost" and the independent variable is the “number of pizzas.” What do you think is meant by those terms? Explain your thinking. Hint: Think of independent as free to choose what you want. A variable is something that changes.

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