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CHAPTER 29 NUCLEAR PHYSICS 662

Dominion Modeling: Scrolls Out of Time

FACETS

This NASA space shuttle, shortly after liftoff, had an increasing speed in a specific direction—straight up.

Vectors and Scalars

4

DOMINION SCIENCE PROBLEM

Accurate Hurricane Models

Hurricanes and cyclones caused almost 2 million deaths in the last two centuries. If you have never experienced one of these storms, consider yourself fortunate. Hurricanes are huge rotating storm systems in the earth's atmosphere that form over warm oceans in tropical or subtropical areas of the world. The winds in these storms exhibit complex motion in three dimensions. The air in a hurricane not only rotates but also spirals into the hurricane's eye from its periphery, rises vertically in the eye, and then spirals outward over the top of the storm. Wind speeds of over 320 km/h (200 mi/h) have been recorded for hurricanes that have made landfall in the United States.

But extreme winds are not the most dangerous aspect of a hurricane. Most deaths are caused by hurricane *storm surges* that inundate coastal areas with many meters of water, allowing huge ocean waves to assault human habitations far from the beach. A storm surge is the abnormally high sea level caused by the adverse combination of low air pressure, high winds piling water onshore, and a high tide. Often in the aftermath of a storm, disease and famine in the disaster area claim even more lives. How can we better model hurricanes to improve forecast accuracy and to more effectively evacuate people out of harm's way?

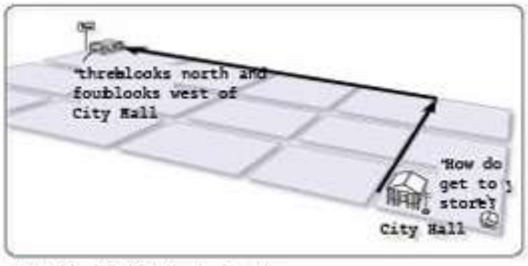
4-1 On May 2, 2008, cyclone Nargis smashed into the country of Myanmar (formerly known as Burma) in Southeast Asia. More than 2000 square miles were flooded, and an estimated 25 000 to 60 000 people were dead or missing.

4A PROPERTIES OF VECTORS AND SCALARS

4.1 Defining Vectors and Scalars

A manager gives a customer directions to her store: "Three blocks north and four blocks west of City Hall." The customer's trip can be represented by two arrows on a map, each having a specific length and direction.

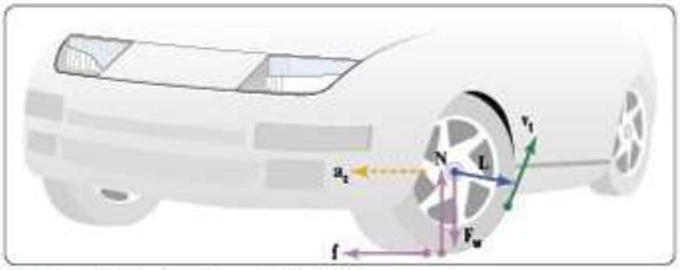
An artillery officer knows that the muzzle velocity of a fired cannon projectile is 930 m/s. He must raise the barrel of his cannon to an elevation of 33° in order to hit a target. The initial speed of the projectile can be represented by a long arrow labeled 930 m/s at an angle of 33° above a horizontal line.



4-2 Each leg of the trip to the store is a vector.

An engineer for a tire manufacturing company is designing a new, low-profile radial tire. In order to understand how the tire will respond to hard braking, he sketches a diagram of the various forces at work on the tire. He draws arrows representing the weight of the vehicle, the angular momentum of the wheel, braking torque, and the tire's friction with the road. Large quantities are represented by long arrows and small ones by short arrows. The diagram helps the engineer understand the dynamics needed to create a computer model of the tire design.

4-3 The velocity vector describes both speed and direction.



4-4 Each quantity in the diagram is represented by a vector.

The *magnitude* of a quantity is the absolute value of its numerical value. As such, magnitudes are always positive. It is important to remember that while all magnitudes of vectors are scalars, not all scalars are magnitudes of vectors.

In each case above, vectors are used to study quantities that have two pieces of information—a specific numerical value and a direction. Physicists use vectors to study dimensions such as displacement, velocity, force, acceleration, and magnetic fields. Vectors are drawn as arrows on *vector diagrams.*

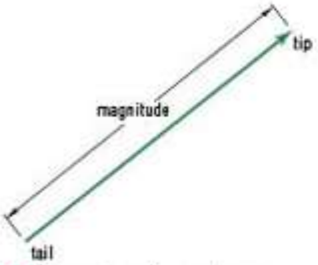
The magnitude of **A** is written A or $|\mathbf{A}|$. The magnitude of a vector is also called its modulus. The magnitude of a vector may be indicated by a numerical label or graphically indicated by the relative length of the vector arrow.

Physicists also work with scalars. A scalar is a quantity that can be described completely by only a single numerical piece of information. Time, mass, volume, and temperature are all scalars because, unlike vectors, they have no direction associated with them. Some scalars are only positive, such as mass and time. Others, such as temperature, may be positive or negative, depending on the instant of measurement.

A vector symbol is printed with boldface roman type, **A**. Since writing boldface letters is difficult, you may use a regular letter with an arrow over it, $\vec{A}$. Upright lines around the vector symbol, $|\mathbf{A}|$, stand for its absolute value and specify its magnitude. As a shorthand notation for the magnitude, either the italicized letter or the absolute value sign will be used; that is,

$$A \equiv |\mathbf{A}| \quad \text{(handwritten, } A \text{ or } |\vec{A}|\text{).}$$

The symbol $\equiv$ means "is defined as."



4-5 The essential parts of a vector diagram

4.2 Vector Angles

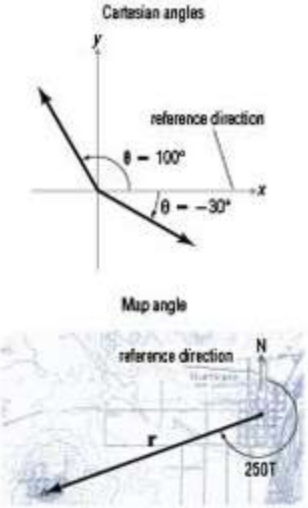
Vectors in diagrams provide important information. The tail of the arrow can indicate the starting point of motion or the point at which a force is physically applied. The length of the arrow is proportional to the magnitude of the vector quantity. Vector diagrams also include directional information indicated by the orientation of the arrow. The vector angle, labeled with the Greek letter θ, is given a numerical value in degrees or radians measured from a fixed direction. This reference direction is usually determined by convention or is selected for convenience in working the problem. This is especially important in two- and three-dimensional vectors. For one-dimensional vectors, direction can be indicated by a plus or minus sign, as stated in Chapter 3.

The vector angle (θ) is the direction of the vector in relation to its reference direction, the direction of zero angle.

There are some *conventions* (generally agreed-upon rules) for establishing reference directions for angular measurement. This will be especially applicable to two-dimensional motion, which you will study in the next chapter. In the

Cartesian (two-dimensional) plane, the reference direction is the positive x-axis. Positive angles are measured counterclockwise and negative angles are measured clockwise around the origin. There is no limit to the size of geometric angles. An angle can be measured in either degrees or radians.

Map directions are always referenced to geographic north, which by convention is located at the *top* of the map or diagram. Angles referenced to true north are indicated by a capital "T" in place of the degree symbol. Map directions are usually measured clockwise from north. For example, 315T is northwest. When using north as the reference direction, angles are always positive and are reported only in degrees from 0T to 360T. This textbook will always give map directions using three digits (which is standard practice for navigational purposes). In this system, north is 000T, northeast is 045T, south is 180T, and so on. On the other hand, if a problem gives an angle with the degree symbol (e.g., 120°), then you know that a Cartesian reference direction is being used.



4-6 The method for measuring vector angles depends on the kind of angle.

4-7 This compass rose shows the cardinal and intercardinal directions, as well as degrees.

4.3 Transporting Vectors

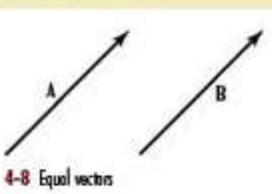
Drawing accurate vector diagrams requires a protractor and a ruler. For most physics problems, it is sufficient to sketch the approximate magnitude and orientation of each vector. Unless specifically requested, sketches don't have to be made exactly to scale.

Vectors are equal if they have both the same magnitude and the same direction. Examine vectors **A** and **B** in Figure 4-8. Even though they are at different locations in the diagram, they have identical magnitudes and they point in the same direction, so **A** = **B**. Remember that vectors are models of the physical quantities that they represent. If necessary for the sake of a problem solution, it is perfectly permissible to transport a vector from one location to another in a diagram as long as you maintain its original length and orientation (see Figure 4-9).

A marine biologist is periodically monitoring the location of a submerged humpback whale (*Megaptera novaeangliae*) using a sonic transponder. He plots its position at three different times on a nautical chart. The whale's position with time is shown in Figure 4-10. The distance the whale moves between the first and second positions is represented by the length of the line segment between P_1 and P_2, written P_1P_2. The whale's *displacement* is indicated by an arrow drawn from P_1 to P_2 (labeled $\overrightarrow{P_1P_2}$). Displacement is a vector quantity because it specifies both distance and direction. Distance is a scalar because only the magnitude of the displacement, a single piece of information, is given.

Map direction angles referenced to true north are followed by the letter "T."

Modern directional compass scales are subdivided into 360°. The four principal compass directions, or *cardinal directions*, are represented by angles that are 90° apart: north is 000T, east is 090T, south is 180T, and west is 270T. *Intercardinal directions* (NE, SE, SW, and NW) are 45° from their associated cardinal directions.

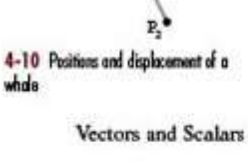


4-8 Equal vectors

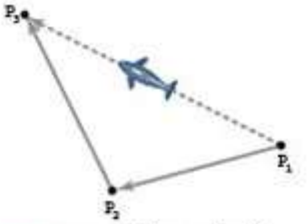
4-9 Vector transport

Problem-Solving Strategy 4.1

Vectors in a diagram can be transported from one point to another as long as their lengths and orientations remain unchanged.



4-10 Positions and displacement of a whale



4-11 Positions and displacement of a whale

If the whale moves on to P_3, the total distance that it traveled through the water from P_1 is the sum of the distances P_1P_2 and P_2P_3. Since the total distance is length alone, it is still just a scalar quantity. The whale's total displacement, however, is the vector from P_1 to P_3 (see Figure 4-11). The length of this vector is clearly different from the distance that the whale traveled. Vectors cannot be added by simple arithmetic. Fortunately, vector addition is not difficult to understand. You will learn this important skill in Section 4B.

4A Objectives

After completing this section, I can

- ✓ differentiate between vectors and scalars.
- ✓ describe the properties of a vector.
- ✓ write the correct symbols for vector and scalar quantities.
- ✓ identify the vector angle according to the kind of reference direction specified.

4A Section Review

1. What is the key difference between scalar and vector quantities?
2. State whether the following quantities are scalars, vectors, or neither.
   - a. force
   - b. one-way sign
   - c. pressure
   - d. π (3.14 . . .)
   - e. distance
   - f. displacement
   - g. velocity
   - h. speed
   - i. mass
   - j. temperature
   - k. Wednesday
   - l. density
3. When dealing with vectors, what is the difference between **B** and *B*? Write another symbol that is equivalent to *B*.
4. Compare and contrast the properties of angles measured in a Cartesian coordinate system to those measured on a map.
5. Why can you transport a vector from one place to another in a vector diagram?

4B OPERATIONS WITH VECTORS: GEOMETRIC TECHNIQUES

When more than one quantity is involved in a problem, vectors often need to be added, subtracted, or multiplied. They may be multiplied and divided by scalars as well. Mathematicians have developed methods to simplify these operations and yield useful results. The geometric techniques of vector math can be done quickly with only a ruler and a pencil.

4.4 Adding Equal Vectors by Scalar Multiplication

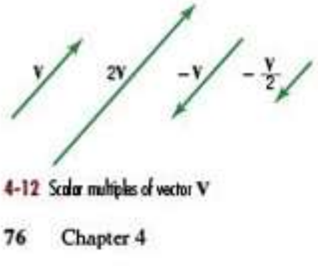
The sum or difference of two or more vectors is called the resultant.

The simplest case of vector addition is adding a vector equal to itself. Graphically, this is accomplished by placing the tail of the second vector at the tip of the first. Their sum is the vector drawn from the tail of the first vector to the tip of the second. The vector sum, called the resultant, is twice as long as the original vector and is oriented in the same direction.

$$\mathbf{V} + \mathbf{V} = 2\mathbf{V}$$

The bold plus sign indicates vector addition. Note that this vector sum is also the product of a scalar number and the original vector. When a vector is multiplied by a positive scalar number, its length (magnitude) changes but its direction stays the same. The magnitude of vector $2\mathbf{V}$ is $2V$.

In Figure 4-12, if **V** is a vector 6 units long, $2\mathbf{V}$ would be a vector 12 units long pointing in the same direction as **V**. If the scalar multiplier is negative, the resulting vector points in the opposite direction from the original. Thus, $-\mathbf{V}$ (that is, $-1 \times \mathbf{V}$) is a vector 6 units long pointing in the opposite direction from **V**. Scalar coefficients may also be fractions, so $-\frac{1}{2}\mathbf{V}$ (or $-\mathbf{V}/2$) is a vector 3 units long oriented opposite to **V**.



4-12 Scalar multiples of vector V