

UNIT 1

FACTORS AND POWERS

1.1 Multiples and Factors

1.2 Least Common Multiple and Greatest Common Factor

1.3 Powers and Exponents

1.4 Negative Exponents

1.5 Laws of Exponents

1.6 Solving Problems Using Exponents

If you need additional help, there are more resources available at math-help.ca/more.

1.1 Multiples and Factors

A **prime number** is an integer greater than 1 whose only integer factors are 1 and itself.

Examples:

2, 3, 5, and 7 are prime numbers since their only factors are 1 and themselves.

6 is not prime since it has two different sets of integer factors: 1 and 6 or 2 and 3

A **factor** of a number is a divisor of that number. It divides evenly into it.

Examples:

1. List all factors of 10.

- 1, 2, 5, and 10 are factors of 10 since they all divide evenly into it.
- Of these factors, only 2 and 5 are prime factors.

2. Show the following numbers as products of prime factors.

- $12 = 2 \times 2 \times 3$
- $50 = 2 \times 5 \times 5$

A **multiple** of a number is the product of that number times another whole number greater than 0.

Examples: Multiples of 5 are $(5 \times 1) = 5$; $(5 \times 2) = 10$; $(5 \times 3) = 15$; $(5 \times 4) = 20$; etc.

A **composite number** is not a prime number and can be factored in more than one way. All numbers that are not prime are composite (with the exception of 1).

Example: 15 is a composite number since it can be factored as 15×1 or 5×3 .

Examples with Solutions:

1. Which of the following numbers are not prime?

1, 3, 4, 5, 7, 9, 11, 15

1 is not prime since it is not greater than 1.

4, 9, and 15 are not prime. They are composite, since they have more than one pair of factors (9 can be factored as 9×1 or 3×3).

2. List all factors of 20.

Factor 20 as follows $2 \times 2 \times 5$ or $2^2 \times 5$.

The set of all factors consists of all numbers that divide evenly into 20.

The numbers are 1 plus all combinations of 2, 2, and 5 shown in step 1.

Answer: 1, 2, 4, 5, 10, and 20.

3. List all multiples of 7 less than 40.

Multiples of 7 consist of numbers that are the product of 7 times 1, 2, 3, 4, ...

We want multiples of 7 less than 40.

$7 \times 1, 7 \times 2, 7 \times 3, 7 \times 4, 7 \times 5$, (7×6 is 42, which is larger than 40).

Answer: 7, 14, 21, 28, 35

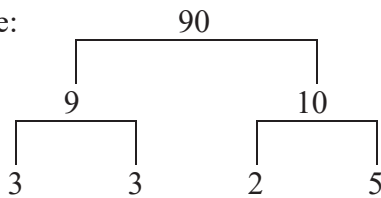
4. Show 90 as a product of prime factors.

Factor 90 until all factors are broken down into prime factors.

$$90 = 9 \times 10 = 3 \times 3 \times 2 \times 5 \text{ or } 3^2 \times 2 \times 5$$

Sometimes a **factor tree** can help in breaking down a number into prime factors.

Example:



The prime factors of 90 are $3 \times 3 \times 2 \times 5$.

Exercises 1.1

1. Identify whether or not each number is prime. Give a reason for your answer.

<u>Number</u>	<u>Yes/No</u>	<u>Reason</u>
a. 22		
b. 31		
c. 77		
d. 57		
e. 43		
f. 51		

2. List all factors of each number. Then list the prime factors.

<u>Number</u>	<u>All Factors</u>	<u>Prime Factors</u>
a. 30		
b. 100		
c. 75		
d. 90		
e. 135		
f. 38		

3. List all multiples of the following numbers that meet each condition.

<u>Number</u>	<u>Multiples of the Number</u>
a. All multiples of 11 that are greater than 40 and less than 100	
b. All multiples of 5 between 11 and 41	
c. All multiples of 9 less than 100	
d. All multiples of 20 less than 200	
e. All multiples of 13 less than 100 that are odd numbers.	

4. Write each number as a product of prime factors.

a. 30	b. 12
c. 26	d. 36
e. 250	f. 1000
g. 90	h. 216
i. 196	j. 242

5. List all factors that are common to both 9 and 30.

6. List all factors that are common to 10, 14, and 70.

7. List all numbers less than 100 that are multiples of both 15 and 10.

8. List all numbers less than 50 that are multiples of both 3 and 5.

9. I am a multiple of both 9 and 15. I am less than 200 and more than 150. Who am I?

10. I am a multiple of 3, 5, and 10. I am less than 100. Who am I?

11. I am a multiple of 3, 5, and 7 between 300 and 400. Who am I?

12. I am a number less than 50. If I am a multiple of both 2 and 14, who am I?

1.2 Least Common Multiple and Greatest Common Factor

The **greatest common factor** (GCF) of two or more numbers is the largest factor that is common to each of them. To find the **GCF** of two numbers, use the following steps.

1. Write each number as a product of prime factors.
2. Select all of the prime factors common to both.
3. The product of those factors is the greatest common factor.

Examples:

1. Find the GCF for 20 and 28.

- $20 = \underline{2} \times \underline{2} \times 5$ 2×2 is common to both numbers
- $28 = \underline{2} \times \underline{2} \times 7$ $\therefore 4$ is the GCF

2. Find the GCF for 30 and 45.

- $30 = \underline{3} \times 2 \times \underline{5}$ 3×5 is common to both numbers
- $45 = \underline{3} \times 3 \times \underline{5}$ $\therefore 15$ is the GCF

The **least common multiple** (LCM) is the smallest multiple of each number that is common to both. To find the **LCM** of two numbers, use one of the following methods.

1. Write multiples of each number.
2. Select the smallest multiple common to both.

OR

1. Write each number as the product of prime factors
2. Select all of the prime factors from the first number and then select only those prime factors from the second that are not already there.
3. Find the product of those factors.

Examples:

Method 1

Find the LCM of 15 and 10

- Multiples of 15 are 15, 30, 45, 60, 75, ...
- Multiples of 10 are 10, 20, 30, 40, 50, 60, 70, ...
- 30 is the smallest multiple of both numbers.

Method 2

Find the LCM of 15 and 10

- $15 = 3 \times 5$ Write as the product of prime factors.
- $10 = 2 \times 5$ Write as the product of prime factors.
- $3 \times 5 \times \underline{2}$ Select all factors of the first and then add factors not there from the 2nd number.
- $\text{LCM} = 30$ The product of the above factors (the smallest number that both 10 and 15 divide into).

Examples with Solutions:

1. Find the GCF of 40 and 50.

Write each number as the product of prime factors: $40 = \underline{2} \times 2 \times 2 \times \underline{5}$

$$50 = \underline{2} \times 5 \times \underline{5}$$

Select those factors that are common to both.

Answer: $2 \times 5 = 10$.

10 is the greatest factor common to both numbers.

2. Find the LCM of 15 and 20.

Method #1

Write multiples of each number until you find the smallest one that is common to both.

Multiples of 15 = 15, 30, 45, 60, 75, ...

Multiples of 20 = 20, 40, 60, ...

Answer: 60 is the smallest number that is a multiple of both.

Method #2

Factor each number as a product of prime factors:

$$15 = 5 \times 3$$

$$20 = 2 \times 2 \times 5$$

Use all of the factors of the first number and then add those from the second number that you do not already have.

Start with 5×3 , add 2×2 .

The $\text{LCM} = 5 \times 3 \times 2 \times 2 = 60$

3. Find (a) the GCF and (b) the LCM of the following numbers:

44, 66

GCF

$$44 = 2 \times 2 \times 11$$

$$66 = 2 \times 3 \times 11$$

Factors common to both are 2×11

GCF = 22 (largest factor that divides into both numbers)

LCM

$2 \times 2 \times 11$ (all prime factors of the 1st)

$2 \times 2 \times 11 \times 3$ (factors of 1st plus factors in the 2nd not already listed)

LCM = 132 (smallest multiple that both numbers divide into)

Exercises 1.2

1. Find the greatest common factor (GCF) for each set of numbers.

a. 20, 70

b. 27, 54

c. 40, 72

d. 14, 42

e. 30, 45, 60

f. 120, 80, 200

g. 580, 145

h. 10, 30, 50, 90

2. Find the least common multiple (LCM) for each set of numbers.

a. 9, 5

b. 14, 35

ABORIGINAL APPLICATIONS THE DRUM



BT Collection

The drum is an essential component in the songs and prayers of Aboriginal people. It represents the voice of the Creator and gives power and resonance to the voices of the singers. It is thought to reflect the heartbeat of “mother earth.”

Large drums, usually played by men, are about a metre across and about 0.7 metres high. They consist of a hide stretched across a circular wooden frame. Smaller drums are played by both men and women. They range from 20 to 50 centimetres across and are about 12 cm high.

Mathematical Applications

The face of the drum is circular. The area of a circle is equal to πr^2 and its circumference is equal to $2\pi r$. (Let $\pi \approx 3.14$.)

1. If the circumference of a drum is 66.602 cm, what is its radius?

2. If the radius of a drum is 15.2 cm, what is its area?

Answers

1. ≈ 10.6 cm

2. ≈ 725.4656 cm²

ANSWERS TO EXERCISES AND UNIT TESTS

UNIT 1

Exercises 1.1 (page 3)

1. a) No; Factors are 1×22 and 2×11
 b) Yes; Only factors are 1 and 31
 c) No; Factors are 1×77 and 7×11
 d) No; Factors are 1×57 and 3×19
 e) Yes; Only factors are 1 and 43
 f) No; Factors are 1×51 and 3×17
2. a) All factors: 1, 2, 3, 5, 6, 10, 15, 30; Prime factors: 2, 3, 5 b) All factors: 1, 2, 4, 5, 10, 20, 25, 50, 100; Prime factors: 2, 5 c) All factors: 1, 3, 5, 15, 25, 75; Prime factors: 3, 5 d) All factors: 1, 2, 3, 5, 6, 9, 10, 15, 30, 18, 45, 90; Prime factors: 2, 3, 5 e) All factors: 1, 3, 5, 9, 15, 27, 45, 135; Prime factors: 3, 5
- f) All factors: 1, 2, 19, 38; Prime factors: 2, 19
3. a) 44, 55, 66, 77, 88, 99 b) 15, 20, 25, 30, 35, 40 c) 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99 d) 20, 40, 60, 80, 100, 120, 140, 160, 180 e) 13, 39, 65, 91 4. a) $2 \times 3 \times 5$ b) $2 \times 2 \times 3$ c) 2×13 d) $2 \times 2 \times 3 \times 3$ e) $2 \times 5 \times 5 \times 5$ f) $2 \times 2 \times 2 \times 5 \times 5 \times 5$ g) $3 \times 3 \times 2 \times 5$ h) $2 \times 2 \times 2 \times 3 \times 3 \times 3$ i) $2 \times 2 \times 7 \times 7$ j) $2 \times 11 \times 11$ 5. 1, 3 6. 1, 2 7. 30, 60, 90 8. 15, 30, 45 9. 180 10. 30, 60, 90 11. 315 12. 14, 28, 42

Exercises 1.2 (page 8)

1. a) 10 b) 27 c) 8 d) 14 e) 15 f) 40
 g) 145 h) 10 2. a) 45 b) 70 c) 75 d) 80
 e) 70 f) 60 3. a) 8 b) 3 c) 9 d) 21 e) 5
 f) 7 4. 60 minutes 5. 35, 70 6. 7 7. Sue 12, Jack 9 8. 9:00 am

Exercises 1.3 (page 12)

1. $3 \times 2^5 = 96$ 2. $7 \times (-2)3 = -56$
 3. $5 \times 104 = 50\,000$ 4. $-6 \times 83 = -3072$ 5. a) 8
 b) 4 6. a) 4 b) -12 7. 35 8. 180 9. 70
 10. 535 11. -392 12. -54 13. -54 14. -8
 15. 1

Exercises 1.4 (page 13)

1. $\frac{1}{9}$ 2. $\frac{9}{4}$ 3. $\frac{10\,000}{81}$ 4. $-\frac{1}{32}$ 5. 125 6. $\frac{625}{1296}$
 7. $\frac{1}{9}$ 8. $\frac{1}{9}$ 9. $-\frac{1}{27}$ 10. $-\frac{1}{27}$ 11. $\frac{1}{5^3}$ 12. $\frac{1}{7^2}$

13. 2^2 14. $\left(\frac{5}{2}\right)^3$ 15. $\left(\frac{5}{7}\right)^4$

Exercises 1.5 (page 18)

1. $35x$ 2. $\frac{-2}{3}x^4y^7$ 3. $a^4b^4c^4$ 4. r^4st^2 5. n^5m^3
 6. $\frac{0.25}{x^6}$ 7. $\frac{6y^4}{rz^3}$ 8. x^{25} 9. 1 10. $\frac{r^6}{8t^9}$ 11. 27
 12. $\frac{x^6}{4}$ 13. $\frac{1}{4x^6}$ 14. $\frac{-27a^9b^{12}}{c^6}$ 15. y^{19} 16. $81b^{32}$
 17. 324 18. 56 19. 180 20. 99 900
 21. $a = 2, b = -1$ 22. $a = -12, b = -7$ 23. $y = \frac{1}{625}$
 24. $t = 25$ 25. All are incorrect.
 26. a) $3^2 \times 3^3 = 3^5$ b) $3x^0 = 3 \times 1 = 3$
 c) $(3x^2)^2 = 3^2 \times x^4 = 9x^4$ d) $(x^4)^5 = x^{20}$
 e) $x^{-1} = \frac{1}{x}$ f) $(3x^2)(4x^3) = 3 \times 4x^5 = 12x^5$
 g) $\frac{10x^6}{5x^4} = \frac{10}{5}x^2 = 2x^2$ h) $\frac{x^{20}}{x^4} = x^{20-4} = x^{16}$
 i) $\frac{x^4}{x^5} = \frac{1}{x}$ j) $\sqrt{9^2 + 16^2} = \sqrt{81 + 256} = \sqrt{337}$

Exercises 1.6 (page 21)

1. 294 cm^2 2. a) 400 b) 1600 c) $200(2)^h$
 3. a) 400π or 1256.64 cm^2 b) 900π or 2827.43 cm^2 4. a) 84.375 m b) 281.25 m 5. \$1817.02

Unit 1 Test (page 24)

1. a) $3 \times 3 \times 2 \times 5$ b. $2 \times 2 \times 3 \times 3 \times 3 \times 3$
 2. a) 32, 36, 40, 44, 48 b) 45, 90 3. a) 10
 b) 15 4. a) 60 b) 180 5. $-3(2)^5$ 6. a) -216
 b) -36 c) -18 d) 9 e) $1\frac{21}{25}$ f) 24 7. a) $\frac{1}{(-8)^3}$
 b) $\left(\frac{2}{3}\right)^2$ 8. a) $-\left(\frac{1}{125}\right)$ b) 625 9. a) $5x^3y^5$
 b) $\frac{m^5}{n}$ 10. 12 150

UNIT 2

Exercises 2.1 (page 29)

1. Novel D has more pages and more words than Novel C. 2. Hockey players E and F played the same number of games, but player F scored more goals. Player K played more games than player F but scored the same number of goals.
 3. Building M has the least number of storeys, but its cost per storey is the highest. Building P has more storeys than building N, but its cost per storey is the same as building N.