The Charlotte Mason Elementary Arithmetic Series

Book 4

by Richele Baburina

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The Charlotte Mason Elementary Arithmetic Series, Book 4

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"The Principality of *Mathematics* is a mountainous land, but the air is very fine and health-giving, though some people find it too rare for their breathing. It differs from most mountainous countries in this, that you cannot lose your way, and that every step taken is on firm ground. People who seek their work or play in this principality find themselves braced by effort and satisfied with truth."

(*Ourselves*, Book 1, p. 38)

Contents

Introduction

Traveling higher up and further in the mountainous land of mathematics takes effort on the part of the climber but offers exhilarating rewards. Charlotte Mason says of arithmetic, "There is no subject in which the teacher has a more delightful consciousness of drawing out from day to day new power in the child. Do not offer him a crutch: it is in his own power he must go" (*Home Education*, p. 261). As a student goes, he grows and discovers new power in himself. This wonderful awareness can be experienced while teaching our own children in a direct, simple, but humble way that respects their ability to work, wonder, and grapple with the ideas found in math.

In this book, as well as the previous in the series, we find a number of practices and principles distinct to Charlotte Mason's philosophy of education. Here are just a few:

Atmosphere. The atmosphere of the math lesson should be free of conflict and anxiety. Irene Stephens, lecturer in math under Charlotte Mason, tells us we often lead our children into a Slough of Despond and continue slogging through this miry swamp year after year rather than leading them into a Delectable Country (*Parents' Review,* Vol. 40, p. 38). This illustration comes from *The Pilgrim's Progress* with the slough being a place where fears, doubts, and discouragement arise. It is a place of descent—the antithesis of the vigorous and delightful ascent into the mountainous land of mathematics pictured by Charlotte, in which each step is taken on firm ground.

Atmosphere also includes the physical environment of the lesson. Special, scientifically developed manipulatives often place the focus on the object and take it off the idea it is meant to represent. This can hinder a student from understanding the concept when met apart from the manipulative. Rather, a variety of simple everyday objects and real-life applications act as handy tools for the investigation of ideas, serving as stepping-stones to more abstract thinking. Used as a means to an end, they are put away as soon as an idea is grasped or a level of comfort working with a process is achieved, and always before boredom sets in.

Discipline. Charlotte Mason valued arithmetic especially for its ability to shape a student's character. Her emphasis on good habits during the math lesson has meant that, over time, careful execution becomes second nature to a student. Short, concentrated lessons help a student maintain focus, allowing him to accomplish more in 20–30 minutes than he would in a long, drawn-out, dawdling lesson. Continuing to embrace daily mental math—that is, work taken orally—for five minutes of this time remains an instrumental part of training good habits such as readiness, promptness, and accuracy. Consistent lessons in which the student is putting forth mental effort will help build habits of thought, and the lessons are written to awaken a student's ability to think mathematically, rather than merely perform mechanically.

Life. Charlotte tells us that "mathematics, like music, is a speech in itself, a speech irrefragably logical, of exquisite clarity, meeting the requirements of mind" (*A Philosophy of Education*, p. 334). Engaging questions, vital ideas, hands-on work, and short, interesting histories that spark the imagination open a realm of beauty and truth for a student to explore. Rather than given rule upon rule and lengthy explanations, a student is led to think and discover rules for himself. As ideas unfold through this living teaching of math, absolute truth will reveal itself as well.

Arithmetic Concepts in Book 4

- Count by numbers greater than 1
- Oral arithmetic, adding rows of five numbers up to 100
- Numeration and notation of numbers up to 1,000,000
- Addition of numbers within 1,000,000
- Oral arithmetic; subtracting rows of five numbers within 55
- Subtraction of numbers within 1,000,000
- Sums using parentheses
- Calculating area
- Drawing plans to scale
- Long multiplication
- Multiplication by a product
- Introduction to factors
- Short division review
- Short division of money review
- Division of one quantity by another with a different unit of measurement
- Averages
- Rounding
- Long division
- Introduction to decimals
- Fractions
- Measures of time
- Roman numerals
- US standard and metric measures of length
- Experiential addition and subtraction of fractions
- Pacing as a form of measurement
- US standard and metric measures of area

- Cubic measure
- US standard and metric measures of weight
- US standard and metric measures of liquid
- US standard and metric dry measures

Overview of Lessons

In Book 3, the student continued the operations of addition, subtraction, multiplication, and division; mastered the multiplication tables for 7 through 10; delved into longer multiplication and division; began working with fractions; and learned to read, write, and work with numbers up to 10,000. He also had a hands-on introduction to weights and measures.

In this fourth book, the lesson length is increased by ten minutes, for a total of 30 minutes per lesson, which includes 5 minutes of mental math. If your student is not able to give focused attention for that amount of time, work gradually to build his concentration. Try to end the lesson on a good note; so if his attention is waning, switch to some lively mental math before putting the book away.

The student will have further practice in the processes of addition, subtraction, multiplication, and division; work with numbers within 1,000,000; learn to draw to scale; delve deeper into long multiplication and long division; and work with averages and rounding. He will also have more exact work with weights and measures in both the imperial standard (US) system and, now, the metric system—including measures of time, weight, distance, area, cubic measure, and volume using an active, hands-on approach. Continued work with fractions, more complex compound addition and subtraction, and a formal introduction to decimals will take place in this book as well.

Independent Work

While math is not considered an independent subject, you gradually want your student to be able to do some work independently. To help him gain proficiency in the ability to work alone, some answers are now placed in the back of the book. Any Mixed Review and Review sections not specified as oral work may also be taken independently, as long as the teacher ensures that the student is working carefully and with understanding.

It is not recommended that the student simply be handed the book to work alone. Care should always be taken that reading and writing never overshadow the ideas of the math lesson, that undesirable habits are not sneaking in, and that accuracy and understanding is maintained. Here are the components that should be kept in balance as you work through these lessons:

- The teacher and student work together on an initial concept and a number of problems until comprehension and comfort is exhibited;
- The student works independently for a set amount of time;
- The student then meets with the teacher again to look over the accomplished work;
- The teacher gives the student lively, oral review in the form of mental arithmetic. (See Mental Arithmetic section below for more details.)

Assign a given amount of time to the student for any independent work. Having a definite finish line will help him stay focused and give his best effort to the task. Be sure to allow time to evaluate this work together in order to address any misunderstandings immediately.

Feel free to arrange the components of each lesson to suit the material to be covered that day, but strive to keep a balance during the week's lessons for "new, review, and mental math too." For example, one lesson might look like this:

- 5–10 minutes on a new concept, with the student working the first three exercises with you at his side
- 10 minutes of independent work
- 2–5 minutes of looking over independent work
- 5 minutes of mental math

and another like this:

- 2–5 minutes going over with the student what is expected in the Review section, noting the amount of time allotted
- 10 minutes of independent work on the Review section
- 10 minutes of work together on the latest concept
- 5 minutes of mental math

yet another like this:

- 25 minutes of work together on a new section
- 5 minutes of mental math

Notation and Writing

Understanding our system of notation is foundational to vital ideas like place value. Students work with numbers only up to where the notation has been learned. Furthermore, when regrouping or exchanging occurs (i.e., borrowing and carrying), the student should say he is working with 5 *thousands,* 7 *tens,* or 2 *hundreds*—not simply 5, 7, or 2.

As your student's ease and ability in reading and writing increases, be sure he maintains physical habits of neatness and order. The orderliness of each number contained in just one box on the grid of graph paper will help him maintain orderliness of thought. Continuing short lessons that alternate between written and mental work will further nurture these habits.

Review

While a student may have mastered or internalized a concept, review is still an important part of the weekly lessons and should not be skipped. Review acts as an aid in solidifying facts, helps concepts and processes become second nature, and encourages the student to make gradual gains in speed while maintaining careful execution.

Regular review is also crucial to secure progress in mathematical skills. Concepts that will be introduced in future books build upon the ideas in this and previous books in the series. It is only through confident fluency in the processes presented thus far that the student will be able to turn his full attention to the next ideas.

Review of material covered in Book 3 as well as review of new material have both been built into this book and spread out over the lessons. You will also find two review sections in the back of this book: one for oral work and one for written. Review sections not specified as oral work may be taken independently as long as the teacher ensures that the student is working carefully and with a good command of the material.

Approach any review section as follows:

- The teacher and student look over the review section to ensure the student understands what is being asked of him;
- The student then works independently at his own pace for a set time;
- The student immediately meets with the teacher to look over the accomplished work and address any concerns.

It is normal to expect to dedicate some time to refresh past concepts in the student's mind after a long break; however, if your student has never worked with the concepts found in the review sections—or you believe he needs additional review—you will find completely guided lessons in the previous books in this series.

Tip: If you are continuing straight from Book 3 without a long break, some of the initial review may be found to be tiresome. In this case, it should be bookmarked to be spread out after work in newer concepts or used as mental arithmetic.

Mental Arithmetic

The student will be expending mental effort throughout the lesson, but there should also be a daily time given to work taken strictly orally—known as *rapid oral work* or *mental math*. Work noted *to be given orally*, Table Work, and additional review found throughout and in the back of the book may all be used for this activity. This type of work should be lively and engaging and is a good way to regain your student's attention if you notice it is waning. It may occur at the end of the lesson or another time during the day, indoors or out, with the student posing his own questions or multiple students posing mental work to each other. This type of mental arithmetic should fall within the scope of a student's learning. Adjust to using lower numbers, if necessary, as he builds proficiency in this area.

If your student has yet to learn his math facts, be sure to spend at least five minutes daily in this endeavor. The Number Sentence Cards (available from Simply Charlotte Mason) are wonderful aids for mental math and attaining fluency with math facts. If your student has never had the opportunity to make connections and investigate the logic of addition,

subtraction, and multiplication tables, please refer to Books 2 and 3 in this series.

Weights and Measures

Your student has met with weights and measures in his daily life; worked simple sums involving measures of value, time, and distance throughout this series; and had a formal introduction to US standard measures in Book 3. In Book 4, more precise measurements will be taken and metric measures will be introduced with short histories then worked with a variety of interesting hands-on activities. Simple tables of weights and measures previously drawn up in Book 3 will be added to and expanded upon. If your student has a new math notebook or is entering the series at Book 4, all previous tables are provided for your convenience. The student should copy each entire table into his new notebook. If your student is continuing on from Book 3 with the same math notebook, he may simply add the new elements to each table.

When to Advance

The number of problems necessary for a student to master or internalize a concept can vary greatly both by the individual student and by the specific concept. You want to give enough work with the newest process or idea so that the student is comfortable and confident working with it, but not so much that he becomes bored. Just as good habits must be reinforced over time, periodic review must also be given to solidify concepts and facts as well as to build some speed in using them. Math concepts often build upon each other, so gaining fluency is vital. The importance of a strong foundation will become increasingly evident as the student meets with more complex math.

This book contains the components necessary to attain and maintain fluency through immediate work in the new material, ample review, and mental math; i.e., rapid oral work. You have the freedom to not use every question given if it is not necessary. Those questions that are skipped may be marked and used for later review. If more are needed, the questions are written in such a way as to allow you to easily change numbers, names, and objects that relate to your own student's life.

Be sure to give each lesson a brief look-through ahead of time so that you have an idea of where you are headed and have all necessary materials on hand. This way, you will be better able to focus on your student during the lesson itself. Precede and follow the arithmetic lesson with something that uses a different part of the brain or body, such as picture study, nature study, handicraft, Swedish drill, reading, or singing.

Pacing

While the concepts found in Book 4 generally take place in a student's fourth year of formal education, you should progress at a pace that ensures each step is taken on solid ground. The pace should be adjusted to the progress of the individual student rather than to a standardized rate of learning or the pace of peers and siblings.

Aim for consistent and regular lessons, and you should find the pace takes care of itself. Help instill a sense of happy assuredness based on your student's own progress and achievements, remembering that communicating the idea that he is somehow "behind" can crush his confidence. The week's work should include work in the newest concept, in review, and in mental arithmetic. The bookmarks provided may also be used to distribute work in these three areas as best fits your student's needs.

The charts given on the next page are for your convenience as a loose planning guide. As always, adjust the chart to your student's pace rather than attempting to make your student fit the charts. The following is based on lessons that are 30 minutes in length with 5 minutes of those given to mental arithmetic. If a student is unable to maintain attention for that length of time, simply make it a target goal to gradually work toward.

Term 1												
Suggested Weeks	1	2	3	4	5	6	7	8	9	10	11	12
Review of Previous Year's Work, Numeration & Notation												
Addition & Subtraction of Numbers within 1,000,000; Sums with Parentheses												
Long Multiplication within 1,000,000												
Area and Drawing Plans to Scale												
Exam												

Term 2												
Suggested Weeks	13	14	15	16	17	18	19	20	21	22	23	24
Multiplication by a Product & Factors												
Division, Averages, Rounding, Division by 10s												
Long Division, Remainders												
Fractions $\frac{1}{10}$ and $\frac{1}{100}$, Introduction to Decimals												
Measures of Time, Roman Numerals												
Exam												

Term 3												
Suggested Weeks	25	26	27	28	29	30	31	32	33	34	35	36
US Standard & Metric Measures of Length, Pacing												
US Standard & Metric Measures of Area, Weight, and Volume												
Work any unused mental math												
Exam												

Exams

Three end-of-term exams are provided in the back of this book. These are customizable to reflect where your student is in the lessons at the end of each term. See page 303 for details.

Supplies Needed

- Gridded math notebook (sized according to the student's handwriting: $\frac{3}{4}$", $\frac{1}{2}$", or $\frac{1}{4}$"). Available from Simply Charlotte Mason.
- Personal chalkboard, dry-erase board, or the like, together with its appropriate writing instrument. For brevity, it will be referred to as *the slate* in the lessons.
- 20 play hundred-dollar bills, 20 play ten-dollar bills, and 20 play one-dollar bills
- 10 quarters, 20 dimes, and 100 pennies, as well as nickels as necessary
- Small objects that can easily be divided into groups, such as candy-coated chocolates, buttons, raisins, nuts, beads, or the like as needed
- Atlas or maps
- Sheets of graph paper with 1" grids
- Sheets of graph paper with $\frac{1}{4}$" grids
- 12-inch ruler with US standard and metric measurements
- Meter stick (or yardstick that also includes metric measurements)
- Yardstick or tape measure
- Analog clock
- Stopwatch, or clock or watch with a second hand
- Low-adhesive tape, such as painter's tape or decorative tape
- Cardboard or heavy card stock with a length of at least 12" (or previously-constructed ruler from Book 3)
- Piece of card stock at least 8" × 10"
- 2 cubes of equal size, or 1 cube and a rectangular box
- 36 one-inch cubes
- Small box(es) that will fit 1-inch cubes evenly
- Kitchen scale, either analog or digital, with both metric and US standard units

- Zip-top bags
- Dry beans or rice
- Felt-tip marker
- Kitchen measurement sets to include:
  - » Liquid measuring cups: 1 cup (8 oz / 250 ml), 2 cup (16 oz / 1 pint / 500 ml), and 4 cup (32 oz /1 quart / 1000 ml)
  - » Standard measuring cups: $\frac{1}{4}$ cup, $\frac{1}{3}$ cup, $\frac{1}{2}$ cup, and 1 cup
  - » Standard measuring spoons: $\frac{1}{4}$ tsp., $\frac{1}{2}$ tsp., 1 tsp., and 1 Tbsp.
- Medicine cup
- Labeled medicine dropper
- Dry ingredients, such as rice or sugar
- Flour
- A variety of household objects
- Number Sentence Cards, Book 2 & Book 3 collections (optional, but highly recommended; see below for details)

Optional

- The parent reference book, *Mathematics: An Instrument for Living Teaching*, explains more fully Charlotte Mason's unique approach in mathematics, including the principles upon which the methods found in this book rest.
- The video series, *Charlotte Mason's Living Math: A Guided Journey*, demonstrates the foundational methods used in elementary arithmetic. Although the student is working with larger numbers than those in the video series, the processes are the same. Recommended if you are new to the method.
- A collection of Number Sentence Cards offers more than 500 ready-to-use equations at your fingertips. Have them on hand so your student can keep working if your math lesson is unexpectedly interrupted, or use them for 5 minutes of mental math to retain math facts, build speed, and cultivate accuracy and concentration. Sets of clearly printed Number Sentence Cards are sold separately or

as part of The Charlotte Mason Elementary Arithmetic Series, Kits for Book 2 & Book 3.

- A kit is available from Simply Charlotte Mason for Book 4, containing many of the items needed for these lessons. A smaller supplemental kit is available for those who already have the Book 3 kit, which contains many of the same items.

Addition & Subtraction

Since the student is working longer equations, we take the operations of addition and subtraction together in order to give more variety to his lessons. He begins with a review of counting forward and backward by numbers greater than one. This will help build the student's sense of number, including identifying patterns and improving his ability to work in multiples, while increasing his feel for how numbers work. The student should remain challenged, so if any of the review exercises seem too simple, they may be made more stimulating by using them to help the student increase his speed and accuracy.

Your student will also look at addition and subtraction in different and new ways in this section, like solving problems with the use of parentheses, adding in tens with larger columns of numbers, and using addition to check his subtraction work. Numeration and notation extend to a million. Working with these larger numbers is not meant to be busy work but to help your student feel confident working with large quantities. The pure number sections will aid in abstract thinking as the student works with numbers not assigned separate meanings.

Your student may also begin increasing his ability to work independently in this section. If he is exhibiting good habits—such as accuracy, attention, and neatness—during the first three problems, then you may assign the remaining problems for independent written work within a set amount of time. Just be sure to allot some time afterward to meet with him and review his work. In this way, you'll be able to address any misunderstandings and keep bad habits from forming.

In the next lesson, pick up wherever he left off. It is important that all work is accomplished in this section so that your student gains an extremely firm understanding and fluency with how numbers behave during the operations of addition and subtraction and so that he builds more speed using the learned addition and subtraction facts before moving on to larger numbers and more complex operations.

Addition Review / Skip Counting

(to be given orally)

The ability to count by numbers greater than one can help a student see how numbers work, identify patterns with numbers, and strengthen his ability to work in multiples. For an extra challenge, have the student work on increasing his speed while maintaining accuracy, or have him begin from different starting points than those given here.

Explain to the student that he will be adding by twos up to 100 and that this "adding by twos" is sometimes called *skip counting*, as he will be skipping numbers in between each as he adds mentally.

Tip: Though a student may balk, review is an important part of solidifying facts, making gradual gains in speed, and helping concepts and processes become second nature.

1. Count by twos from 2 to 100.

Tip: Rather than saying "2 plus 2 equals 4. . ." have your student say "2, 4, 6, 8, . . ." without saying anything between the numbers.

2. Beginning at 3, add by twos to 100 like so: "3, 5, 7, 9, . . ."

Tip: With this challenge (and some of the others in this section), your student will not be able to count to exactly 100, so tell him to simply count as near to 100 as possible.

3. Count by threes from 3 to 100. (3, 6, 9, 12, . . .)

4. Beginning at 4, count by threes to 100. (4, 7, 10, 13, . . .)

5. Count by fours from 4 to 100. (4, 8, 12, . . .)

6. Beginning at 5, count by fours to 100. (5, 9, 13, 17, . . .)

Tip: Following the initial lesson, the remainder of these may be bookmarked and used during the five minutes set aside for mental math or rapid oral work until they're achieved with relative ease and rapidity. For an extra challenge,

remember to have the student begin from different starting points than those given here.

7. Count by fives to 100. (5, 10, 15, . . .)
8. Beginning at 6, skip count by fives to 100. (6, 11, 16, 21, . . .)
9. Count by sixes to 100. (6, 12, 18, . . .)
10. Beginning at 1, add by sixes to 100. (1, 7, 13, 19, . . .)
11. Count by sevens to 100. (7, 14, 21, . . .)
12. Beginning at 8, count by sevens to 100. (8, 15, 22, . . .)
13. Count by eights to 100. (8, 16, 24, . . .)
14. Beginning at 9, add by eights to 100. (9, 17, 25, . . .)
15. Count by nines to 100. (9, 18, 27, . . .)
16. Beginning at 10, skip count by nines to 100. (10, 19, 28, . . .)
17. Count by tens to 100. (10, 20, 30, . . .)
18. Beginning at 11, skip count by tens to 100. (11, 21, 31, . . .)

Answers

1. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94, 96, 98, 100
2. 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99
3. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, 78, 81, 84, 87, 90, 93, 96, 99
4. 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70, 73, 76, 79, 82, 85, 88, 91, 94, 97, 100
5. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100
6. 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89, 93, 97

7. 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100
8. 6, 11, 16, 21, 26, 31, 36, 41, 46, 51, 56, 61, 66, 71, 76, 81, 86, 91, 96
9. 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
10. 1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, 79, 85, 91, 97
11. 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98
12. 8, 15, 22, 29, 36, 43, 50, 57, 64, 71, 78, 85, 92, 99
13. 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96
14. 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97
15. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
16. 10, 19, 28, 37, 46, 55, 64, 73, 82, 91, 100
17. 10, 20, 30, 40, 50, 60, 70, 80, 90, 100
18. 11, 21, 31, 41, 51, 61, 71, 81, 91

Subtraction Review / Skip Counting Backward

(to be given orally)

The ability to count backward by numbers other than one can help a student see how numbers work, identify patterns with numbers, and strengthen his ability to work with multiples. For an extra challenge, have the student work on increasing his speed while maintaining accuracy, or have him begin from different starting points than those given here.

Tip: Following the initial counting during the main lesson, you may want to bookmark these for review in the five minutes set aside for rapid oral work until answers come quickly and easily from various starting points.

1. Count backward from 20 to 0, subtracting 2 each time: "20, 18, 16, . . ."

2. Count backward from 30 to 0, subtracting 3 each time. (30, 27, 24, . . .)

3. Count backward from 40 to 0, subtracting 4 each time. (40, 36, 32, . . .)

4. Count backward from 50 to 0, subtracting 5 each time. (50, 45, 40, . . .)

5. Count backward from 60 to 0, subtracting 6 each time. (60, 54, 48, . . .)

6. Count backward from 70 to 0, subtracting 7 each time. (70, 63, 56, . . .)

7. Count backward from 80 to 0, subtracting 8 each time. (80, 72, 64, . . .)

8. Count backward from 90 to 0, subtracting 9 each time. (90, 81, 72, . . .)

9. Count backward from 100 to 0, subtracting 10 each time. (100, 90, 80, . . .)

Answers

1. 20, 18, 16, 14, 12, 10, 8, 6, 4, 2, 0
2. 30, 27, 24, 21, 18, 15, 12, 9, 6, 3, 0
3. 40, 36, 32, 28, 24, 20, 16, 12, 8, 4, 0
4. 50, 45, 40, 35, 30, 25, 20, 15, 10, 5, 0
5. 60, 54, 48, 42, 36, 30, 24, 18, 12, 6, 0
6. 70, 63, 56, 49, 42, 35, 28, 21, 14, 7, 0
7. 80, 72, 64, 56, 48, 40, 32, 24, 16, 8, 0
8. 90, 81, 72, 63, 54, 45, 36, 27, 18, 9, 0
9. 100, 90, 80, 70, 60, 50, 40, 30, 20, 10, 0

Addition Review with Longer Rows of Figures

(to be given orally)

Tip: As a help, pull a Number Sentence Card out before a lesson begins so your student's work can continue uninterrupted if you get called away for a moment.

Have your student add the following rows of figures as you read them aloud, taking pauses as you would in dictation. For example, the first row would be read as follows: 4 + 3 are (student answers "7"), + 6 (student answers "13"), + 2 (student answers "15"), + 7 (student says "22"). You may also cover the answers section and have the student read each problem aloud, solving as he goes.

1. 4 + 3 + 6 + 2 + 7
2. 8 + 2 + 5 + 9 + 4
3. 6 + 4 + 1 + 8 + 2
4. 5 + 7 + 3 + 9 + 2
5. 9 + 2 + 7 + 3 + 5
6. 12 + 9 + 12 + 6 + 3 =
7. 15 + 3 + 11 + 6 + 4 =
8. 19 + 4 + 12 + 7 + 5 =
9. 8 + 14 + 9 + 17 + 3 =
10. 13 + 9 + 18 + 4 + 7 =

Answers

1. 7, 13, 15, **22**
2. 10, 15, 24, **28**
3. 10, 11, 19, **21**
4. 12, 15, 24, **26**
5. 11, 18, 21, **26**
6. 21, 33, 39, **42**
7. 18, 29, 35, **39**
8. 23, 35, 42, **47**
9. 22, 31, 48, **51**
10. 22, 40, 44, **51**

Mixed Review of the 2's Table *(to be given orally)*

This type of review will help keep math facts fresh while also helping to increase speed and concentrated attention. If your student's accuracy needs

improving, have him state the answer as a fully worded number sentence; i.e., *2 times 9 is 18* in response rather than just *18*.

Simple fractions were introduced and worked with in Book 3 as an aspect of division. If your student has never encountered simple fractions or the idea of fractional parts, he might be led to it by dividing concrete objects (such as candies, pennies, buttons, and the like) into any number of equal parts by first divvying them up into equal piles and ascertaining how many objects are in one part. For a fuller treatment along with notation of fractions see Book 3, pp. 165-180.

1. 2 rows of chairs with 8 chairs in each row are how many chairs?
2. What is 2 × 7?
3. How many 2s in 20?
4. How many ears do 3 kittens have altogether?
5. How many ears have 5 kittens in all?
6. What is half of 18?
7. What is 12 ÷ 2?
8. What is half of 12?
9. Is half of 12 the same as 12 divided by 2?
10. James caught 5 fish and John caught twice as many. How many fish did John catch?
11. What is 16 ÷ 2?
12. What is half of 16?
13. Amber ate 4 slices of pizza. If Philip ate half as much, how many slices did Philip eat?
14. Casey's grandmother lives 7 miles away. If he rides his bike there and back, how many miles will he have ridden?

15. What is 9 × 2?

16. What is $\frac{1}{2}$ of: 4? 12? 6? 20? 8? 10? 18?

17. If you picked 8 peaches and ate $\frac{1}{2}$ of them, how many would you have eaten?

18. Last year the Dunlap family planted 2 acres of strawberries. This year, their strawberry patch doubled in size. How many acres of strawberries have they?

19. If you divide 10 pancakes equally amongst 2 people, how many pancakes would each person receive?

20. Say the 2s table up to 2 × 10 (or 2 × 12 if you prefer).

Answers

1. 16 chairs
2. 14
3. 10
4. 6 ears
5. 10 ears
6. 9
7. 6
8. 6
9. yes
10. 10 fish
11. 8
12. 8
13. 2 slices
14. 14 miles
15. 18
16. 2, 6, 3, 10, 4, 5, 9
17. 4 peaches
18. 4 acres
19. 5 pancakes
20. 2 × 1 = 2, 2 × 2 = 4, 2 × 3 = 6, 2 × 4 = 8, 2 × 5 = 10, 2 × 6 = 12, 2 × 7 = 14, 2 × 8 = 16, 2 × 9 = 18, 2 × 10 = 20, 2 × 11 = 22, 2 × 12 = 24.

More multiplication and division tables review can be found on page 253.

Multiplication

The work in this section covers long multiplication, area, and drawing to scale in a hands-on way, as well as multiplication by a product and factors. A review of multiplication with a small multiplier, multiplication by zeros, and double-digit multiplication is given before long multiplication is formally worked with thoroughly detailed step-by-step instruction.

If your student has completed the previous books in the Charlotte Mason Elementary Arithmetic Series, he will have already had a significant amount of work in multiplication that includes the ideas of multiplication as repeated addition, what "times" indicates, and multiplication as the complement of division. He will have constructed multiplication tables in a way that allowed him to find patterns and see the rationale behind them, and he will have learned them front-to-back, committing them to memory.

Some students take longer to learn their math facts than others. If your student has yet to gain fluency, be sure to include 5 minutes of daily work on the multiplication/division tables. Students who have already learned their facts should still review them as a part of their weekly mental math schedule to maintain fluency and increase speed. Learning the multiplication tables will free students up to attend to new ideas found in the arithmetic lessons as well as to focus on more complex ideas they'll meet in the future.

Review of Larger Multiplication with a Small Multiplier

(to be given orally)

Though this review is to be given orally, allow the student to use written work to find the solution whenever necessary.

1. Gumballs are 25¢ each. How much for 8 gumballs?
2. Bookmarks at the craft fair cost 62¢ each. How much are 3 bookmarks altogether?
3. A stamp costs 55¢. How much do 7 stamps cost in all?
4. Three friends gathered 75 pinecones each. How many did they collect in all?
5. Olivia has 6 puppies that each fetch 16 sticks. How many sticks did they fetch altogether?
6. Pencils are 20¢ each. How much for 6 pencils?
7. If 9 friends did 54 jumping jacks each, how many jumping jacks did they do in all?
8. Mark made 7 flower arrangements, each containing 33 flowers. How many flowers did he need to complete the arrangements?
9. Robbie and 3 friends read 27 comic books each. How many comic books did the 4 friends read altogether?
10. If each package contains 17 gummy worms, how many gummy worms do 8 packages contain?

Answers

1. $2.00
2. $1.86
3. $3.85
4. 225 pinecones

5. 96 sticks
6. $1.20
7. 486 jumping jacks
8. 231 flowers
9. 108 comic books
10. 136 gummy worms

More written review of larger multiplication by a small multiplier can be found on page 285.

Multiplication with Numbers Ending in Zero

(to be given orally)

Simpler work should be taken orally whenever possible. However, if an equation is outside your student's ability for oral work, then written work may be used.

In Book 3, the student was led step by step to discover the short way to multiply by 10s, 100s, 1000s, etc. If this technique is unknown to the student, he may be led to it by the questions included here that study the shift in place value.

1. What is 10 × 2? (20)
2. What is 2 × 10? (20)
3. What happens to 2 *units* if you place a zero to the right? (The *2* becomes two *tens*, which make 20.)
4. What happens to the 2 *tens* in *20* if you take the zero away? (The two *tens* become two *units*.)
5. If placing a zero to the right of *2* multiplies it by 10, how may you multiply 5 by 10? (Place a zero to its right.)
6. What is 100 × 2? (200)
7. What is 2 × 100? (200)

8. What happens to the *2* if you place two zeros to the right of it? (The *2* moves to the hundreds place to become *200*.)

9. What happens to the 2 hundreds in *200* if you take one zero away? (The *2* moves to the tens place to become *20*.)

10. What happens to it if you take both zeros away from *200*? (The *2* moves to the units place and becomes *2*.)

11. What is 24 × 1? (24)

12. What is 24 × 10? (240)

13. What is 24 × 2? (48)

14. What is 24 × 20? (480)

15. Is multiplying by 20 the same as multiplying by 2 and then by 10? (yes)

16. Can you tell a quick method of multiplying by 20? (To multiply a number by 20, multiply by 2 and place a zero to the right.)

17. If you want to multiply a number by 200, what is a quick method you could use? (To multiply a number by 200, multiply by 2 and place two zeros to the right.)

18. Would multiplying by 50 be the same as multiplying by 5 and then by 10? (yes)

19. Multiply 12 × 5 and then × 10. Multiply 12 × 50. Is the answer the same? (12 × 5 = 60; 60 × 10 = 600; 12 × 50 = 600; yes)

20. Can you tell a quick method of multiplying a number by 50? by 500? (To multiply a number by 50, multiply by 5 and place a zero to the right. To multiply a number by 500, multiply by 5 and place two zeros to the right.)

Exercises

These exercises may be given orally, written as independent work in the math notebook, or a mixture of both.

1. Have your student multiply the following numbers by 10: 5; 8; 10; 17; 21; 36; 100; 130; 192; 1,000; 1,070; 1,488
2. Multiply by 100: 7, 15, 62, 93, 100
3. Multiply by 20: 4; 9; 12; 53; 210; 1,000
4. Multiply by 200: 3, 11, 35, 48, 126
5. Multiply by 30: 6, 11, 40, 102, 333
6. Multiply by 40: 4, 40, 9, 90, 100
7. Do the following multiplication sums:

25	25	11	60	60
× 5	× 50	× 50	× 6	× 60

8. Multiply by 500: 2, 3, 6, 10, 20
9. Multiply by 1,000: 4, 8, 15, 26, 100

Answers

1. 50; 80; 100; 170; 210; 360; 1,000; 1,300; 1,920; 10,000; 10,700; 14,880
2. 700; 1,500; 6,200; 9,300; 10,000
3. 80; 180; 240; 1,060; 4,200; 20,000

Answers to 4–10 are on page 312.

Tip: Remember, the final 5 minutes of each 30-minute math lesson should be dedicated to mental math.

Introduction to Area

(instructions to the parent)

Objects used: sheets of graph paper with 1" grids, low-adhesive tape such as painter's tape or decorative tape, 12-inch ruler, and yardstick or measuring tape

Read through this section so you are familiar with it and feel free to put it into your own words. Spread this lesson out in as many daily lessons as necessary.

Tip: Linear measurement began in Book 3. If your student has never worked with measures of length, you may want to work the Measures of Length section on US standard measures found in the Weights and Measures section of that book, beginning on page 183.

Area refers to how large a surface is and may also be called *surface area.* We calculate area in our daily lives to find out how much paint is needed for a room, how much flooring to install, how much grass seed will cover a lawn, or whether a family (and its books) can comfortably fit inside a new home. Other practical applications for area are found in gardening, farming, architecture, interior design, and science, just to name a few. In this section, your student will learn to calculate the area of squares and rectangles. He will also learn the notation of square units. It is important that the student gets in the habit of always specifying the unit of measurement used, in both his oral and written work.

1. Ask: You've previously learned how to tell how long and how wide something is by measuring it with a ruler. What if we want to tell how big the *surface* of something is altogether, like a wall or a floor?

2. Show a sheet of graph paper with 1" grids and say: Since this sheet of graph paper is divided up into squares, we can tell how many *squares big* it is by counting them.

   Have your student count all the whole squares. (This should equal 70 if using the sheet included in Kit 4.)

3. Next, have him draw on the gridded paper the front of a book cover that is 5 squares long by 4 squares wide. Ask, "How big is it?" (20 squares or 20 squares big)

4. Now ask him to draw the front of a pencil case that is 10 squares long and 3 squares wide, then ask, "How big is it?" (30 squares big)

   This idea might have already struck your student, but you might say: It takes awhile to count all the squares. Can you think of a faster way to determine how many squares there are?

   Be sure to allow your student plenty of time in this step to observe the drawings and consider for himself.

   Explain: In the first figure, we have 5 rows, each containing 4 squares; that is, it is 5 squares long by 4 squares wide, or 5 squares × 4 squares make 20 squares. In the second figure, we have 10 rows of 3 squares; that is, the front of the pencil case is 10 squares long by 3 squares wide, or 10 × 3 to make 30 squares.

5. Next, have your student measure the length and width of just one of the grid squares using a standard ruler, saying something like:

   Since each of the squares is one inch long and one inch wide, we call it a *square inch* and give our measurements in square inches. If each square was 1 foot long by 1 foot wide, we would call it a *square foot.* Square feet are used to measure larger things, like a room or house. When we measure in squares, we call what we are measuring the *area.*

   If you're using graph paper or a gridded dry erase board with squares of a different size, adjust the lesson to reflect it. Have your student measure and note the size of the grids and how many grid squares make up a square inch.

6. The student will now perform a number of exercises. Have him draw the following figures on square-inch grid paper, then find the area of each by means of multiplication. The student need draw these only until he can either visualize them in his mind or perform the calculations without having to draw each one.

If using graph paper with smaller grids, make sure your student has determined how many squares make a square inch in order to calculate his figure correctly in inches.

Draw:

1. A rectangle 6 inches long and 4 inches wide

   area = 6 rows of 4 squares each
   = 6 × 4 squares
   = 24 squares or 24 square inches

2. 5 in. × 3 in.
3. 4 in. × 2 in.
4. 7 in. × 5 in.
5. 3 in. × 9 in.
6. 9 in. × 2 in.
7. 8 in. × 3 in.
8. 7 in. × 6 in.
9. 5 in. × 4 in.
10. 6 in. × 5 in.

Answers

1. 24 square inches
2. 15 square inches
3. 8 square inches
4. 35 square inches
5. 27 square inches
6. 18 square inches
7. 24 square inches
8. 42 square inches
9. 20 square inches
10. 30 square inches

7. Explain that we measure the surface of small things, like the top of a table or nightstand, in square inches and larger things, like a room or house, in square feet. Flooring—like carpet and tile—is sold in square yards, and property is measured in acres. Large territory, like a town or county, is measured in square miles.

   We must have a way to notate these units of measurement on paper so they're not confused with other units of measurement.

   Write the following terms and abbreviations on the slate as you explain:

   square inches, which can be abbreviated as sq. in.

   square feet, which can be abbreviated as sq. ft.

square yards, which can be abbreviated as sq. yd.

square miles, which can be abbreviated as sq. mi.

8. Next, the student should make a *square foot* on the floor using a ruler and painter's tape to mark out all four sides.

   Ask: If a square foot is 12 inches long by 12 inches wide, how many square inches are in a square foot? (12 × 12 = 144 square inches)

9. Now have him make a *square yard* on the floor with a yardstick or measuring tape and painter's tape to mark out all four sides. Then have him divide the square yard into square feet using the painter's tape.

   Ask: How many feet long and how many feet wide is a square yard? (3 feet by 3 feet)

   If a square yard is 3 feet long by 3 feet wide, how many square feet are in a square yard? (3 x 3 = 9 square feet)

10. Ask: Can you come up with a rule for finding the area of a rectangle or square if we know the lengths of its sides? (The answer should be something like: *To find the area, multiply its length by its width.*)

11. Your student will now draw up a table of Measures of Area in the back of his notebook. You may use the one below as a guide. Leave some space below it for any additions made at a later time. He should also write the rule he gave in Step #10 for finding the area of a rectangle or square.

    Be sure to have your student put this table on a fresh page as it will be added to later in the book.

    Measures of Area

    1 square inch

    1 square foot = 144 square inches

    1 square yard = 9 square feet

Tip: Your student will have more work with square measures (and metric square measures) in the Weights and Measures section later in this book.

12. Once he has completed writing the table, give your student some time to study it, say it through a few times, and try to visualize it in his mind. He may refer to the table as necessary.

13. Now, have your student find the area of a number of things around the house in square inches, square feet, or square yards, depending on the size, e.g., various table tops, a door, window, desk, picture, map, blackboard, etc. Measure to the closest foot, explaining that we will round the measurement—that is, if the length or width is so many feet plus less than 6 inches over, we won't count those extra inches; but if there are 6 or more inches over, we will count them as another foot.

    If the student wants to figure area more precisely by using fractions of inches, allow him to think it through and do so.

Additional Exercises Using Area

These exercises may be worked either orally, in writing, or a mix of both.

1. A piece of paper has a length of 8 inches and a width of 5 inches. What is its area in square inches?

2. A backyard has a length of 50 feet and a width of 40 feet. What is its area in square feet?

3. A deck has a width of 7 feet and a length of 9 feet. What is the area of the deck in square feet?

Tip: If the student is working with ease, you may use 4–10 for independent work.

4. A book has a length of 9 inches and a width of 6 inches. What is the area of the book in square inches?

Decimals

Your student has been introduced to the idea of fractions as part of a whole and the idea of a fraction as a division of a quantity in his previous work with fractions of halves, thirds, fourths, and fifths. As he meets with the fractions $\frac{1}{10}$ and $\frac{1}{100}$ these ideas are further advanced, leading into a formal introduction to decimals, along with their numeration and notation. He has already had life experience with decimals due to our decimalized monetary system. The use of hands-on and written work with money in this and the previous books has also provided him with much practical experience with decimals.

Decimals will be explored more fully in Book 5.

Understanding the Fractions $\frac{1}{10}$ *and* $\frac{1}{100}$

(instructions to the parent)

Objects used: 100 pennies, 10 dimes, 4 quarters, and a one-dollar bill as necessary

If at any time this process is unclear, have your student physically exchange a one-dollar bill for coins, divide the coins into equal parts (or piles), and look at one of these parts.

1. Ask: If I have $1.00 and give away half of it, how much would that be? (50 cents)

   Ask: What do we have to do to give away one-half of anything? (Divide it into 2 equal parts, and give 1 part away.)

   How many halves are in a whole dollar? (2 halves)

   Ask: How many quarters are in a dollar? (4 quarters)

   How would I give away one-quarter of a dollar bill? (Divide it into 4 equal parts, or four quarters, and give one part, or one-quarter, away)

2. Ask: If I have $1.00 and want to give away *one-tenth* of it, what would I do? (Divide it into 10 equal parts, and give one part or one-tenth away.)

   Ask: How much is one-tenth of a dollar? (10 cents or one dime)

   How many dimes make one dollar? (10 dimes)

   Ask: How many pennies make one dollar? (100 pennies)

   If I were to give away one-hundredth of a dollar, what must I do? (Divide it into 100 equal parts, and give one part, or one-hundredth, away.)

   What is one-hundredth of a dollar? (1 cent or 1 penny)

   Which is larger, one-tenth or one-hundredth of a dollar? (one-tenth)

3. Review while writing on the slate: One-half may be written ½ or $\frac{1}{2}$ and means 1 of 2 equal parts.

   One-fourth may be written ¼ or $\frac{1}{4}$ and means 1 of 4 equal parts.

4. Allow your student to write on the slate while asking: How would you write one-tenth? (1/10 or $\frac{1}{10}$) What does one-tenth mean? (1 of 10 equal parts)

   How would you write one-hundredth? (1/100 or $\frac{1}{100}$) What does one-hundredth mean? (1 of 100 equal parts)

   If your student cannot come up with the notation on his own after giving it some thought, then lead him to it by asking him to write the fractions he's already worked with: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{10}$ and then asking again how to write $\frac{1}{100}$.

   Explain that while both formats are correct, he will be writing fractions with a straight horizontal line between the numbers for ease in working with them as he continues his study of math.

5. Write on the slate: $\frac{1}{10}$ of 100. Ask: One-tenth of 100 equals how many? (10)

   If you divide 100 by 10, do you get the same answer? (yes)

   Is $\frac{1}{10}$ of 100 the same as 100 ÷ 10? (yes)

   Write on the slate: $\frac{1}{100}$ of 100.

   Ask: How much is $\frac{1}{100}$ of 100? (1)

   Is $\frac{1}{100}$ of 100 the same as 100 ÷ 100? (yes)

Oral Exercises

Now that your student has articulated the correlation between these fractions and division, you will give him a number of small problems to work orally.

1. What is $\frac{1}{10}$ of 20?
2. What is $\frac{1}{10}$ of 50?
3. What is $\frac{1}{100}$ of 300?
4. What is $\frac{1}{10}$ of 110?
5. What is $\frac{1}{100}$ of 1,200?
6. What is $\frac{1}{10}$ of 500?
7. What is $\frac{1}{100}$ of 500?
8. Which is larger, $\frac{1}{10}$ or $\frac{1}{100}$ of 500?
9. What is $\frac{1}{10}$ of 10?
10. 1 is $\frac{1}{10}$ of what number?

Answers

1. 2
2. 5
3. 3
4. 11
5. 12
6. 50
7. 5
8. $\frac{1}{10}$ is larger
9. 1
10. 10

Written Exercises

These may be assigned as independent work, with your student working as many as he is able in a given amount of time. In the next lesson, pick up wherever he left off.

1. What is $\frac{1}{10}$ of 1,000?
2. What is $\frac{1}{100}$ of 1,000?
3. Which is larger, $\frac{1}{10}$ or $\frac{1}{100}$ of 1,000?
4. What is $\frac{1}{100}$ of 700?
5. What is $\frac{1}{10}$ of 450?
6. What is $\frac{1}{10}$ of 600?
7. What is $\frac{1}{10}$ of 80?
8. 8 is $\frac{1}{10}$ of what number?
9. What is $\frac{1}{100}$ of 900?
10. 9 is $\frac{1}{100}$ of what number?

Answers

1. 100
2. 10
3. $\frac{1}{10}$

Answers to 4–10 are on page 318.

Weights and Measures

A formal introduction to US standard weights and measures was given in Book 3 of The Charlotte Mason Elementary Arithmetic Series, including hands-on exploration of measures of time in hours, days, months, and a year; measures of length in feet, inches, and yards; measures of weight in ounces and pounds; of liquid measures; and of dry measures.

We continue our investigation here using smaller and more precise measures in all these areas, as well as adding larger measures to the tables of weights and measures previously drawn up. Roman numerals are introduced; and metric measures are presented, compared to US standard measures, and worked with for a significant amount of time so that your student can begin to feel at home with them. He will receive a formal introduction to shapes in this section in preparation for cubic measure and volume. Shapes and forms will be explored further in the study of geometry to come.

The student will also explore conversion between units of measurement and resume and extend his work with fractions in an experiential hands-on way in this section.

Measures of Time

(instructions to the parent)

Objects used: analog clock, clock, or watch with a second hand, or stopwatch

Your student should have previously explored the relationship between minutes, hours, days, months, and a year in order to continue the more advanced work in this section.

1. Add interest to the lesson with a short history, saying something like:

   Have you ever heard something extremely small described as being *minute* [mī-nüt], such as a *minute detail* in a painting? Notice how similar it is to the word *minute* [mi-nət], a 60th part of an hour of time—what we mean when we say, "Lunch will be ready in a minute." Originating from the Latin *pars minuta prima*, this little unit of one sixtieth of an hour literally translates as "first very small part." Now, a *minute* [mi-nət] is, of course, a *minute* [mī-nüt] amount of time to wait for lunch, unless one is particularly hungry, but there is an even smaller unit of time we call a *second*. The *second* is a sixtieth part of a minute, that is, a minute divided into 60 parts, and its name comes from the Latin *pars minuta secunda*, which translates as "second very small part."

   If you have a clock or a watch that ticks off the seconds as well as the minutes, you may see just how much time it measures. Some children are taught that the amount of time to say a number with the word *Mississippi* added to it closely resembles a *second,* so they sometimes use it when counting for games like hide-and-seek. You can see for yourself how accurate it is by counting "one-Mississippi, two-Mississippi, three-Mississippi" and on, with hardly a breath in between, while watching the second hand of a clock or stopwatch.

   Historical records tell us that we have the Babylonians to thank in large part for our 60 seconds in a minute and 60 minutes in an hour. When these ancient astronomers began a quest to more