11 Inequalities

Basic Practice

1. In each of the following, determine whether the specified value of x is a solution of the given inequality.

(a) $x > 10$; $x = 19$

(b) $x < 5$; $x = 5$

(c) $x < -12$; $x = -2$

(d) $x > -23$; $x = -6$

(e) $x \geqslant 34$; $x = 34$

(f) $4x \leqslant 20$; $x = -1$

(g) $5x > 7$; $x = 1.2$

(h) $3x \geqslant -2$; $x = -\frac{1}{2}$

(i) $\frac{x}{2} \geqslant -5$; $x = -11$

(j) $\frac{3}{5}x < 9$; $x = -\frac{3}{5}$

2. Solve the following inequalities.

(a) $2x > 12$

(b) $4x > 32$

(c) $3x < -18$

(d) $5x < 22.5$

(e) $6x \geqslant 27$

(f) $8x \geqslant -30$

(g) $16x \leqslant 36$

(h) $24x \leqslant -64$

3. **(a)** List all the positive even integers that are smaller than or equal to 20.

(b) Find all possible values of x in each of the following inequalities if x is a positive even integer that is smaller than or equal to 20.

(i) $x < 10$

(ii) $x > 12$

(iii) $x \leqslant 9$

(iv) $x \geqslant 14$

(v) $2x < 8$

(vi) $3x > 51$

(vii) $5x \leqslant 21$

(viii) $4x \geqslant 71$

4. **(a)** List all the prime numbers that are smaller than or equal to 30.

(b) Find all possible values of x in each of the following inequalities if x is a prime number that is smaller than 30.

(i) $x < 15$

(ii) $x > 23$

(iii) $x \leqslant 17$

(iv) $x \geqslant 22$

(v) $4x < 28$

(vi) $5x > 45$

(vii) $2x \leqslant 7$

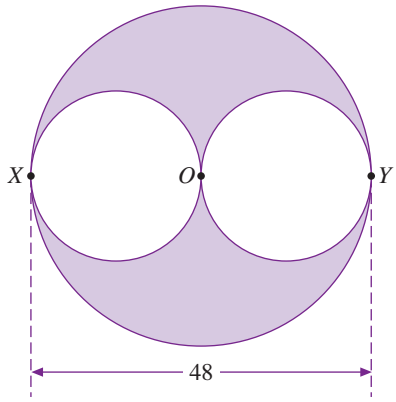
(viii) $3x \geqslant 43$

Further Practice

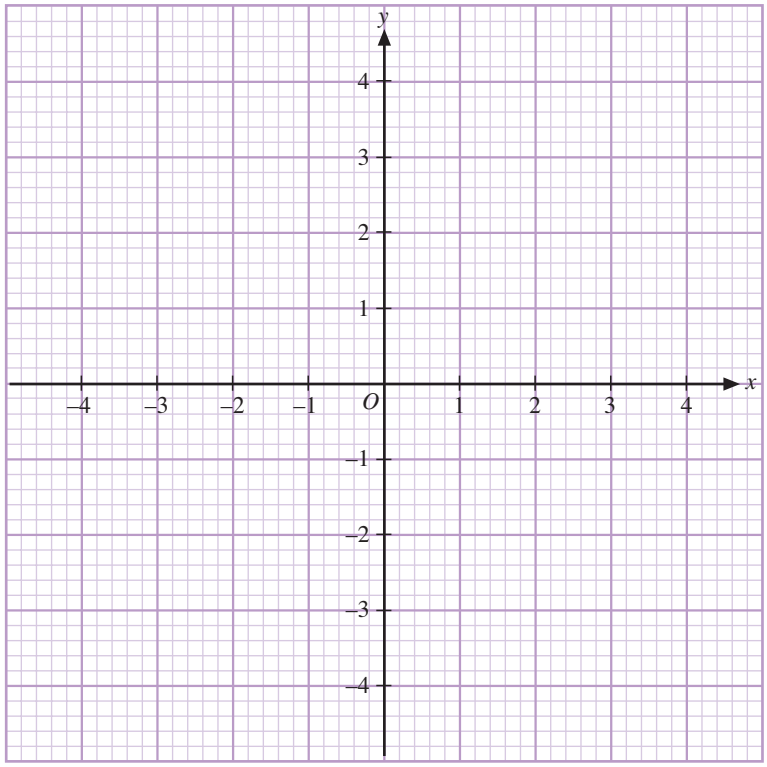
11. **(a)** A wire is bent into a square of area 81 cm^2. Find
(i) the length of a side of the square, **(ii)** the perimeter of the square.
(b) Suppose the same wire is bent into an equilateral triangle. Find the length of a side of the equilateral triangle.

12. **(a)** The length and width of rectangle $ABCD$ are 25 cm and 32 cm respectively. Find
(i) the perimeter of the rectangle, **(ii)** the area of the rectangle.
(b) A square is formed when the sides of $ABCD$ are extended. If the length of $ABCD$ is extended by 60%, find
(i) the length of the square,
(ii) the percentage increase in the width of the rectangle.
(c) Express the area of the square as a percentage of the area of the rectangle.

13. Two small circles are cut out from a large circle of diameter 48 cm. The point O is the center of the large circle and the diameters of the small circles are OX and OY respectively. The points X and Y are on the circumference of the large circle.
Find, in terms of π,
(a) the area of the resulting plane figure,
(b) the perimeter of the resulting plane figure.

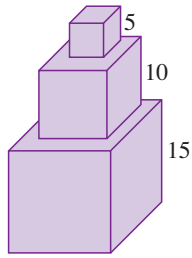


14. **(a)** Plot each of the following points in the given diagram.
(i) $A(-3, -4)$, $B(2, -4)$, and $C(4, 1)$
(ii) $P(-4, 1)$, $Q(-4, -2)$, and $R(2, 3)$
(b) Hence, find the area of
(i) $\triangle ABC$,
(ii) $\triangle PQR$.
(c) The points D and S lie on the x-axis and y-axis respectively. Find the area of
(i) $\triangle ABD$,
(ii) $\triangle PQS$.



Enrichment

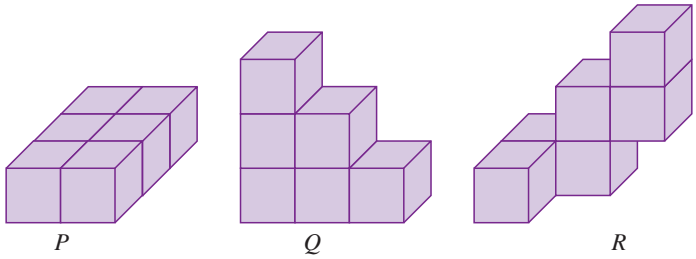
26. In the diagram, three cubical building blocks are stacked up on a table. The lengths of the sides of the blocks are 5 cm, 10 cm, and 15 cm respectively.



(a) Find the total area of the exposed surfaces of the stack, excluding the contact surface with the table.

(b) If a cylinder of height 30 cm has volume equal to the total volume of the blocks, find the base radius of the cylinder.

27. Six cubes of side 1 cm are glued together to form a solid. Three possible solids P, Q, and R are shown below.



(a) Determine the total surface area of solid

(i) P,

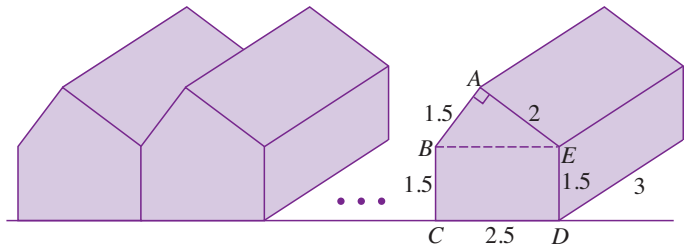
(ii) Q,

(iii) R.

(b) Form a solid with the least total surface area.

(c) Form a solid with the greatest total surface area.

28.



A developer builds a row of identical semi-detached huts along a beach as shown in the diagram above. $ABCDE$ is the cross-section of a hut. $\triangle ABE$ is a right-angled triangle with $AB = 1.5$ m, $AE = 2$ m, and $m\angle BAE = 90°$. $BCDE$ is a rectangle with $CD = 2.5$ m and $BC = 1.5$ m. The length of each hut is 3 m. The thickness of each side wall is 30 cm.

(a) Find the total surface area of each hut, excluding the floor.

(b) Find the volume of space of each hut (ignore the thickness of the walls).

(c) If n huts are in a row, find, in terms of n,

(i) the total roof area,

(ii) the total volume of the side walls.

Challenging Practice

30. Let ξ be the set of employees in a company,

P = {employees in the company who earn more than \$2,000 a month}

and Q = {employees in the company who earn at least \$3,000 a month}.

(a) Describe the sets P' and Q'.

(b) Describe using '$\subset$', the relationship between

(i) P and Q,

(ii) P' and Q'.

31. The frequency table below shows the number of dogs owned by a group of children.

Number of dogs	0	1	2	3	4	5
Number of children	7	10	5	5	x	1

(a) The mean number of dogs owned by each child is 1.6. Form an equation in x and solve it.

(b) Hence, find the number of children in the group.

(c) A child is randomly selected. Find the probability of selecting a child with more than 3 dogs.

(d) A dog is randomly selected. Find the probability of selecting a dog that belongs to a child who has at most 3 dogs.

32. A box contains 200 buttons that are either blue or green. A button is randomly selected from the box.

(a) Find the number of each type of button if the probability of selecting a blue button is $\frac{11}{25}$.

(b) How many blue buttons must be removed from the 200 buttons so that the probability of selecting a green button will become $\frac{8}{13}$?

(c) How many blue buttons must be added to the 200 buttons so that the probability of selecting a green button will become $\frac{14}{27}$?

(d) When x blue buttons are added and x green buttons are removed from the 200 buttons, the probability of selecting either a blue or green button is the same. Find the value of x.

33. Jeffrey bought a grey (G), a red (R), a blue (B), and a yellow (Y) T-shirt. He also bought a blue (B), a white (W), and a grey (G) pair of jeans. Suppose that Jeffrey randomly matches a shirt with a pair of jeans.

(a) List all the possible ways of matching a shirt with a pair of jeans.

(b) Find the probability of Jeffrey wearing

(i) a yellow T-shirt,

(ii) a white pair of jeans.

(c) Let M be the event that Jeffrey matches a shirt with a pair of jeans of the same color. Find P(M) and P(M').