CHAPTER 1
NUMBER CONCEPTS

$\sqrt{7}$ π 2^3

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1.1 The Real Number System

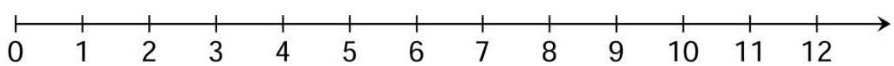
The system of **real numbers** consists of a collection of smaller sets of numbers that has evolved over several centuries. It began with numbers used to count objects, which were used for trading. It was then extended and refined as a need for numbers to represent parts of objects and locations on the number line became important. Below is a discussion of the sets of numbers that make up the real number system. Each of these sets builds on those contained in the preceding set.

Natural Numbers

Natural numbers can be thought of as counting numbers. They can be used to identify how many objects are in a collection. Since a collection of objects has at least one item, the natural numbers begin with the number 1 and then continue to represent additional objects in the set. Counting numbers can be listed as follows: 1, 2, 3, 4, 5, 6, 7, 8, …

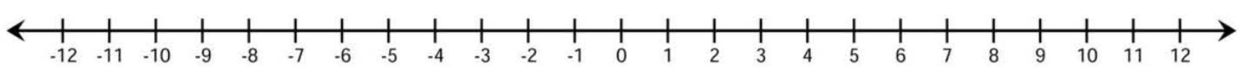
Whole Numbers

Whole numbers consist of the natural numbers with the addition of the number 0. Although 0 does not represent an object in a set, it is an important addition to the number system. Whole numbers can be listed as follows: 0, 1, 2, 3, 4, 5, 6, 7, 8, … They correspond to locations on the number line as follows:



Integers

The set of **integers** builds on the set of whole numbers by adding the negative values of each. It includes numbers such as -1, -2, -3, -4, -5, … (Note: There is no negative value for 0.) Negative values of whole numbers are used in many situations, such as to represent a minus temperature (-22° C), distance below sea level (-8 m below the sea), or a golf score that is under par (-4 strokes under par). Integers correspond to locations on the number line as follows.



Rational Numbers

The set of **rational numbers** builds on the set of integers by including parts of the counting objects discussed earlier (such as, one-half of an item or quantity, or one quarter of a degree in temperature).

A **rational number** is any number that can be written in the form $\frac{a}{b}$ where a and b are integers and b $\neq 0$. This includes the natural numbers (1, 2, 3, …), the whole numbers (0, 1, 2, 3, …), and the integers (...-3, –2, -1, 0, 1, 2, 3 …). Natural, whole, and integer numbers are rational since each can be written in the form $\frac{a}{b}$ ($1 = \frac{2}{2}$, $-3 = -\frac{3}{1}$, $0 = \frac{0}{4}$).

The set of rational numbers begins to fill in many locations on the number line, but there still remain locations that do not have corresponding numbers associated with them.

Examples of Rational Numbers:

All fractions and mixed numbers, both positive and negative

$\frac{2}{3}$, $-\frac{3}{4}$, $\frac{5}{2}$, $-3\frac{1}{4}$ Note: $\frac{0}{7} = 0$ is rational but $\frac{7}{0}$ is **not rational** since it is not defined when the denominator equals 0.

All integers

(-11, -3, 0, 1, 5, 68)

All terminating and repeating decimals, both positive and negative

0.8, -0.32, $0.\overline{3}$, $7.\overline{12}$

Irrational Numbers

To complete our system of real numbers, it is necessary to add an additional set. These additional numbers are called **irrational numbers**. Irrational numbers are any numbers that cannot be written as the quotient of two integers $\frac{a}{b}$, where b $\neq 0$.

Examples of Irrational Numbers:

Numbers that are roots of whole numbers that cannot be simplified to obtain a rational number

$\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$ (Note: $\sqrt{9}$ is rational since it is equal to 3.)

Numbers whose decimal representation does not repeat in a pattern

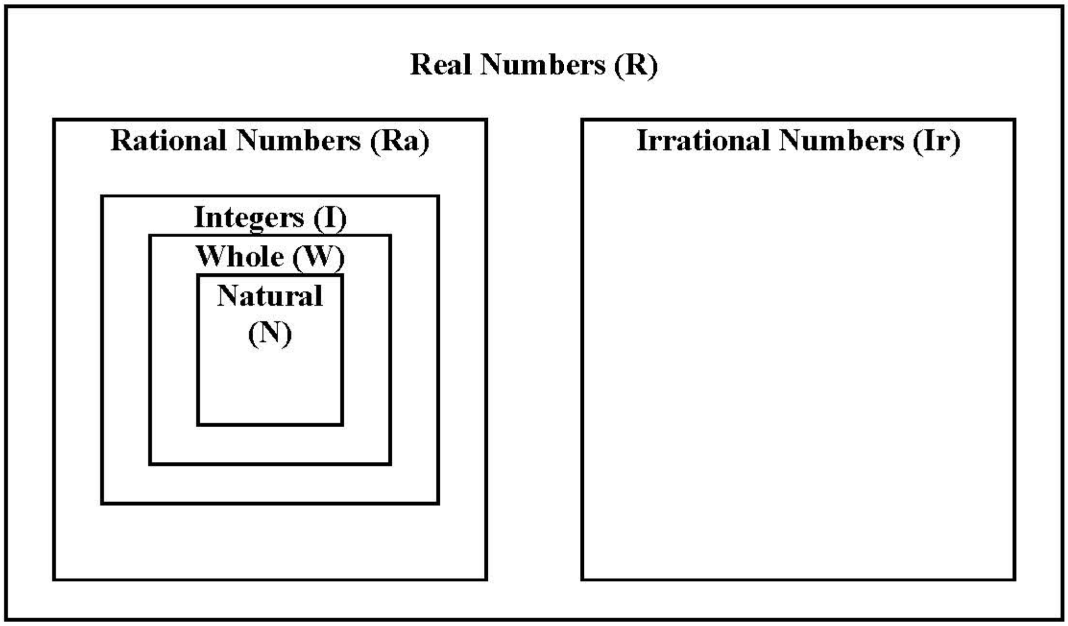
0.1357421… (Note: $0.33\overline{3}$ is rational since it repeats a pattern and is equal to $\frac{1}{3}$.)

Special numbers such as π (which is equal to 3.1415926…)

Real Numbers

The set of **real numbers** consists of all rational and irrational numbers. All locations on a number line correspond to a real number. We can think of the set of real numbers as filling all locations on the number line.

The diagram below shows the relationship between the sets of numbers discussed so far.



Note:

As shown in the above diagram, the set of rational numbers includes the following.

Natural Numbers: N = $\{1,2,3,4,...\}$

Whole Numbers: W = $\{0,1,2,3,4,...\}$

Integers: I = $\{...-3,-2,-1,0,1,2,3,4,...\}$

Rational: Ra = All of the above plus any other number that can be written in the form $\frac{a}{b}$, b $\neq 0$

All rational and all irrational numbers make up the set of real numbers.

Identification of Rational and Irrational Numbers

Rational numbers can be shown in several different formats, as long as they can be rewritten in the form $\frac{a}{b}$, b ≠ 0.

1. Natural numbers, whole numbers, and integers
   Examples: 7, -43, 0, 2761, - 403
2. Proper fractions, mixed numbers, or improper fractions
   Examples: $\frac{3}{11}$, $-\frac{2}{9}$, $3\frac{1}{4}$, $\frac{7}{5}$, $-8\frac{1}{10}$, $-\frac{7}{3}$
3. Decimals (terminating or repeating)
   Examples: 0.8, -0.25, $0.22\overline{3}$, $2.\overline{61}$

Irrational numbers cannot be shown as common fractions.

1. Decimals that do not terminate or repeat in a pattern (0.12323569…)
2. Roots of numbers that are not rational ($\sqrt{2}, \sqrt{11}, -3\sqrt{5}$, …
3. Special numbers like π

Examples with Solutions

Identify which of the following are rational and which are irrational numbers. Give a reason for your answer.

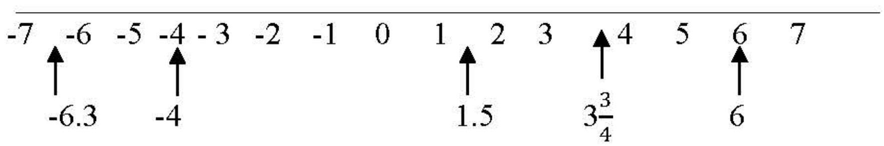
	Rational or Irrational?	Reason
1. -1.25	Rational	It can be written as $-\frac{125}{100}$ or $-\frac{5}{4}$
2. 0.6010347…	Irrational	The decimal doesn't terminate or repeat the same pattern.
3. $-\sqrt{25}$	Rational	It can be written as -5.
4. 0.1111 …	Rational	It repeats the same pattern and can be written as $\frac{1}{9}$.
5. $\sqrt{13}$	Irrational	The decimal version doesn't repeat the same pattern $\sqrt{13}$ = 3.6055513…
6. 4.01	Rational	It has a terminating decimal. It could be written as $4\frac{1}{100}$ or $\frac{401}{100}$.
7. $-7\frac{1}{2}$	Rational	It could be written as $\frac{-15}{2}$.
8. $2125\frac{1}{4}$	Rational	It could be written as $\frac{8501}{4}$.

9. 2.333… Rational The decimal repeats the same pattern and is equal to $2\frac{1}{3}$ or $\frac{7}{3}$.

10. 5.0100382… Irrational The decimal doesn't terminate or repeat the same pattern.

Comparing and Ordering Rational Numbers

Each rational number corresponds to a point on the number line. Below are several examples.



Numbers **increase in** size as you go from left to right on the line.

$1 < 3$; $2.1 < 4$; $-7 < -6$; and $2 > 1.8$; $-3 > -5$; $-1 > -10.5$

Examples: To compare the size of rational numbers when one is written in decimal and the other in common fraction form, write both either in decimal or in common fraction form and then compare.

1. Compare 0.1 with $\frac{3}{20}$ (convert both to fractions first).

   Change 0.1 to $\frac{1}{10}$.

   The common denominator is 20, $\therefore \frac{1}{10} = \frac{2}{20}$.

   $\frac{2}{20} < \frac{3}{20}$ or $0.1 < \frac{3}{20}$

2. Compare 3.15 with $3\frac{1}{11}$ (convert both to decimals first).

   Change $3\frac{1}{11}$ to a decimal $\rightarrow 3.\overline{09}$.

   $3.15 > 3.\overline{09}$ or $3.15 > 3\frac{1}{11}$

Exercises 1.1

Identify which of the following are rational and which are irrational numbers. Give a reason for your answer.

	Rational or Irrational	Reason
1. 0.013		
2. $5\frac{1}{2}$		
3. 7.0900134...		
4. 0.666…		
5. -10.001		
6. $\sqrt{49}$		
7. 0.122357…		
8. 0.212121…		
9. 210.013		
10. $\sqrt{8}$		
11. -5.999…		
12. 3.009		
13. $-345\frac{1}{3}$		

Use a check mark to indicate the set(s) to which each number belongs.

Set of Numbers

		N	W	I	Ra	Ir
14.	0.7					
15.	-45					
16.	$\frac{15}{7}$					
17.	0.13243…					

		N	W	I	Ra	Ir
18.	$0.\overline{7}$					
19.	2					
20.	$1\frac{5}{8}$					
21.	-160					
22.	0					
23.	$\sqrt{81}$					
24.	0.93					
25.	$\sqrt{15}$					

Note: N = Natural Numbers, W = Whole Numbers, I = Integers, Ra = Rational Numbers, Ir = Irrational Numbers

26. Locate the following numbers on the number line: 3.1, $2\frac{5}{8}$, $-\frac{13}{12}$, $-\sqrt{6}$, $-\sqrt{16}$

-4 -3 -2 -1 0 1 2 3 4

27. Arrange the following numbers from smallest to largest.

a. -0.57, -0.507, -5.07, -5.70

b. 3.4, $-\frac{11}{3}$, -3.4, -3.5

c. $-\frac{3}{8}$, $-\frac{2}{3}$, - 0.6, -0.4

28. Put the correct symbol (>, =, <) between each pair of numbers.

a. 0.15 ☐ $\frac{7}{40}$

b. -1.8 ☐ $-\frac{9}{5}$

c. -2.8 ☐ $-\frac{13}{5}$

29. Express each term in common fraction form (as a quotient of two integers).

a. 0.17

b. $-0.\overline{5}$

c. $-1\frac{2}{3}$

d. 3.07

30. Which rational number is greater?

a. $-0.\overline{6}$ or -0.6?

b. -0.25 or $-\frac{1}{3}$?

c. $-\frac{2}{3}$ or $-\frac{4}{5}$?

Extra for Experts

31. List the set of all integers greater than -4 and less than $\frac{1}{2}$.

32. Is $\sqrt{\frac{4}{9}}$ rational or irrational? Give a reason for your answer.

33. Is the sum of the following numbers rational or irrational? Give a reason for your answer.
0.1 + 0.01 + 0.001

34. Is the sum of the following numbers rational or irrational? Give a reason for your answer.
0.333… + 0.666… + 0.999…

35. If the sum of 3.82 + 12<u>ab</u> is an integer, what digits must go in place of *ab*?

36. If the sum of -12 + $-8\frac{1}{2}$ + *n* is a natural number, what is the smallest number that can replace *n*?

ANSWERS TO EXERCISES AND CHAPTER TESTS

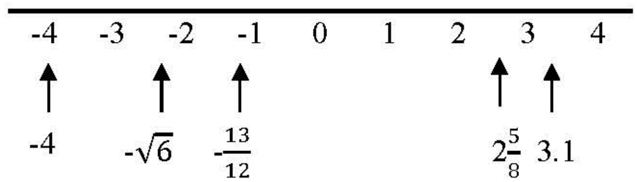
CHAPTER 1

Exercises 1.1 (page 7)

	Rational/ Irrational	Reason
1. 0.013	Rational	Can be written as 13/1000
2. $5\frac{1}{2}$	Rational	Can be written as 11/2
3. 7.0900134...	Irrational	Decimal doesn't terminate or repeat
4. 0.666….	Rational	Repeating decimal equal to $\frac{2}{3}$
5. -10.001	Rational	Can be written as $-\frac{10\,001}{1000}$
6. $\sqrt{49}$	Rational	Equal to 7
7. 0.12231353…	Irrational	Decimal doesn't terminate or repeat
8. 0.212121…	Rational	Repeating decimal equal to $\frac{21}{99}$
9. 210.013	Rational	Terminating decimal
10. $\sqrt{8}$	Irrational	Decimal value doesn't terminate or repeat 2.8284271…
11. -5.333…	Rational	Decimal repeats and can be written as $-\frac{16}{3}$
12. 3.009	Rational	Decimal terminates and can be written as 3009/1000
13. $-345\frac{1}{3}$	Rational	Can be written as $-\frac{1036}{3}$

	Number	Set of Numbers N	W	I	Ra	Ir
14.	0.7				√	
15.	-45			√	√	
16.	$\frac{15}{7}$				√	
17.	0.13243…					√
18.	$0.\overline{7}$				√	
19.	2	√	√	√	√	
20.	$1\frac{5}{8}$				√	
21.	-160			√	√	
22.	0		√	√	√	
23.	$\sqrt{81}$	√	√	√	√	
24.	0.93				√	
25.	$\sqrt{15}$					√

26.



27. a) -5.70, -5.07, -0.57, -0.507
b) $-\frac{11}{3}$, -3.5, -3.4, 3.4 **c)** $-\frac{2}{3}$, -0.6, -0.4, $-\frac{3}{8}$
28. a) < **b)** = **c)** < **29. a)** $\frac{17}{100}$ **b)** $-\frac{5}{9}$ **c)** $-\frac{5}{3}$
d) $3\frac{7}{100} = \frac{307}{100}$ **30. a)** -0.6 (It is to the right of -$0.\overline{6}$ on the number line.) **b)** -0.25 **c)** $-\frac{2}{3}$ (change to $-\frac{10}{15}$ and $-\frac{12}{15}$) **31.** -3, -2, -1, 0
32. Rational; can be written as $\frac{2}{3}$
33. Rational; can be written as $\frac{111}{1000}$
34. Rational; can be written as 2 **35.** 18
36. $21\frac{1}{2}$

Exercises 1.2 (page 15)

1. a) 3 **b)** 15 **c)** $\frac{5}{7}$ **d)** $\frac{9}{4}$ **e)** 0.9 **f)** 0.02
2. a) 2.6 **b)** 5.3 **c)** 8.4 **d)** 14.1 **3. a)** $x = \pm 6$
b) $x = \pm 7.1$ **c)** $x = \pm 0.9$ **d)** $x = \pm 0.25$
e) $x = \pm 200$ **4.** B, D, E **5. a)** ±9 **b)** ±7
c) ±0.3 **6. a)** 2.75 **b)** 11.38 **c)** 0.55
7. a) ±10.25 **b)** ±6.18 **c)** ±5.91 **d)** ±6.90
8. 7.42 cm **9. a)** 4.53 cm **b)** 0.57 m
10. 40.16 m **11. a)** 10.83 cm^2 **b.** 7.35 cm^2
c) 9.07 cm **12. a)** 942 cm^3 **b)** 1.82 cm

Exercises 1.3 (page 23)

1. a) 8 **b)** 9 **c)** 64 **d)** 81 **e)** 25 **f)** 32 **g)** 216
h) 729 **i)** 125 **j)** 243 **2. a)** 25 **b)** 49 **c)** 16