Geometry

Fifth Edition

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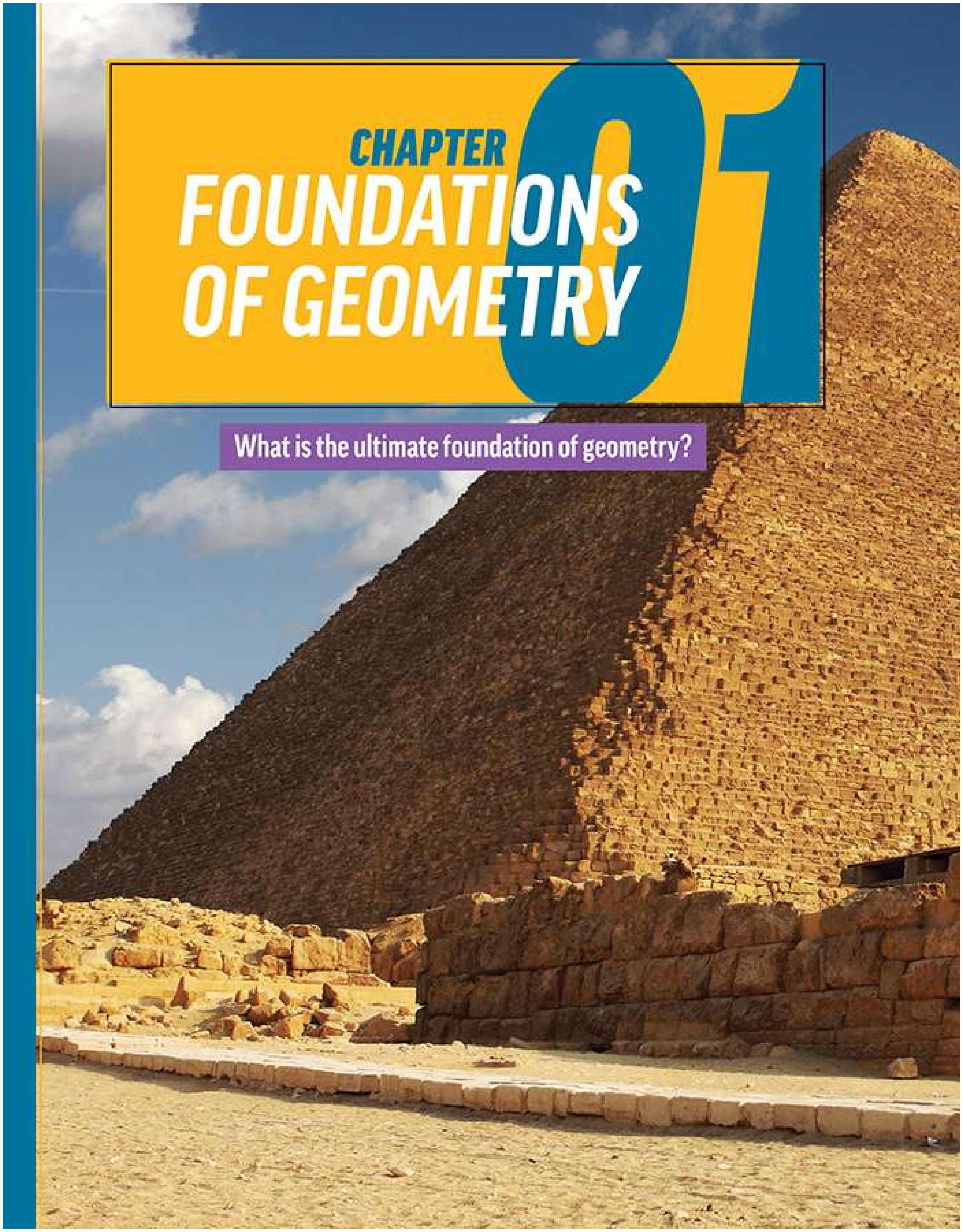
Greenville, South Carolina

CHAPTER 01

FOUNDATIONS OF GEOMETRY

What is the ultimate foundation of geometry?



How would you like the job of designing the Great Pyramid? The construction of the Pyramid of Khufu, the Great Pyramid of Giza, was one of the greatest architectural feats of history. Modern historians estimate that it took over 20,000 men 30 years to arrange the 2,300,000 blocks of stone averaging 2.5 tons each.

The Egyptians had to be experts in geometry. The base of the 484.4 feet high pyramid covers 13.1 acres. The difference between the longest and shortest sides is only 7.9 inches. The sides align with the true compass points, forming almost perfect right angles. Each exterior face rises at an angle of 51°52′, and the gradients of inside corridors are almost identical.

Any structure capable of standing the test of time must be built on a firm foundation. Concrete, steel, and wood may not appear to be related until they are brought together to form a building. This first chapter lays the foundation for your study of geometry. You'll learn about points, lines, planes, angles, and other "building blocks" that are used in later chapters to build a more sophisticated structure.

CONTENTS

1.1 SETS

How can we show relationships between groups of people?

I will be able to

- describe relationships between sets, subsets, and their elements.
- perform set operations.
- represent set relationships and operations with Venn diagrams.

How do sets relate to geometry? Sets help us visualize, explore, and analyze mathematical relationships between different groups of objects. Set theory unifies the many branches of mathematics.

A **set** is any collection of objects. Sets are typically named with a capital letter and designated using braces, { }. Objects in a set are called **elements** or **members** of the set.

Sets are described by listing the elements or by stating a rule. **Set builder notation** is often used to state the rule describing the elements in a set. For example, $C = \{x \mid x$ is a letter in the word *complement*$\}$ describes the set $C = \{c, o, m, p, l, e, n, t\}$. The x represents an arbitrary element of the set and $\mid$ is read "such that." Note that each member is listed only once and that, while the elements can be listed in any order, they are often listed in numerical or alphabetical order.

EXAMPLE 1: Describing Sets

Describe set P whose elements are cell phone accessories pictured above using both a list and set builder notation.

Answers

P = {AC adapter, cable, case, DC adapter, earbud, holder, smartwatch}

$P = \{x \mid x$ is one of the illustrated cell phone accessories$\}$

SKILL CHECK **EXERCISE 3** ✓

NUMBER SETS

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$
$\mathbb{W} = \{0, 1, 2, 3, 4, \ldots\}$
$\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \pm4, \ldots\}$

The symbol $\in$ means "is an element of." The fact that the smartwatch is an element of set P can be written as

smartwatch $\in P$.

A set containing no elements is called the **empty set** or the **null set** and is denoted by either { } or $\varnothing$. For example, $D = \{x \mid 2 < x < 3$, where $x \in \mathbb{Z}\} = \varnothing$.

The following table summarizes basic set relationships.

	Equal Sets =	Subset $\subseteq$	Proper Subset $\subset$
Definitions	The sets contain the same elements.	Every element in B is also in A.	B is a subset of A, but B is not equal to A.
Examples	If $A = \{1, 3, 5, 7, 9\}$, $B = \{3, 7\}$, and $C = \{x \mid 0 < x < 10$, where x is an odd integer$\}$		
	$A = C$	$B \subseteq A$ and $C \subseteq A$	$B \subset A$

The definitions imply that the empty set is a subset of every set and that every set is a subset (but not a proper subset) of itself.

Adding a slash to a symbol indicates that the relationship is not true.

$\neq$ not equal to $\nsubseteq$ not a subset of

$\notin$ not an element of $\not\subset$ not a proper subset of

EXAMPLE 2: Analyzing Statements Relating Sets

Determine whether each statement is true or false if $D = \{1, 3, 5, 7, 9\}$, $E = \{3, 7\}$, and $F = \{x \mid 0 < x < 10$, where x is an odd integer$\}$. Explain your reasoning.

a. $\{1\} \in D$ **b.** $D \not\subset F$ **c.** $E \subset F$ **d.** $E \subseteq \varnothing$

Answers

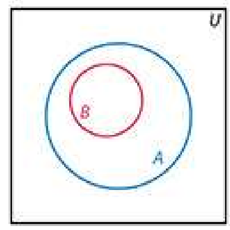
a. False; the set $\{1\}$ is not an element of D. Both $1 \in D$ and $\{1\} \subseteq D$ are true.

b. True; while D is a subset of F, it is not a proper subset because $D = F$.

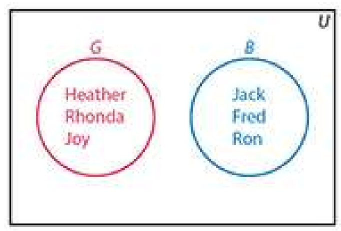
c. True; all the elements of E are in F and $E \neq F$.

d. False; neither element of E can be found in the empty set.

SKILL CHECK **EXERCISE 33**

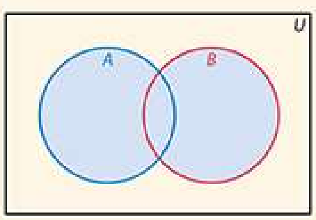
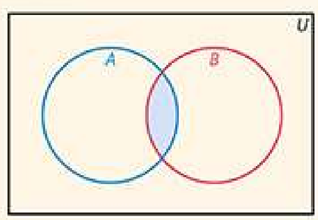
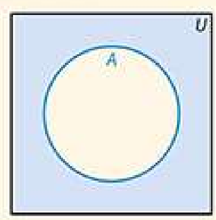


Venn diagrams are used to illustrate the relationships between sets and the set operations of *union*, *intersection*, and *complement*. The **universal set**, U, contains all the elements within the problem's context. In Example 2 the universal set might be the set of integers, $\mathbb{Z}$, or real numbers, $\mathbb{R}$. In geometry the universal set is often the set of points in a plane or in space. This text assumes that the universal set is not empty.



Equivalent sets are sets that have the same number of elements. Since the set of boys $B = \{$Jack, Fred, Ron$\}$ and the set of girls $G = \{$Heather, Rhonda, Joy$\}$ are equivalent, there is a **one-to-one correspondence** between them. This means each element of B can be paired with one and only one element of G. Even though the sets are equivalent, they clearly are not equal. However, if sets are equal, they are also equivalent.

Venn diagrams can be shaded to illustrate set operations.

UNION $A \cup B$	INTERSECTION $A \cap B$	COMPLEMENT A'
the set of all the elements in both sets	the set of all the elements common to both sets	the set of all the elements in the universal set that are not in A
		

EXAMPLE 3: Performing Set Operations

If $R = \{1, 2, 3, 4\}$, $S = \{1, 3, 5, 7\}$, and $U = \{x \mid x \text{ is a single-digit natural number}\}$, find the result of each set operation.

a. $R \cup S$ **b.** $R \cap S$ **c.** R'

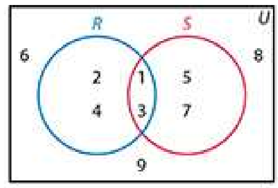
Answers

Placing the elements on a Venn diagram may help you visualize the relationships.

a. $R \cup S = \{1, 2, 3, 4, 5, 7\}$

b. $R \cap S = \{1, 3\}$

c. $R' = \{5, 6, 7, 8, 9\}$



SKILL CHECK **EXERCISE 25** ✓

AFTERSCHOOLHELP

Sets

Union and intersection are **binary operations** because each operation is applied to 2 sets, while complement is a **unary operation** because it is applied to just 1 set.

Sets whose intersection is the empty set are called **disjoint sets**. The sets of boys and girls illustrated earlier are disjoint because the sets have no common element.

When combining several set operations, be careful to use the correct order of operations. Complete operations within grouping symbols first and perform complements before unions and intersections.

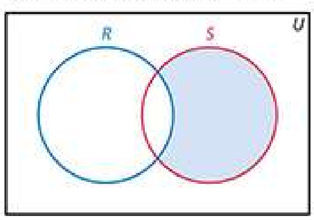
EXAMPLE 4: Performing & Illustrating Set Operations

Find each set and draw a Venn diagram illustrating the operations if $R = \{1, 2, 3, 4\}$, $S = \{1, 3, 5, 7\}$, and $U = \{x \mid x \text{ is a single-digit natural number}\}$.

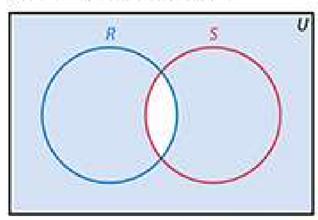
a. $R' \cap S$ **b.** $(R \cap S)'$

Answers

a. $\{5, 6, 7, 8, 9\} \cap \{1, 3, 5, 7\} = \{5, 7\}$



b. $\{1, 3\}' = \{2, 4, 5, 6, 7, 8, 9\}$



SKILL CHECK **EXERCISE 39** ✓

VOCABULARY

- binary operation
- complement
- disjoint sets
- element
- empty (null) set
- equal sets
- equivalent sets
- intersection
- member
- one-to-one correspondence
- proper subset
- set
- set builder notation
- subset
- unary operation
- union
- universal set

A. EXERCISES

Describe each set using a list of its elements.

1. $L = \{x \mid x \text{ is a letter in the word } \textit{grace}\}$

2. $C = \{y \mid y \text{ is a color in the US flag}\}$

Describe each set using set builder notation.

3. $A = \{2, 4, 6, 8\}$

4. $B = \{0, 1, 4, 9, 16, 25, \ldots\}$

State whether each pair of sets is *equal, equivalent,* or *neither.*

5. $A = \{\text{a, b, c}\}$, $B = \{1, 2, 3\}$

6. $K = \{5, 7, 8\}$, $M = \{1, 9, 2, 7\}$

7. $G = \{3, 1, 9\}$, $H = \{9, 3, 1\}$

8. $L = \{\text{man, son, brother}\}$, $N = \{\text{woman, daughter, sister}\}$

9. $Q = \{\text{Santa Fe, Boise}\}$,
$R = \{\text{New Mexico, Idaho, Virginia}\}$

Translate each sentence into symbolic notation.

10. Set A is a subset of set L.

11. b is an element of set K.

12. Set A is a proper subset of set B.

13. The set containing k, l, and m is not a subset of set T.

Translate each symbolic expression into a written sentence.

14. $3 \in A$

15. $\{\text{a, b, c}\} \not\subset D$

16. $(A \cup B)' \not\subset S$

17. $\varnothing \subset U$

18. Determine whether each statement is *true* or *false*.

a. $\varnothing \subseteq U$

b. $\varnothing \subset U$

c. $\varnothing \subseteq \{\ \}$

d. $\varnothing \in \{1, 2, 3, \ldots\}$

19. *True* or *false*: The empty set is a proper subset of every set.

B. EXERCISES

**Draw a Venn diagram illustrating the relationship of each pair of sets.
State a probable universal set in each case.**

20. $C = \{\text{Bible, English, math}\}$,
$D = \{\text{history, science}\}$

21. $L = \{\text{Kansas, Kentucky}\}$,
$M = \{\text{Alabama, Arizona, Arkansas, Kansas}\}$

22. $A = \{-1, 2, 4, 9\}$, $B = \{1, 2, 4, 6\}$

23. $M = \{2, 3, 4, 5, 6\}$, $P = \{3, 5\}$

24. Essential Question: How can we show relationships between groups of people?

Find each set if $A = \{1, 3, 5, 9\}$, $B = \{2, 3, 7, 9\}$, and $U = \{x \mid x \text{ is a natural number}\}$. Then draw and shade a Venn diagram illustrating each result.

25. $A \cap B$
26. $A \cup B$
27. B'
28. $(A \cap B)'$
29. $(A \cup B)'$
30. $(A \cup B) \cap B'$
31. $(A \cap B) \cup B'$

Determine whether each statement is *true* or *false* if $A = \{1, 3, 8\}$, $B = \{1, 3, 9\}$, $C = \{3, 8, 9\}$, and $D = \{3, 9\}$.

32. $A \subseteq B$
33. $D \subset C$
34. $D \in B$
35. $3 \in (A \cap D)$
36. $(A \cup D) \subset (B \cup C)$

MP5 Find each set if $K = \{0, 1, 2, 5\}$, $L = \{0, 1, 3, 4\}$, $M = \{0, 2, 3, 6\}$, and $U = \{x \mid 0 \leq x \leq 10, x \text{ is a whole number}\}$. Then draw and shade a Venn diagram illustrating each result.

37. $K \cap L$
38. $M \cup K$
39. $L \cap M'$
40. $M' \cup L$
41. $(M \cap K') \cup (M' \cap K)$
42. $(K \cap M) \cup (K' \cap M')$
43. $(K \cup L) \cup M$
44. $(M \cap K) \cup L$

MP3 Determine whether each statement is *always*, *sometimes*, or *never* true. If sometimes, explain when the statement is true.

45. Equal sets are also equivalent sets.
46. A set and its complement are disjoint sets.
47. $A \cup B = A \cap B$
48. If A and B are disjoint sets, then $B = A'$.
49. $A \cap A' = U$

C. EXERCISES

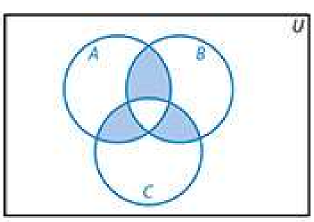
An infinite set is called *countable* if it is equivalent to the set of natural numbers. While the counting may never end, the elements can be counted one at a time. The following pairing shows a one-to-one correspondence between the natural numbers and the odd positive integers, which demonstrates that the odd positive integers are countable and that the sets are equivalent.

$$\begin{array}{l}\{1, 2, 3, 4, 5, \ldots, n, \ldots\} \\ \ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow \qquad \downarrow \\ \{1, 3, 5, 7, 9, \ldots, 2n-1, \ldots\}\end{array}$$

KEYWORD SEARCH

infinite countable sets

50. MP4 Show that the even positive numbers are countable.
51. MP4 Show that the integers are countable.
52. Use the set operations of intersection, union, and complement to write an expression describing the shaded area.



1.2 DEFINITIONS & UNDEFINED TERMS

What do geometry and a person's worldview have in common?

I will be able to

- describe good definitions.
- evaluate definitions.
- illustrate points, lines, and planes.
- describe the relationships between points, lines, and planes.
- explain why human reasoning is not sufficient to determine eternal truths.

The study of geometry requires an understanding of many terms. Definitions provide a solid foundation for the building blocks needed to succeed in geometry. Good definitions exhibit the following characteristics.

1. **Accurate**: The definition states the term being defined and clearly communicates the concept while avoiding ambiguous language.
2. **Concise**: The definition avoids unnecessary wording while being grammatically correct.
3. **Understandable**: The definition uses only words that have been previously defined or are clearly understood without being defined.
4. **Objective**: The definition avoids personal opinions, emotionally charged words, and figures of speech.
5. **Reversible**: The definition identifies the class the object belongs to and its distinguishing characteristics.

The peacock fans his tail, forming a geometric arrangement of points, lines, and circles in a plane.

The definition of a spoon as "a utensil used for eating, stirring, and serving food" lacks reversibility. A fork would fit because the definition lacks the spoon's distinguishing characteristic of "consisting of a small, shallow bowl attached to a handle."

EXAMPLE 1: Evaluating a Definition

Evaluate the definition "Space is the set of all points."

Answers

1. Accurate: The defined object is named, and the definition is not ambiguous.
2. Concise: The definition is grammatically correct and uses only necessary words.
3. Understandable: While "set" was defined in the previous section, "point" has not been defined.
4. Objective: The definition uses appropriate, objective language.
5. Reversible: The definition is reversible since "the set of all points is space."

SKILL CHECK EXERCISE 21 ✓

The stated definition of space would be acceptable if the word *point* were defined. How can a point be defined? While we generally know what a point is, some words in that definition would also need to be defined, and this process could continue indefinitely. Mathematicians have agreed to accept 3 undefined terms as basic building blocks for geometry.

GEOMETRY IN HISTORY
NOT YOUR USUAL MATH CLUB (PART 1)

Italy, 529 BC

The sun was sinking in the west, and Philip felt pretty tired. He'd spent the whole day helping Ninon survey a plot of land outside Croton, a Greek-speaking city in southern Italy. Ninon could be grouchy and demanding at times, but Philip still felt proud that Ninon had accepted him as a student-apprentice three years ago. Widely regarded as one of the best geometers in Croton, Ninon rarely accepted students. Philip knew that with a lot of hard work he'd one day have a fine career as a geometer as well. That thought pleased him because he loved solving problems.

Every now and then, when Ninon felt less cranky than usual, he liked to talk about geometry and its history. Philip learned that geometry ("earth measuring") was used in Babylonia and Egypt at least 1500 years earlier. Just like Philip and Ninon, these ancient cultures used geometry to solve everyday problems. For example, Egyptian geometers used it to establish land boundaries. Since the Nile River overflowed its banks every year, property boundaries had to be surveyed and reestablished regularly. The pharaoh charged rent based on the area of each plot, so surveyors helped keep everything fair.

Egyptian geometers had other problems to solve as well, such as calculating the volumes of food containers used in trade. And, of course, pyramid construction required all sorts of geometric techniques to make sure that the pyramids turned out right. Philip didn't want to think how the pharaoh, who claimed to be a god, would react if someone made an error in the calculations for his sacred burial place.

The Babylonians, Philip discovered, used geometry to solve problems involving circles and triangles. They also applied geometry to stargazing and calculating astronomical events. Like the Egyptians, they used it for construction since many of their kings were city builders. In fact, that's where Ninon got his training. When he was a young man, he traveled almost two thousand miles to Babylon, where he studied under a famous geometer.

"I wish Melissa would hurry up," Philip muttered to himself. "I'm starving!" His stomach rumbled in agreement. A few days ago, he'd agreed to go with Melissa, a casual friend, to visit a secret geometry club that she had recently joined. She claimed that these new friends of hers were doing things with geometry that had never been done before. Despite his hunger, Philip was curious. "And maybe," he thought, "they'll offer some snacks!"

"Hi, Philip!" a pleasant voice called. A rather attractive girl dressed all in white walked up to Philip and took his arm. "Let's go. It's this way."

To be continued in Chapter 2.

DISCUSSION

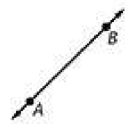
1. What does geometry mean?
2. List several ways that ancient Egyptians used geometry.
3. In ancient Egypt, the pharaoh claimed to be a god. How might he react if someone made an error in the calculations for his sacred burial place?
4. List several ways that ancient Babylonians used geometry.

TECHNOLOGY CORNER
DYNAMIC GEOMETRY SOFTWARE

Dynamic geometry apps such as *GeoGebra Geometry* or *Desmos Geometry* can help you learn geometry. Navigate to a dynamic geometry software application to complete these activities.

Many dynamic geometry applications have similar tools that allow you to create objects and shapes to illustrate different relationships in geometry. Explore the tools by scrolling through the different menu options.

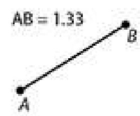
A. Illustrating the Line Separation Postulate

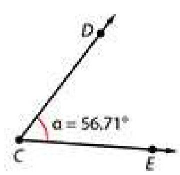


1. Draw a line by selecting the line tool and clicking in the window to place 2 points. Move the line by dragging either point or the entire line around the window.
2. Label the points as A and B.
3. Place point C on $\overleftrightarrow{AB}$ so that C-A-B.
4. Use the ray tool to draw $\overrightarrow{AC}$. You may need to first click on the ray's endpoint, A, and then a second point on the ray, C. Color $\overrightarrow{AC}$ (including point C) red.
5. Draw $\overrightarrow{AB}$ and color $\overrightarrow{AB}$ (including point B) green. Be sure that point A is a third color.
6. Save your work in Part A.

B. Constructing & Measuring a Segment & an Angle

1. Clear all the objects in the application.
2. Construct a segment, $\overline{AB}$, and measure its length. AB may be displayed as the length of a segment or as a distance between 2 points.
3. Construct $\angle DCE$ by drawing 2 rays with a common endpoint. Then find $m\angle C$. Notice how the measure changes as you drag each point around.
4. Save and print your work in part B. Measure the segment and angle from your printout and compare the actual and stated measures.





GEOMETRY AROUND US
DNA: LIFE'S BLUEPRINT

When you look in the mirror, you see many of the features that make you unique. You may have red hair, blue eyes, and freckles, or you may be tall with black hair and brown eyes. God created you unlike any other human in history, and He knew you even before you were born (Ps. 139:13–18; Eph. 1:4). But what lies beneath the surface of what you look like? How do scientists study the intricate workings of the human body? The composition of each cell is determined by deoxyribonucleic acid, better known as DNA. In fact, this complex biomolecule serves as the blueprint for all plants, animals, and humans. Each strand of DNA writes the story of what makes you . . . you!

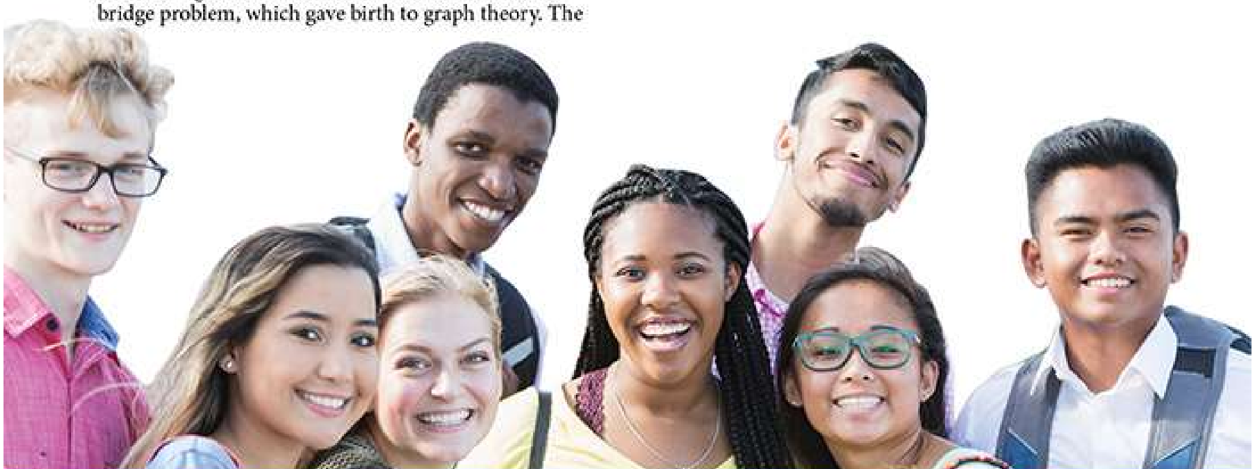
If you look at the shape of a DNA model, you may notice that it is composed of what looks like curves and line segments. These curves are called *helices*, from the Greek word meaning "twisted or curved." In the 1700s Leonard Euler contributed to today's understanding of DNA with his famous solution to the bridge problem, which gave birth to graph theory. The technique used to solve this problem became the stepping stone researchers needed when sequencing DNA in the 2000s. Mathematicians and scientists spent 13 years analyzing and sequencing human DNA in what was called the Human Genome Project. While the project was declared complete, there is still much that scientists don't know about the human body.

God created life with complexity (Rom. 1:20; Col. 1:16; Heb. 11:3). Inspired by God's creation, scientists have used DNA as a model when researching computer storage needs of the future. In fact, researchers experimented with using the DNA of living bacteria to store external information. Albert Einstein's famous equation that was published in 1905 became one of the first items of stored information retrieved through gene sequencing as the message "$E = mc^2$ 1905."

So, next time you look in the mirror, consider the millions of gigabytes of information stored inside a single gram of your DNA!

DISCUSSION

1. If uncoiled, how long is a single human DNA molecule?
2. Sketch a DNA double helix structure and identify the geometric figures used for it.
3. Describe several ways that God created life with complexity.



1 CHAPTER REVIEW

Key Concepts

Sets [1.1]

- Use set notation to precisely describe sets, their elements and subsets, and set operations.
  - union: combine the elements
  - intersection: find the common elements
  - complement: list the universal elements not in the set
- Venn diagrams can illustrate the relationship between sets and set operations.

Definitions & Undefined Terms [1.2]

- Good definitions are accurate, concise, understandable, objective, and reversible.
- Point, line, and plane are the 3 undefined terms in geometry.
- Any system of thought has foundational assumptions.

An Ideal Geometry [1.3]

- Geometric theorems are proven statements based on assumed postulates which describe the relationships between defined and undefined terms.
- An ideal postulate system is consistent, independent, and complete.
- The 6 incidence postulates and 3 incidence theorems describe basic relationships between points, lines, and planes.

Subsets of Lines & Planes [1.4]

- separation postulates
  - Every point on a line separates the line into 3 disjoint sets: the point and 2 half-lines.
  - Every line in a plane separates the plane into 3 disjoint sets: the line and 2 half-planes.
- An angle is the union of 2 distinct rays with a common endpoint.
  - Every angle divides a plane into 3 disjoint sets: the angle, its interior, and its exterior.

Segment & Angle Measures [1.5]

- The Ruler and Protractor Postulates define segment and angle measures.
- The Segment and Angle Addition Postulates describe sums of the measures of collinear segments and adjacent angles.
- Congruent segments have the same length, and congruent angles have the same measure.
- Angles are classified by their measure as acute, right, obtuse, or straight.

2-Dimensional Figures [1.6]

- Many curves can be further classified as simple, closed, or both.
- Simple closed curves define regions that can be concave or convex.
- The perimeter of a polygon is the sum of the measures of its sides.
  - The perimeter of a regular n-gon with sides of length s is $P = ns$.
- The circumference of a circle can be found using $C = \pi d$ or $C = 2\pi r$.

3-Dimensional Figures [1.7]

- Pyramids are cones with a polygonal base and prisms are cylinders with polygonal bases.
- Cones and cylinders are classified by the shape of their bases.
- A polyhedron is classified by the number of its faces.
- There are 5 regular polyhedra (or Platonic solids).

Sketches, Drawings & Constructions [1.8]

- Sketches are neat freehand drawings illustrating the general characteristics of the figure.
- Drawings utilize a variety of tools to create accurate, detailed representations.
- Constructions utilize an unmarked straightedge to draw straight lines and a compass to measure and copy distances.
- In geometry, do not assume an unmarked figure is drawn to scale.

Biblical Worldview Shaping

Foundations of Geometry

What is the ultimate foundation of geometry?
Scripture is the ultimate foundation of geometry.

EXERCISES

Translate each sentence into symbolic notation. [1.1]

1. The set containing a, b, and c is a proper subset of set G.
2. The number 3 is not an element of set H.

Find each set using $U = \{x \mid x \text{ is a digit}\}$, $A = \{0, 2, 4, 6\}$, and $B = \{1, 2, 3, 4, 5\}$. Then draw and shade a Venn diagram illustrating each result. [1.1]

3. $A \cap B$
4. $A \cup B$
5. A'
6. $(A \cup B) \cap B'$
7. $(A \cap B) \cup (A' \cap B')$

Evaluate each definition. Either state that the definition is good or cite the characteristic of a good definition that it is lacking. [1.2]

8. Parallel lines are lines that do not intersect.
9. Parallel planes are planes that do not intersect.
10. Skew lines are lines that are not parallel, do not intersect, and are noncoplanar.
11. They are points that lie on the same line.

Identify each described undefined term. [1.2]

12. denoted by a lowercase letter or with 2 points that lie on it
13. a flat surface extending infinitely in 2 dimensions
14. a location in space named with a capital letter

Briefly describe each characteristic of an ideal postulate system. Then state how frequently we should expect a postulate system to exhibit that characteristic. [1.3]

15. complete
16. consistent
17. independent

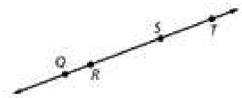
Name the postulate that verifies each statement. [1.3]

18. If $\overleftrightarrow{CD}$ intersects $\overleftrightarrow{DE}$, then the intersection is point D.
19. The only plane containing noncollinear points U, V, and W is plane UVW.

Fill in the blanks to complete each postulate. [1.3–1.4]

20. Expansion Postulate: A line contains at least _______ points, a plane contains at least _______ noncollinear points, and space contains at least _______ noncoplanar points.
21. Plane Intersection Postulate: _______ planes intersect in exactly _______ _______.
22. Line Separation Postulate: Every _______ on a line divides the line into 3 disjoint sets: the _______ and 2 _______.
23. Plane Separation Postulate: Every _______ in a plane divides the plane into 3 disjoint sets: the _______ and 2 _______.

State the result of each set operation. [1.4]



24. $\overleftrightarrow{RS} \cup \{R\}$

25. $\overrightarrow{SR} \cap \overrightarrow{ST}$

26. $\overline{QR} \cap \overrightarrow{TS}$

27. $\overrightarrow{RS} \cup \overrightarrow{RT}$

28. $\overrightarrow{TR} \cap \overleftrightarrow{QS}$

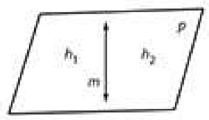
Use the figure illustrating the Plane Separation Postulate for exercises 29–32. [1.4]

29. Name the edge of each half-plane.

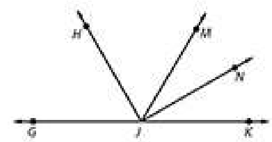
30. Find $h_2 \cap m$.

31. Find $(h_1 \cup h_2)'$.

32. Find $(h_1 \cup m)'$.



Use a protractor to find the measure of each angle in the figure. Then classify the angle as *acute, right, obtuse,* or *straight.* [1.5]



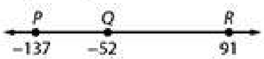
33. $\angle GJM$

34. $\angle NJK$

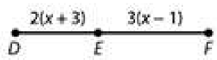
35. $\angle HJN$

Find each distance or angle measure. [1.5]

36. PQ and QR



37. DE and EF if $DF = 138$

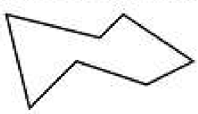


38. LM and LN if M is the midpoint of $\overline{LN}$

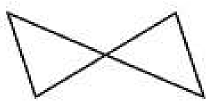
39. $m\angle 2$ and $m\angle 3$ if $\angle 1 \cong \angle 2$ with $m\angle 1 = (4x + 7)°$ and $m\angle 2 = (5x - 4)°$

If the figure is a polygon, classify it by its number of sides. If it is not, explain why not. Then classify any region formed by a closed figure as *convex* or *concave*. [1.6]

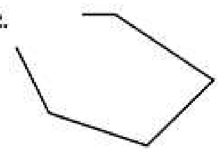
40.



41.

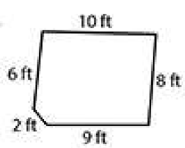


42.



Find the perimeter of each polygon. [1.6]

43.



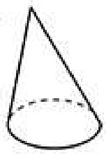
44. an equilateral triangle with sides of 15 in.

45. Find the circumference (to the nearest tenth) of a circle with a diameter of 20 ft.

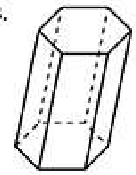
46. Find the radius (to the nearest tenth) of a circle with a circumference of 61 m.

Classify each cylinder or cone with the most specific designation possible and as a polyhedron if possible. [1.7]

47.



48.



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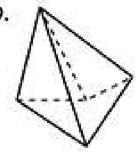
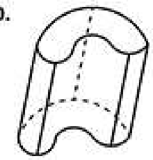


50.



51. Which kind of regions form the lateral faces of (a) any pyramid and (b) any prism? **[1.7]**

52. State the total number of vertices for (a) a pyramid with an n-gonal base and (b) a prism with an n-gonal base. **[1.7]**

53. State the total number of edges for (a) a pyramid with an n-gonal base and (b) a prism with an n-gonal base. **[1.7]**

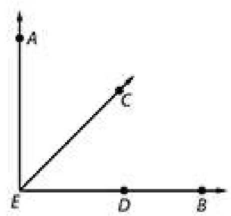
54. State whether each statement can be assumed from the drawing. **[1.8]**

a. $\angle AEB$ is a right angle

b. C is in the interior of $\angle AEB$

c. E-D-B

d. $\angle AEC \cong \angle CED$



Sketch each figure. [1.8]

55. a regular pentagon

56. a concave hexagonal pyramid

Draw each figure. [1.8]

57. an octagonal prism

58. a regular hexahedron

Draw $\angle XYZ$ as illustrated for each construction in exercises 59–61. [1.8]

59. a segment with length $4XY$

60. a circle with a radius of $2XY$

61. an angle with a measure of $3m\angle XYZ$

62. Geometry in History: List several ways that ancient Egyptians and Babylonians used geometry.

63. Geometry around Us: Describe several ways that God created life with complexity.

64. BWS Essential Question: What is the ultimate foundation of geometry?