

The CHARLOTTE MASON
ELEMENTARY ARITHMETIC

S E R I E S

SAMPLE



Richele R. Baburina

BOOK 6

The Charlotte Mason
Elementary Arithmetic Series

Book 6

by Richele Baburina

Special thanks to Alicia Behm for her help in writing additional sums.

Answers for all written work in this book, along with step-by-step solution information, can be found in *The Charlotte Mason Elementary Arithmetic Series, Book 6 Solutions Guide*, published by Simply Charlotte Mason.

The Charlotte Mason Elementary Arithmetic Series, Book 6
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“The Principality of *Mathematics* is a mountainous land, but the air is very fine and health-giving, though some people find it too rare for their breathing. It differs from most mountainous countries in this, that you cannot lose your way, and that every step taken is on firm ground. People who seek their work or play in this principality find themselves braced by effort and satisfied with truth.”

(*Ourselves*, Book 1, p. 38)

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Introduction

The realm of Number is one of vast beauty, and this book hopes to help both you and your student capture more of this vision. As the work becomes more involved, one can fall into the mindset that math is a hardship to bear rather than a creative pursuit to cherish. This is the time to step back for a moment and reflect on the structure, patterns, and harmony found in numbers and to view each section of work as part of a unified and wondrous whole. Irene Stephens, Lecturer in Mathematics for Charlotte Mason's House of Education, likened it to an embroiderer's thoughts while working on the Bayeux Tapestry, "I am so weary of sewing in these hundreds of little green stitches,' we hear her say, 'they are not so very beautiful after all'; and then there comes to this maiden someone with the beautiful picture of the finished tapestry—and the green patch is a patch no longer, it has its own place and its own value; and she, realizing this, thinks no more of green stitches but only of the landscape, part of which it is her high privilege to work upon the canvas" ("Mathematics," *The Parents' Review*, 1912, p. 138).

In this book, as well as the others in the series, we find a number of Charlotte Mason's guiding principles and practices to help our children form a personal relationship with numbers while maintaining a healthy relationship with each other. Here are a few:

Atmosphere. The student should never be pushed or pulled through a math lesson. Rather, he should be given the space needed to think, wonder, and grapple with the presented ideas and concepts. A student should never be given the idea that he is somehow behind, due either to your own fears or to outside standards. If the student has lost confidence, return to a place where he is secure in his understanding until that confidence returns. Your own right relationship with the subject, along with consistent daily work, will go far in cultivating a healthy atmosphere surrounding the math lesson.

The timing of the Arithmetic lesson can also make it more enjoyable. Place the Arithmetic lesson when your child is mentally fresh, and switch

to a lively round of mental math if he begins to weary. While we don't necessarily have to follow Charlotte's own schedule of Bible, Arithmetic, then Grammar or Dictation, keeping her principle of alternating subjects with those that use a different part of the brain or body is its own form of rest and refreshment.

Discipline. Carefully graduated teaching and daily mental effort on the part of the student are just a few of the methods that make the study of arithmetic essential in fostering good habits. Helping the child over every difficulty with little effort on his part will cultivate the opposite.

Charlotte tells us, "it is in his own power he must go" (*Home Education*, p. 261). If we imagine Mathematics as a mountainous land, think how little stamina a hiker would gain if his guide carried him over every rock encountered on the trail.

Short, concentrated lessons of just 25–30 minutes allow a student to accomplish more than he would in long, drawn-out ones. Interesting example questions, along with the continuation of 5 minutes of mental math (oral work), keep the student engaged and further the development of accuracy, promptness, and attention. Careful execution is greatly helped by a gridded notebook in which the work is kept as neatly as possible, while the beginning stages of any new notation involve simpler equations to secure neatness and orderly arrangement.

Life. Vital ideas, hands-on activities, and related bits of history involving groundbreaking thoughts and thinkers all provide the material and stimulation for mental growth and mathematical thinking. Rather than giving lengthy explanations or algorithms to memorize, engaging example questions stir the imagination and lead the student to discover rules for himself—making him for a moment another Euclid, Ramanujan, Lovelace, or Johnson.

Arithmetic Concepts in Book 6

- Review of Fractions
- Review of Greatest Common Factor and Least Common Multiple
- Multiplication of Fractions
 - Multiplication of Proper Fractions by Whole Numbers
 - Multiplication of Mixed Numbers by Whole Numbers
 - Multiplication of Fractions by Fractions
 - Multiplication of Fractions by a Mixed Number
 - Multiplication of Mixed Numbers by Mixed Numbers
- Division of Fractions
 - Division of a Whole Number by a Fraction
 - Division of a Fraction by a Whole Number
 - Division with Mixed Numbers
 - Division of a Fraction by a Fraction
- Repeating Decimals
- Converting between Decimals and Fractions
- Ratios and Proportion
 - Meaning
 - Numeration and Notation
 - Finding Ratios of Different Units
 - Comparing Numbers
 - Comparing Ratios
 - Equal Ratios or Proportion
 - Golden Ratio
- Percentages
 - Meaning
 - Numeration and Notation

- Converting between Decimals and Percents
- Converting between Fractions and Percents
- Converting between Fractions, Decimals, and Percents
- Adding and Subtracting Percents
- Percents Greater than 100
- Finding the Percent of a Number
- Finding Totals with Percents
- Finding What Percent a Number is of Another Number

Overview of Lessons

In the previous book, the student advanced his work with decimals and fractions, worked with prime numbers, factors, and both greatest and least common denominators. He continued his work in drawing to scale along with measures in decimals as well as fractions.

In Book 6, the student will advance his work in fractions to include the operations of multiplication and division. He will explore ratios and proportion and enter the world of percentages—converting between percents, decimals, and fractions—along with finding the percent of a number using various means. Both US standard and the metric systems will occur with questions of a concrete nature throughout the book.

The arithmetic work for each week contains three important components: New, Review, and Mental Math too. New refers to work in the newest concept; Review encompasses work in previous ideas and concepts to help the student maintain fluency, build habits of thought, and help prepare him for the next step; and Mental Math—oral work that is free from pencil, paper, or concrete objects—lends enthusiasm to the lesson while developing promptness, speed, and accuracy. The daily lesson length is 25 minutes per lesson with an additional 5 minutes given to mental math. Note that many sections in the book may be used for oral work, and additional mental math is found at the back of the book. If your child is unable to give focused attention for a full 30 minutes, work gradually to build his power of concentration. If paying attention becomes difficult, try switching to some lively mental math before returning to the main work. On days when a student is unable to give his full attention even after this change, it might mean it is time to change subjects, picking up where you left off either later in the day or even the next day.

This book is recommended for use with *Charlotte Mason Practical Geometry, Part 2*. When used with Practical Geometry, four days should be given to Arithmetic and one day to Geometry. The recommended schedule is the traditional one followed by Miss Mason's own students: Monday through Thursday given to Arithmetic lessons with Friday reserved for the hands-on Geometry lesson, though you might find another schedule works best for your family's needs.

Independent Work

Independent work entails the student putting forth mental effort without the teacher helping him over every difficulty by telling or offering crutches. Arithmetic is not considered a wholly independent subject, but you do want your student to progress in his ability to work independently. As an aid in nurturing this ability, answers to problems (outside of the introduction to a new concept) are available in the Solutions Guide as a companion to this book. A good balance looks like this:

- the teacher and student work on an initial concept and a number of problems together until comprehension and comfort is exhibited,
- the student then works independently for a set amount of time,
- the student then meets with the teacher again to look over the accomplished work.

Any Mixed Review and Review sections not specified as oral work may also be taken independently as long as you ensure the student's work is done with care and understanding. It is not recommended to simply hand the book over for the student to work through alone. Care should always be taken that neatness and orderliness are maintained in written work; reading and writing never overshadow the ideas of the math lesson; accuracy, clear thinking, and confidence are maintained; and poor habits are not taking hold.

Assign a given amount of time for independent work, while also allowing time to evaluate the work together in order to immediately address any confusion or misunderstanding. A lesson might look like:

- 10–15 minutes in new concept, student working initial exercises with you at his side
- 8–10 minutes of independent work
- 2–5 minutes looking over independent work
- 5 minutes mental math

and another lesson might look like this:

- 2–5 minutes going over with student what is expected in the Review section, noting the amount of time allotted

- 10 minutes independent work of Review section
- 10 minutes work together in latest concept
- 5 minutes mental math

or

- 15 minutes of work together in new section
- 5 minutes mental math
- 10 minutes continued work in new section

Notation and Writing

Understanding our system of notation is foundational. When working with fractions, it is important the student says the value when he is working with thirds, fourths, fifths, tenths, etc.— not simply 3, 4, 5, or 10. The same is true when working with decimal numbers, with the student saying he is working with tenths, hundredths, thousandths, etc. Units should always be written except when working with ratios.

Equations are kept simple in the early stages of learning any new notation. As your student's ease and ability in reading and writing increases, be sure he continues to maintain physical habits of neatness and order, even while working independently. The orderliness of each number contained in just one box on the grid of graph paper—or, perhaps, two when dealing with fractions—will help him maintain careful execution. It is also a good idea to have your student outline final answers with a box to keep it clear from his written work. Alternating between oral and written work during the lesson will further cultivate these habits.

Mental Arithmetic

A daily time given to work taken strictly orally, known as oral work or mental math, is integral to Charlotte's philosophy of math education. Mental math excites enthusiasm and increases a student's power of imagination, concentration, readiness, and accuracy. Work noted as Oral Work, to be taken orally, and additional review that is found throughout or in the back of the book, may all be used for this activity. Mental math may occur at the end of the lesson, when you need to regain your student's

responsiveness, or another time during the day—indoors or out—with the student adding to the questions or multiple students posing mental work to each other. The work should be lively and engaging, while the questions posed should always fall within a child’s comprehension. Adjust the questions to lower numbers, fewer operations, or reduced speed as he builds skill in this area. Likewise, his mental work should become more challenging as he advances in proficiency—perhaps involving more than one operation, higher numbers, or increased speed.

This 5-minute activity also helps a student learn or maintain fluency and speed in math facts. The Number Sentence Cards found at simplycharlottesmason.com are a wonderful addition to mental math time as they help secure math facts while building speed, promptness, and accuracy. Oral table work provided in and at the back of previous books in this series may be used as well.

Review

Review is an integral part of the weekly lessons as it helps the student maintain fluency in concepts and processes previously covered. Both a review of new material covered and a review of Book 5 are provided throughout the main section of the book, along with extra written review of new concepts found in the back. Review sections not specified as oral work may be taken independently as long as the teacher first ensures the student is working carefully and with a good understanding of the material.

The target balance is:

- the teacher and student look over the review section to ensure the student understands what is being asked of him,
- the student then works independently at his own pace for a set time,
- the student immediately meets with the teacher to look over the accomplished work and address any concerns.

Review is completely natural after a long break or holiday, and it is usually found that anything forgotten comes back with a time of refreshing. If, however, your student has never worked with the concepts found in the

initial review sections, or a fuller review is deemed necessary, you will find completely guided lessons in the previous books in this series.

Tip: If you are moving into this book straight from Book 5 without a break, the initial review may be tiresome. If this is the case, explain to your student that you will go at his pace, which may be quite quick, or simply bookmark the review to be used during future weeks.

When to Advance

Each child is unique, which means the number of problems necessary to master or internalize a concept will vary with each individual child as well as the particular concept. You may find the student advances quickly through one concept while he takes quite a bit longer with the processes of another. A good rule of thumb is to have the child work with the newest concept or idea until he has gained both comfort and confidence in his work, but not so much that he becomes bored or agitated. Periodic review will solidify concepts and processes as well as build relative speed in using them. Work with the view that you are preparing the soil “from which in after years it may be possible for a good mathematician to spring” (Stephens, *The Teaching of Mathematics to Young Children*, p. 17).

The new material, with its corresponding exercises, abundance of review work, along with the oral work, i.e., mental math, found in this book are all parts of the whole in attaining and maintaining fluency. The provided bookmarks, or your own page markers, help you easily adapt and customize the lessons according to the needs of each individual child. Watching your child’s comfort level and understanding with each process allows you to determine how many questions he will need at a given time. Not every question may be found to be necessary. In that case, you may bookmark it for future use. Or, if further work is needed in an area, you may find extra material at the back of the book. Questions are also written in such a way that you may easily change numbers and names to expand the number of questions at your disposal or relate the problems to your own child’s life and interests.

You are encouraged to look over each lesson in advance and familiarize yourself with it. This will give you a heads up on any necessary materials that might need gathering, allow you to deliver the lesson in a more

relaxed natural way, and give you the additional ability to maintain focus on your student.

Practical Geometry

The study of Arithmetic in this book is meant to be taken together with Practical Geometry. This approach allows the student to see for himself that differing branches of mathematics are not entirely separate entities, but part of a wondrous whole where even shapes are beholden to the laws of number. The lessons entail a hands-on approach in which the student becomes comfortable with handling mathematical tools—such as the compass, protractor, and set square—while exploring the behavior of geometrical forms and developing his drawing skills. Following Charlotte Mason’s traditional schedule, the 30-minute weekly lesson is taken in conjunction with the arithmetic lessons with four days given to Arithmetic and one day to Practical Geometry.

For more information, see *Charlotte Mason Practical Geometry, Part 1* and *Part 2*. While the work in these books may be taken separately, it is recommended that *Practical Geometry, Part 2* continues in tandem with this book—replacing one day of arithmetic.

Pacing

Although the ideas and work found in Book 6 generally take place in a child’s sixth year of formal education, the scope and pace of the work in mathematics should always be adjusted to the progress of the individual child rather than a standardized rate of learning or the pace of siblings and peers.

Never injure a student’s confidence with the idea that he is somehow behind, but allow his confidence to grow based on his own accomplishments and progress. Consistent and regular lessons, combined with daily mental effort, will keep your student advancing further up and further in the mountainous land of Mathematics. Varying the weekly elements of New, Review, and Mental Math not only helps keep the lessons from becoming stale, but gives you the freedom to tailor the lessons to the needs of your student. For example, you might find that not all the questions in an exercise are needed at the moment, so these may be bookmarked for review

at a later date. Or, if your student is enthusiastic about the newest concept being introduced, feel free to continue this work for the remainder of the lesson and take the 5 minutes of mental math later in the day.

The charts are provided here for your convenience and should be viewed as a loose planning guide only. Adjust the charts to your child's pace rather than pushing or pulling him in an attempt to make him fit the charts. The following is based on lessons that are 25 minutes in length with 5 minutes of mental arithmetic. If your student is unable to stay focused for this length of time, make it a target goal and remember that there is value in gradually working toward it. Your student may also find it helpful to have his weekly or daily schedule posted where he can see it. Knowing that there is a finite time given to each lesson can help spur concentrated attention.

Term 1												
<i>Suggested Weeks</i>	1	2	3	4	5	6	7	8	9	10	11	12
Review of Previous Year's Work												
Multiplication of Fractions												
Exam												

Term 2												
<i>Suggested Weeks</i>	13	14	15	16	17	18	19	20	21	22	23	24
Division of Fractions												
Decimals Review, Repeating Decimals, Additional Review												
Ratios and Proportion												
Exam												

Term 3													
<i>Suggested Weeks</i>	25	26	27	28	29	30	31	32	33	34	35	36	
Ratios and Proportion													
Percents													
Extra Written Work													
Exam													

Exams

Three end-of-term exams are provided in the back of this book. These are customizable to reflect where your child is in the lessons at the end of each term. See page 189 for details.

Supplies Needed

- Gridded math notebook

Note: If your student will also be completing Charlotte Mason Practical Geometry, Part 2 alongside this course, the 5 mm grid size recommended for the Geometry course will also work for the exercises in this book.

- *The Charlotte Mason Elementary Arithmetic Series, Book 6 Solutions Guide.* This guide includes answers to all written work presented throughout the book, as well as full step-by-step solutions which can serve as a teaching aid when needed.
- Personal chalkboard, dry-erase board, or the like, together with its appropriate writing instrument. For brevity, it will be referred to as the *slate* in the lessons.
- 12-inch ruler with US standard and metric measures
- (optional) Pennies
- Number Sentence Cards, Book 2 & Book 3 collections (optional, but highly recommended; see below for details)

Optional

- The handbook, *Mathematics: An Instrument for Living Teaching*, contains a fuller explanation of Charlotte Mason's unique approach in mathematics, including the principles upon which the practices found in this book rest. Available from Simply Charlotte Mason.
- Number Sentence Cards. A collection of number sentence cards with over 1,000 ready-to-use equations at your fingertips. Valuable to have on hand for use in the 5 minutes of mental math to retain math facts, build speed, and promote accuracy and concentration. Two sets of clearly printed Number Sentence Cards are sold separately or as part of The Charlotte Mason Elementary Arithmetic Series, Kits 2 & 3. Each set contains two packs of cards.

- Place Value Chart from Book 5 (visit simplycm.com/cmeas5-supplies for download and instructions)

Note: Concrete objects, commonly called manipulatives, may no longer be necessary for a child this far along. Feel free to allow their use as needed as it will save time in the long run.

Multiplication of Fractions

Multiplication of a Fraction by a Whole Number

(instructions to the parent)

Objects used: ruler or student-made ruler from previous books' exercises showing quarter-inches

1. Ask: How many $\frac{1}{4}$ " (quarter-inches) in one inch? (4)

Tip: Allow your student to use the ruler to see the fractions of inches.

If you take $\frac{1}{4}$ " and multiply it by 3, how many quarter-inches would you have? ($\frac{3}{4}$ ")

When looking at the ruler, you were adding $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ to get $\frac{3}{4}$. That is, you were adding the fraction the same number of *times* as the whole number.

2. Explain: You'll remember that multiplication is simply repeated addition, so we say " $\frac{1}{4}$ inch times 3."
3. Write on slate:

$$\frac{1}{4} \times 3 = \frac{1 \times 3}{4} = \frac{3}{4} \quad \text{Multiply the numerators, then place the product over the denominator}$$

Ask: Why do you suppose we are multiplying the numerator by 3 and not the denominator? (Because the numerator tells us *how many* quarter-inches; the denominator doesn't change.)

Tip: The above answer will be in the student's own words. If an answer isn't forthcoming, lead the student through the walk-through below.

State: If you have 1 inch and divide it into 4 equal parts, you are dividing it into fourths.

Write on the slate:

$$\frac{1}{4}$$

Ask: How many fourths make one whole inch? (4 fourths make one whole inch.)

Write on the slate:

$$\frac{4}{4}$$

Ask: Does the denominator change? (no)

Why? (The denominator shows how many equal parts the whole has been divided into.)

State: The denominator names the fractional units with which we are dealing. In this case, we are dealing with quarter-inches. So, if we take $\frac{1}{4}$ a certain number of times, the denominator doesn't change and it is the numerator that tells us how many *quarters* or how many *fourths*.

Ask: If you take $\frac{1}{4}$ " and multiply it 7 times, how many quarter-inches do you get? ($\frac{7}{4}$ ")

State: Again, while looking at the ruler, you might have solved this as repeated addition by taking $\frac{1}{4}$ seven *times*. We can solve this with multiplication by notating it like this:

$$\frac{1}{4}'' \times 7 = \frac{1 \times 7}{4}'' = \frac{7}{4}'' \quad \text{Multiply the numerators. Place the product over the denominator.}$$

$$\frac{7}{4} = 1 \frac{3}{4}$$

Next, we turn the improper fraction into a mixed number and simplify as necessary.

4. Ask: Looking at the ruler, how many are $\frac{1}{8}$ " times 3? ($\frac{3}{8}$)

Have the student write the question as a multiplication problem on his slate.

$$\frac{1}{8} \times 3 = \frac{1 \times 3}{8} = \frac{3}{8}$$

Ask the student to tell the steps taken to solve this problem. (Multiply the numerators, then place the product over the denominator.)

Next, ask the student to draw a rectangle in his gridded notebook that is 1 row of 5 squares. Now have him shade in $\frac{1}{5}$ of the squares.

Ask him to solve the following, allowing him to shade the squares to find the answers.

How many are 3 times $\frac{1}{5}$? ($\frac{3}{5}$)

Now ask him to write 3 times $\frac{1}{5}$ as a multiplication problem and tell the steps in multiplying a fraction by a whole number.

$$3 \times \frac{1}{5} = \frac{3 \times 1}{5} = \frac{3}{5}$$

Multiply the numerators, then place the product over the denominator.

Have the student notate the following questions as multiplication problems and solve. He may draw rectangles in his gridded notebook and shade in squares as above in order to obtain the answer until the process is clear.

How many are 2 times $\frac{1}{3}$? ($\frac{2}{3}$) $2 \times \frac{1}{3} = \frac{2 \times 1}{3} = \frac{2}{3}$

How many are 6 times $\frac{1}{7}$? ($\frac{6}{7}$) $6 \times \frac{1}{7} = \frac{6 \times 1}{7} = \frac{6}{7}$

How many are $\frac{1}{4}$ times 3? ($\frac{3}{4}$) $\frac{1}{4} \times 3 = \frac{1 \times 3}{4} = \frac{3}{4}$

How many are $\frac{1}{9}$ times 2? ($\frac{2}{9}$)

How many are $\frac{1}{8}$ times 7? ($\frac{7}{8}$)

Exercises

To be taken orally.

How many are $\frac{1}{7}$ times 3? ($\frac{3}{7}$)

How many are $\frac{1}{10}$ times 5? ($\frac{5}{10} = \frac{1}{2}$)

How many are $\frac{2}{9}$ times 4? ($\frac{8}{9}$)

How many are 5 times $\frac{1}{8}$? ($\frac{5}{8}$)

How many are $\frac{2}{7}$ times 2? ($\frac{4}{7}$)

To be worked in the math notebook.

1. $\frac{3}{10} \times 9$ 2. $\frac{7}{8} \times 5$ 3. $\frac{5}{7} \times 6$ 4. $\frac{9}{10} \times 11$ 5. $\frac{2}{3} \times 8$

5. Ask: Suppose we were to take $\frac{1}{4} \times 8$. Could we use the process of canceling in finding the solution?

Have the student notate on the slate: $\frac{1 \times 8}{4}$

Notice that we write the whole number on the same line as the numerator because the denominator of 1 is understood.

This time, use the process of cancelation as a step in solving.

Tip: The steps are written out for your convenience. Give the student time to solve on his own.

Ratios, Proportion, and Percent

Ratio

(instructions to the parent)

Say: On the first day of co-op, it was found that 4 out of every 5 students had already read *Macbeth*.

How would you express the relationship between the number of students who have read *Macbeth* to the number of students in the co-op?

Tip: Your student may readily state that four-fifths of the students have read Macbeth. If, after some thought, he does not give the fraction, simply continue on with the following question.

How would the numbers be written as a fraction? ($\frac{4}{5}$)

When we find out what part one number is of another, or want to express their relationship, we are finding what is called the *ratio* of one number to another. That is, when we compare two numbers by division, the result is called their ratio. We can think of ratio as the relation of numbers.

You have already been dealing with ratios throughout your study of fractions. For example, when dividing 1 pizza among 8 people in the form of the fraction $\frac{1}{8}$, this was a ratio—the relationship between a quantity of pizza and a quantity of people through division. When you drew plans to scale in the previous math books, each scale you worked with was a ratio.

Here's another familiar example of a ratio expressed as a fraction: Norma makes bouquets of 10 flowers of which 7 are dahlias and 3 are carnations. $\frac{7}{10}$ of the flowers are dahlias.

In this case, we see the relationship between the part of the bouquet that is dahlias (numerator) to the whole of the bouquet, that is 7 flowers + 3 flowers (denominator). This relationship could tell us that for every 20 flowers, Norma uses 14 dahlias.

It's important to note that, while a ratio can be expressed as a fraction, not every fraction is a ratio. For example, your family might eat $\frac{1}{2}$ of a pan of lasagna for supper. Here both the numerator and denominator are referring to the 1 pan of lasagna.

This is different than expressing a relationship between two different things in fraction form, though. For example, if we had the ratio of 1 pan of lasagna for every 2 families, we could express it as $\frac{1}{2}$.

We can also express the relationship between a part and a part.

For example, there are 4 quarts in a gallon, hence, the relationship between 3 quarts and 4 quarts can be expressed as a ratio in the form of a fraction: $\frac{3}{4}$.

Ask: There are 9 boys and 15 girls in the co-op. How do you suppose we would find the ratio of boys to girls?

Tip: Allow plenty of time for your student to think through this. Since he has had much work with fractions, he may be able to tell you the steps on his own. If not, you may lead him through, using the following steps:

$\frac{9}{15}$ Write the numbers as a fraction. Since we are comparing boys to girls, the number of girls will be the denominator.

$\frac{3}{5}$ Simplify the fraction to lowest terms.

This means that for every 3 boys at co-op there are 5 girls.

Ratios are often found in real-life situations. For example, a hot chocolate mix might show in its instructions to use 2 tablespoons of mix per 1 cup milk or water; and under the full price of products at the grocery store, ratios such as 35 cents per 100 grams are often given. A popular ratio in

interior design is $\frac{60}{40}$, which compares the amount of furnishings to free space in order to not overcrowd a room and achieve visual balance. In the automotive trade, the compression ratio of an engine compares the cylinder's expansion volume to its compressed volume; and in roofing, the pitch of a roof is the ratio of *rise to run*, which is basically the vertical rise of the roof height to the horizontal span of its width (run).

Be on the lookout for ratios in daily life, as relationships between two quantities are of more importance than you might think.

Numeration and Notation of Ratios

(instructions to the parent)

Say: Now that you've been introduced to ratios, you'll learn a bit more about how to express them in reading and writing. You've already seen that ratios may be written as fractions, but a ratio may also be expressed by a $:$ (write the symbol on the slate) symbol, which you might know as a *colon*. This symbol is placed between two quantities to show their order of comparison.

Ask: These symbols all denote division of quantities. What do you notice about these three symbols?

(to be written on the slate) \div $-$ $:$

Tip: Allow your student time to study the symbols and tell what he notices himself.

You might notice that the fraction bar and the ratio symbol look like the parts of the division symbol you first learned. The division symbol may also be seen as a fraction bar with the two points visually showing the placement of each term, or the dividend and divisor. \div or $\frac{3}{4}$ or 3:4