

CHARLOTTE MASON  
PRACTICAL GEOMETRY

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PART II

SAMPLE



*Dr. Julie Ryle* AND *Tabitha Wirges*



# **CHARLOTTE MASON PRACTICAL GEOMETRY**

Part 2

By Julie Ryle and Tabitha Wirges

Charlotte Mason Practical Geometry, Part 2  
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“The Principality of *Mathematics* is a mountainous land, but the air is very fine and health-giving, though some people find it too rare for their breathing. It differs from most mountainous countries in this, that you cannot lose your way, and that every step taken is on firm ground. People who seek their work or play in this principality find themselves braced by effort and satisfied with truth.”

*Ourselves*, Book 1, p. 38

## NOTES FOR USERS OF THE DIGITAL DOWNLOAD:

1. For many of the figures in this book, it will be necessary to view the document on your device at 100 percent scale to ensure the measurements are accurate.
2. We recommend printing individual pages for your student to work with when they are directed to trace figures from the book.

# CONTENTS

<b>GEOMETRY CONCEPTS IN PART 2</b> .....	7
<b>OVERVIEW OF LESSONS</b> .....	9
<b>SUPPLIES NEEDED</b> .....	12
<b>CONTINUING THE STUDY OF GEOMETRY</b> .....	13
<b>MAPS AND PLANS</b> .....	15
Lesson 1—Maps or Plans to Scale .....	15
Lesson 2—Perimeter .....	17
Optional Lesson — Floor Plans .....	20
Lesson 3—Using a Compass for Maps .....	25
Lesson 4—Navigational Compass .....	27
Lesson 5—Navigational Bearing .....	29
<b>TRIANGLES</b> .....	32
Lesson 6—Side-Angle-Side Triangles .....	32
Lesson 7—Angle-Side-Angle Triangles .....	38
Lesson 8—Triangle Congruence .....	44
Lesson 9—Similar Triangles .....	48
Lesson 10—Circumscribed and Inscribed .....	51
Optional Lesson—Creative Extra .....	53
Lesson 11—Rough Sketches and Scale Problems .....	56
Lesson 12—Exam I .....	59
<b>QUADRILATERALS</b> .....	60
Lesson 13—Parallelogram .....	60
Lesson 14—Rectangle, Square, Rhombus .....	63
Lesson 15—Diagonals of a Quadrilateral .....	65
Optional Lesson—Creative Extra .....	68
Lesson 16—Constructing a Quadrilateral .....	69
Lesson 17—Sum of the Angles of a Quadrilateral .....	73
Lesson 18—Quadrilaterals in Maps .....	75
<b>CIRCLES — TANGENTS</b> .....	78
Lesson 19—Finding the Center of a Circle .....	78
Lesson 20—Finding the Center of a Circle Given an Arc .....	81
Lesson 21—Perpendicular Lines from Semicircles .....	84

Lesson 22—Tangent Lines . . . . .	88
Lesson 23—Equality of Tangents . . . . .	90
Optional Lesson—Creative Extra . . . . .	94
Lesson 24—Exam II . . . . .	95
<b>POLYGONS</b> . . . . .	<b>96</b>
Lesson 25—What is a Polygon? . . . . .	96
Lesson 26—Regular Polygons . . . . .	100
Lesson 27—Drawing Regular Polygons . . . . .	104
Optional Lesson—Creative Extra . . . . .	107
<b>AREAS</b> . . . . .	<b>110</b>
Lesson 28—Area . . . . .	110
Lesson 29—Square Units . . . . .	115
Optional Lesson—Creative Extra . . . . .	119
Lesson 30—Area of Rectangles . . . . .	121
Lesson 31—Area of Triangles . . . . .	124
Lesson 32—Area of Parallelograms . . . . .	129
<b>THREE-DIMENSIONAL SHAPES</b> . . . . .	<b>132</b>
Lesson 33—Nets . . . . .	132
Optional Lesson—Creative Extra . . . . .	137
Lesson 34—Nets of Other Solid Figures . . . . .	138
Lesson 35—Surface Area . . . . .	143
Lesson 36—Exam III . . . . .	148
<b>GLOSSARY</b> . . . . .	<b>149</b>
<b>REFERENCES</b> . . . . .	<b>151</b>
<b>PRACTICAL GEOMETRY SOLUTIONS</b> . . . . .	<b>153</b>

# GEOMETRY CONCEPTS IN PART 2

- Maps
- Drawing to scale
- Perimeter
- Navigation
- Drawing triangles with construction tools given specific measurements
- Congruent shapes
- Similar shapes
- Circumscribed circles
- Inscribed circles
- Polygons
- Internal angles
- External angles
- Quadrilaterals
- Center of a triangle
- Center of a circle
- Tangents
- Perpendiculars
- Area
- Area of rectangles
- Area of triangles
- Area of parallelograms
- Nets
- Surface area
- Perimeter of 3D objects



# OVERVIEW OF LESSONS

Welcome to the study of practical geometry! Since ancient times, people have been utilizing geometric concepts with only two simple tools: a straightedge and a compass. These lessons incorporate those same simple tools to not only make learning basic elements of geometry a fun, hands-on experience, but also to give a better foundation of geometric principles before students begin a formal study of geometry in high school.

Although the lessons are designed to only take approximately 30 minutes each to complete, their length really depends on each student. As with part 1 of this course, if you complete one lesson per week, then it will take one school year to complete this book. You could, of course, choose to take an accelerated path, doing more lessons each week. Regardless of the chosen pace, the lessons begin with maps and plans and progress to more challenging work with triangles, circles, and various polygons.

## LESSON STRUCTURE

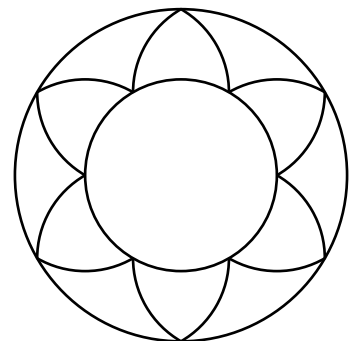
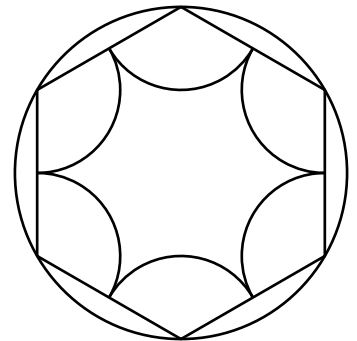
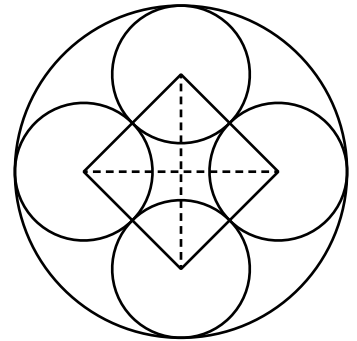
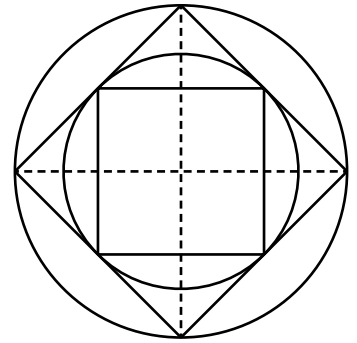
Each lesson begins with a few Review questions. These are meant to be answered quickly and orally. If your student struggles with a question, provide a quick reminder and move on. Following the Review, new ideas are explored with engaging exercises found throughout the lesson. At the conclusion of each lesson is a Math Notebook Entry prompt. The prompt provides your student an opportunity to recall what was learned by writing a short narration in her own words. These narrations could be written in paragraph form or by drawing and labeling the ideas. All three aspects of the lesson are vital, but the bulk of lesson time should be spent on the new ideas and exercises. Answers for all exercises are provided in the back of the book. Exercise steps are numbered, and questions that have corresponding answers are marked with letters.

There are six Euclidean constructions presented in this book, each used in different ways to aid in the understanding of many geometric concepts. These lessons will provide an opportunity for students to work with drawing tools and explore relationships between ideas.

## WRITTEN WORK AND GOOD HABITS

While your student may be able to visualize the solutions of the exercises, it is the process of constructing the figures that is the key. These constructions are what will take your student from knowing about these geometric concepts as abstract ideas to understanding them personally for herself. Doing the constructions will instill these geometric concepts in her mind and allow her to know, without doubt, the truth of them. Encourage her to have fun creating these geometric ideas for herself.

Precision and accuracy are foundational to the study of mathematics, and both habits apply to life outside of math as well. Some students may struggle with a lack of caring about accuracy, but some struggle with finding a balance between accuracy and



perfection. Encourage your student to keep a growth mindset as part of the atmosphere of the practical geometry lesson. In order to maintain such an atmosphere, it can be helpful to think of the lessons as more of a discussion between you and your student(s). If a student gives an incorrect answer, instead of directly correcting her, continue the conversation of the lesson by asking a question that guides her toward the correct answer which, in turn, will build her confidence in her mathematical abilities.

For instance, suppose your student says that a cube has seven sides. Instead of saying something like, “No, actually, a cube has six sides,” you could say, “Let’s grab the dice from Monopoly and count the sides!” We suggest taking a die yourself, along with your student. Inspecting the die in your hands, ask, “How can we make sure we don’t count one of the sides twice?” Listen to your student’s ideas for this, and go along with her if possible. If not, offer a suggestion, such as using the dots on the die or marking off the sides with a dry erase marker. This is similar to the process of thinking of a hypothesis in a science lesson and then experimenting to see if the hypothesis is correct.

This kind of discussion can be used for any exercise, especially with the use of the Solutions Guide in the back of this book. As the teacher, you can reference the solution to an exercise in order to know the direction these discussions need to go. As students give their answers, keep in mind the vocabulary or terms found in the solutions. For instance, if your student answers a question correctly by stating that an angle was cut exactly in half, then you can reply, “Yes! The angle has been cut in half. You *bisected* it. Great job!” If the solution does not help you understand, or your student needs more than the coaching you are able to give, you can even look at the solution with your student. Then you both can determine how this answer can be achieved given the information from the exercise. Remember: as the teacher, you are not the bearer of all knowledge. Rather, you are the guide into this wonderful world of numbers and patterns.

## VIDEO LIBRARY

In addition to the exercises and solution guide available in this book, your purchase includes access to our Practical Geometry Video Library, which offers visual walk-throughs of many components of the course. You can watch these videos independently before you teach your student, or you can watch them together as part of your lesson. You can access these videos through your account at [simplycharlottesmason.com](http://simplycharlottesmason.com). If you have any trouble accessing the files, reach out to Simply Charlotte Mason at [contact@simplycharlottesmason.com](mailto:contact@simplycharlottesmason.com).

## SKILL DEVELOPMENT

As you guide your student with these rich discussions, remember that she is developing the skills to use geometric tools and to do each construction. Thus, some of her measurements may be slightly different from the solutions. In fact, some small amount of error is almost certain to occur on a regular basis, as we are imperfect beings using imperfect tools. We have included discussions and reminders about possible errors in

measurement and good expectations for the student throughout the text. If your student struggles with achieving an accurate measurement with her ruler or with using a compass well, but otherwise demonstrates an understanding of the exercise, simply encourage her to keep working on her skills and precision. There is no need to keep reworking the exercise until the measurement or construction is *exactly* correct. Practice using the tools is more important.

## EXAMS

There are three exams in this text. Each takes place at the end of a 12-week term, following 11 geometry lessons. The exams are meant to be a chance for the student to demonstrate what she has learned. It is a chance to both celebrate progress as well as assess understanding.

## OPTIONAL LESSONS

In addition to the collection of 33 lessons and 3 exams that make up this book, we have included optional lessons for families who wish to explore the concepts in more depth. These Creative Extras appear throughout the book, and you can do these lessons on the same day as a geometry lesson or save it for another time during the week.

*TIP: Keep the lessons short and fun. Remember your student will see these geometric ideas again in a high school geometry study. Join in with your student and enjoy the beauty of mathematics!*

## LESSON ICON KEY


 REVIEW

 CONSTRUCTIONS

 MATH JOURNAL ENTRY

 VIDEO

# SUPPLIES NEEDED

 A visual overview of some of the required supplies is available with the Practical Geometry Video Library.

- Metric graph paper with 5-mm squares, either as loose sheets or as a notebook. Available from Simply Charlotte Mason.
- Geometry tools
  - US standard ruler with measurements marked to  $\frac{1}{16}$  of an inch
  - Metric ruler with measurements marked to 1 mm
  - Drawing compass
  - Protractor
  - Set squares (triangular rulers)
- Examples of various shapes
  - Cylinder (canned good, oatmeal container)
  - Cone (ice cream sugar cone, party hat)
  - Pyramid
- (optional) Highlighters
- Colored pencils or pens in at least two distinct colors
- Tracing paper (approximately 10 sheets)
- Blank sheets of paper (approximately 5 sheets)
- Scissors
- Round object to trace a circle (such as a drinking glass or can)
- Cardstock (two 8.5x11" sheets)
- Tape

# CONTINUING THE STUDY OF GEOMETRY

Imagine standing at the base of the Great Pyramid of Giza. As you stare up at this magnificent structure, consider its age, its magnitude, and its grandeur. Looking from side to side, note that the four faces are oriented exactly in line with the four cardinal directions (north, south, east, and west). It spans an area of nearly 13 acres. Its base measures 755 feet on each side, and it is 454 feet tall.



The Great Pyramid of Giza

It was over 4500 years ago, in the year 2570 B.C., that the great pharaoh Khufu stood here. It was for him that this largest pyramid was built. Herodotus, the Father of History, tells us that Khufu was a frightening and cruel tyrant. The only intact, 3-dimensional representation of the pharaoh, known as the Khufu Statuette, is small enough to fit in the palm of your hand; it is a 3-inch ivory figurine.



The Khufu Statuette

# LESSON 3—USING A COMPASS FOR MAPS

OBJECTS USED: metric grid paper, compass, metric ruler, and US standard ruler

## REVIEW

---

1. A collection of points that are an equal distance from a center point is a \_\_\_\_.
2. If two towns are drawn to be 2" apart on a scale of 100 miles to 1 inch, what is the actual distance between the towns?
3. What does it mean if a line segment is bisected? an angle?
4. Construct an isosceles triangle with base 2" and legs 1.5".

Your drawing compass has proven to be quite useful to draw circles. But, its ability to draw *a collection of points that are all an equal distance from a center point* can be used for more than just drawing circles. For instance, your compass can be used to mark off points equidistant from a given point in order to

- construct triangles with given side lengths,
- bisect a line segment, and
- bisect an angle.

### Exercise 1

Draw a line segment 3.5" long. Then, find two points, each of which is 2" from one end and 2.5" from the other. How was your compass useful in finding these two points?

.....

Since the compass is useful in marking off points that are an equal distance from a given point, it is very useful in drawing plans.

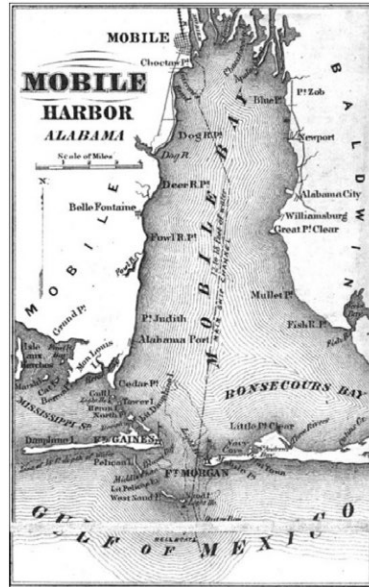
### Exercise 2

Sawyer and Emery live in houses that are 100 yards apart. Their favorite meeting spot is under a tree that is 65 yards from Sawyer's house and 55 yards from Emery's.

- a. Draw a line segment (scale 10 yards to 1 cm) to show the position of their homes. Label the line segment  $\overline{SE}$ .
  - b. Is there more than one possible position for the tree? If so, how many?
  - c. Choose one position to be the location of the tree and mark it  $T$ .
  - d. If Sawyer walks from his house to the tree, then to Emery's house, and finally, directly back to his house, how far will he have walked?
  - e. What is the perimeter of the triangle created between their houses and the tree?
- .....

### Exercise 3

Mobile Bay in coastal Alabama was once defended by two forts, Ft. Gaines and Ft. Morgan, on opposite sides of the mouth of the bay. Today, both forts still stand as historic sites. There is a ferry that travels the 3.5 miles between them, carrying tourists from one site to the other.



Suppose each fort's guns had an effective range of 2 mi. Draw a plan showing the location of the forts, labeling the points  $G$  and  $M$  (scale 1 mile to 1 inch). Add an indication of north on your map and label the area that is the Gulf of Mexico and the area that is Mobile Bay.

- Draw the boundaries of the area that each fort could defend based on their gun range.
- Draw the perpendicular bisector of  $\overline{GM}$  and determine the territory each fort should defend. Shade one region with horizontal lines and the other with vertical lines.
- Determine what part of the Gulf of Mexico was covered by fire from *both* forts. Shade this region in completely.
- Very lightly shade the region of the Gulf of Mexico that could not be defended by either fort.
- Create a key on your map indicating the meaning of each of the shaded regions.

*TIP: You could also used colored pencils for this exercise.*

## MATH NOTEBOOK ENTRY

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Explain how a compass can be used for drawing scale maps. Include an example using any of the exercises from the lesson, or create your own.

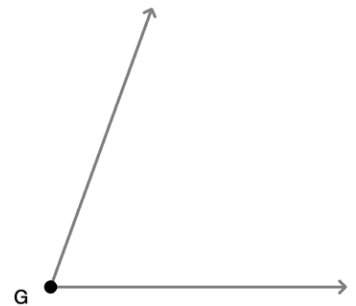
# LESSON 4—NAVIGATIONAL COMPASS

OBJECTS USED: straightedge and compass

## REVIEW

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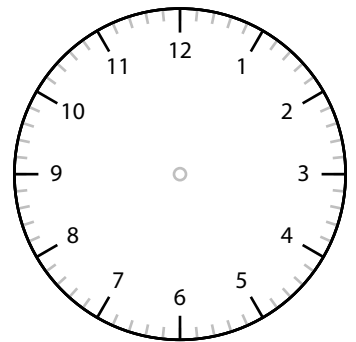
1. How many faces does a square pyramid have?
2. About how many centimeters are in 3 inches?
3. How many degrees are in a circle? in a semicircle? in complementary angles? in supplementary angles?
4. Copy the angle shown here by construction. Then bisect it.



## Exercise 1

Consider an ordinary clock.

1. Through how many degrees does the minute-hand revolve
  - a. in 1 hour?
  - b. in 2 hours?
  - c. in  $\frac{1}{2}$  hour?
  - d. in 5 minutes?
  - e. in 1 minute?
  - f. in 25 minutes?
2. How long will it take the minute hand to turn
  - g. through  $48^\circ$ ?
  - h. through  $102^\circ$ ?
  - i. through  $9^\circ$ ?



---

On a navigational compass, the four cardinal directions of north, east, south, and west are the four main rays. Between the four cardinal directions are the four ordinal directions of northeast (NE), southeast (SE), southwest (SW), and northwest (NW).



### Exercise 3

Determine the number of degrees for each of the following directions. Use a combination of your knowledge of angles with the illustration from exercise 2 and addition / subtraction as needed.

Direction	# of degrees
S	
W	
NE	
SE	
SEbE	
WSW	
WbN	

---



#### MATH NOTEBOOK ENTRY

---

Draw a compass like the one in exercise 2 in your notebook. Label all 32 points with their directions and the cardinal and ordinal directions with their degree measures.

## LESSON 5—NAVIGATIONAL BEARING

**OBJECTS USED:** metric grid paper, protractor, compass, and ruler with metric and US standard markings



#### REVIEW

---

1. What are the cardinal directions of the navigational compass? The ordinal directions?
2. How many degrees from N is NE?
3. What does the sum of the angles of a triangle equal?

# TRIANGLES

## LESSON 6—SIDE-ANGLE-SIDE TRIANGLES

OBJECTS USED: protractor, compass, set square, metric ruler

### REVIEW

---

1. How many edges does a rectangular prism have?
2. For  $\triangle ABC$ ,  $\angle A = 70^\circ$  and  $\angle C = 50^\circ$ . What does  $\angle B$  equal?
3. What kind of triangle has two sides that are congruent? What kind of triangle has an angle that is larger than  $90^\circ$ ?
4. If three segments are 2", 3", and 4.5" long, can they form a triangle? Why or why not? If it is possible, construct such a triangle.

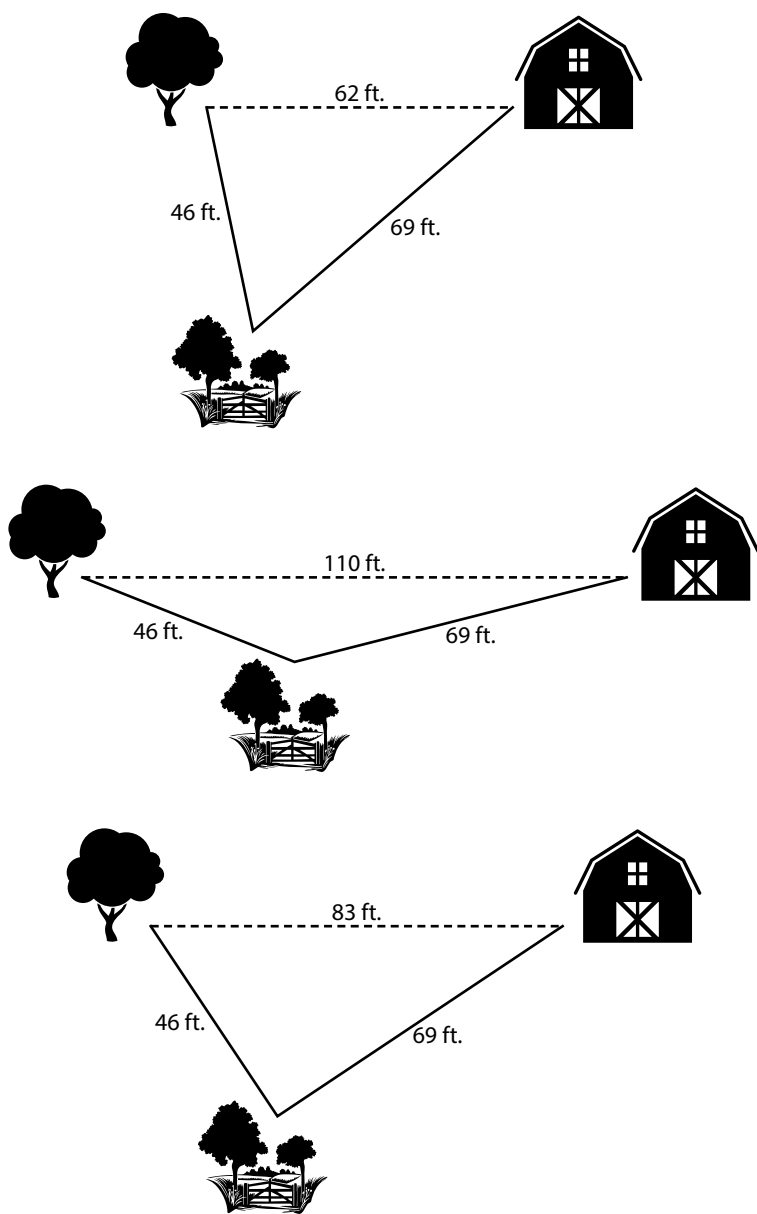
While discovering that the sum of any two sides of a triangle are always longer than its third side, you learned how to construct a specific triangle given the length of its three sides.

Such a construction is a handy tool! But, must you always know all three sides in order to construct a desired triangle? Just how much information does one need in order to accomplish this? What if the length of one side was unknown?

For instance, suppose there is a gate, an oak tree, and a barn at your grandparents' farm, and you'd like to run a slackline from the oak tree to the barn. (A slackline is similar to a tightrope, but close to the ground.) You know the oak tree is 46 feet from the gate, and the barn is 69 feet from the gate. You need to know the distance from the oak tree to the barn to know how long of a slackline to purchase.



This scenario creates a triangle, with the gate where you're standing, the tree, and the barn at its vertices. What type of triangle is it? Though you know the lengths of two sides, there are many possibilities for the length of the third side, as shown in the following sketches.



*NOTE: These drawings have been scaled to fit the page.*

Since you don't know the length of the third side of the triangle, you will need more information about the triangle in order to draw it and find the length of the third side. Notice that as the length of the third side changes, so does the angle opposite it.

## Exercise 1

Using a protractor, measure the angle at the gate in each of the triangles above.

---

The angle of the gate is between the two sides whose lengths we know. The angle between two sides of a triangle is referred to as the sides' *included angle*. If the measure of this included angle were given, then you could draw the required triangle.



By pointing at the tree and barn, your arms would form the included angle needed for this triangle and could be measured with the help of a friend. Say it is measured to be  $65^\circ$ .

You now have the measure of two sides and their included angle for the triangle. The length of the third side will be the distance from the barn to the tree!

In the following construction, you will draw a scale model (using a scale of 10 feet to 1 centimeter) in order to find the length of the desired slackline. Recall that, when doing constructions, only a compass and straightedge are used. You will not take measurements using a protractor or ruler.



### CONSTRUCTION 9—DRAW A TRIANGLE GIVEN TWO SIDES AND THE INCLUDED ANGLE

---

**TIP:** *Constructions are numbered sequentially in continuation from part 1 of this series.*

\_\_\_\_\_  $b = 4.6$  cm

\_\_\_\_\_  $t = 6.9$  cm

# THREE-DIMENSIONAL SHAPES

## LESSON 33—NETS

**OBJECTS USED:** metric ruler, US standard ruler, protractor, solid figures of cubes and prisms, scissors, tape, and cardstock

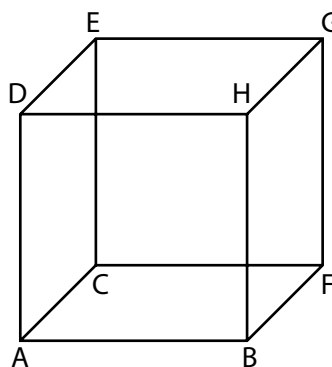
### REVIEW

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1. What are the parts of a cube? How many edges does a cube have? How many vertices does a cube have? Draw a rough sketch of a cube.
2. How many faces does a square pyramid have? What shapes are the faces of a square pyramid?
3. Draw  $\triangle ABC$  with  $b = 7.8$  cm,  $c = 6.2$  cm, and  $\angle A = 118^\circ$

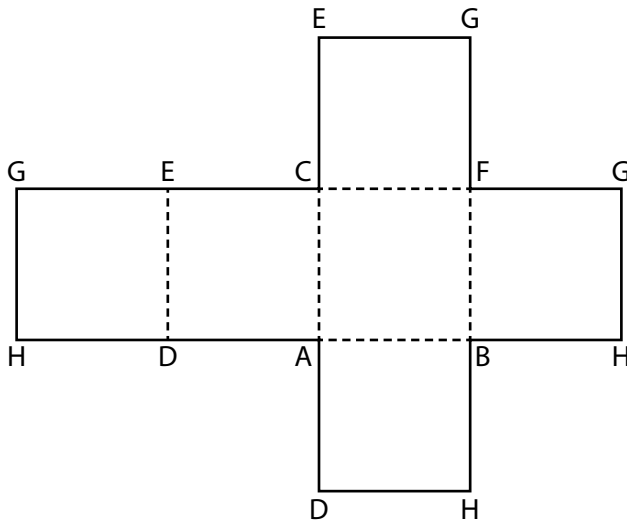
In this book, you have learned a great deal about all sorts of shapes, including circles, triangles, quadrilaterals, and other polygons. Thus far these shapes have had two dimensions, length and width, and are thus referred to as two-dimensional, or 2D, shapes. What about a shape that has three dimensions: length, width, and height? Three-dimensional, or 3D, shapes are the same as solid figures. You may recall from *Practical Geometry, Part 1* that solid figures, such as a cube, cone, pyramid, or cylinder, have surfaces that consist of curved or flat sides. If the side of a figure is flat, it is called a *face*. The faces of a solid figure are often circles, triangles, quadrilaterals, or other polygons—in other words, the very shapes you have been studying!

We will now see how models of three-dimensional shapes may be constructed, beginning with the simple figure of a cube.



Suppose the surface of the cube is cut along the vertical edges, and also along the edges  $\overline{DH}$ ,  $\overline{HG}$ , and  $\overline{GE}$ .

Now suppose the sides are unfolded and flattened out like when you unfold a cardboard box. The surface would then be represented by a figure consisting of six squares arranged as below.



This figure is called a *net* of a cube. A *net* of a figure is the term used to describe what a three-dimensional object would look like if it were opened up and laid flat.

### Exercise 1

1. Draw a sketch of a cube and its net using the squares of your grid paper as a guide.
2. Label the vertices of the cube and its net as shown in the example above. How many edges does the net have?

You'll recall that perimeter is the total length of the outside edge of a shape. The net of a cube creates a rather funny looking dodecagon, or 12-sided polygon. Since it is a polygon, its perimeter can be found by adding up the length of all its outside edges. If the cube has edges that are 2" long, the perimeter of its net would be 2 added up 14 times, or 28".

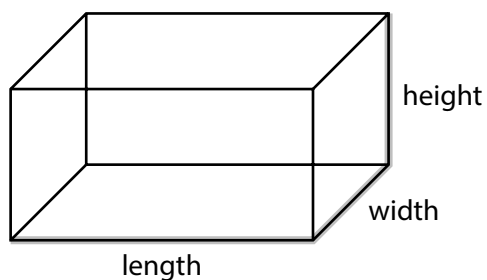
### Exercise 2

Make a model of a cube with edges 2" long in the following way.

1. Draw its net on cardstock. Find the perimeter of its net.
2. Cut out the net along the outside lines.

3. Then, fold and crease along each of the dotted lines.
  4. Fold the faces over until the edges come together; fix the edges in position using strips of tape.
- 

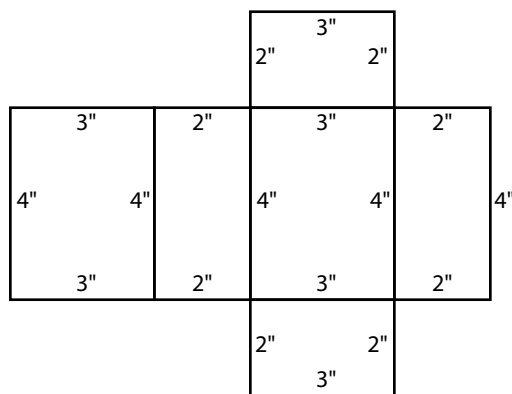
What about a rectangular box that is not a cube? What is that shape called, and what would its net look like? It is similar to a cube, but instead of six faces that are congruent squares, a *rectangular prism* has six rectangular faces.



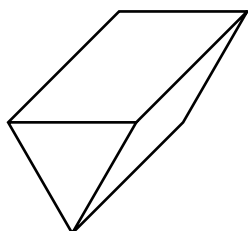
### Exercise 3

Make a model of a rectangular prism, whose length is 4", width 3", and height 2", in the following way:

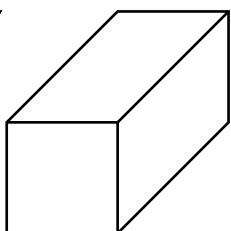
1. On cardstock, draw the net, which will consist of six rectangles arranged as shown below and having the dimensions marked in the diagram.
  - a. What is the perimeter of its net?
2. Now cut the net out, fold the faces along the dotted lines, and secure the edges with tape, as explained in exercise 2.
  - b. Are any of the rectangles congruent to one another?



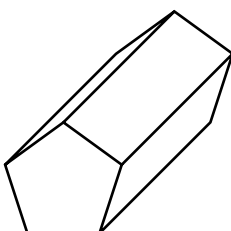
Each of the shapes shown below are also known as *prisms*. Notice that they all have several rectangular faces, in addition to two congruent polygons, known as the prisms' *bases*.



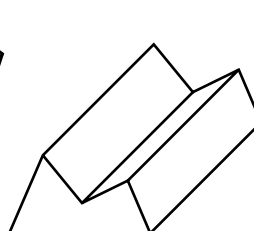
triangular prism



square prism



pentagonal prism



pentagonal prism

#### **Exercise 4**

- a. What shapes are the faces of a triangular prism?
  - b. How many faces does it have of each shape?
  - c. What shapes are the faces of a pentagonal prism?
  - d. How many faces does it have of each shape?
- 

The faces of a prism will consist of two congruent polygon bases, as well as several rectangles. In fact, it will have the same number of rectangles as its polygon base has edges. You can draw the net of a prism by first drawing one of the bases, followed by the correct number of rectangles (each having the correct width and equal length), and then the final base, like the net shown on page 136: