

WELCOME TO *GEOMETRY!*

"And God said, Let us make man in our image, after our likeness."
— Genesis 1:26a

Since we are created in God's image, we have attributes similar to His. While we cannot create from nothing as God did, we can create beautiful art and buildings. God created the universe by simply speaking it into existence, but our creations require existing materials, careful thought, frequent collaboration, and hard work.

The Creation Mandate in Genesis 1:28 authorizes us to manage the earth's resources and develop them to their fullest potential. Since people first fell into sin (Gen. 3), we have struggled to fulfill this God-given mission. The powerful tool of mathematics helps us describe and understand His creation in our attempts to exercise wise dominion.

The word *geometry* comes from two Greek words that mean "earth measure." Around 300 BC the Greek mathematician Euclid organized the known geometric principles into 13 books called the *Elements*. These 13 books formed the foundation for the definitions, postulates, theorems, proofs, and constructions in today's Euclidean geometry that provides abstract models of the world around us.

Initial attempts at any new skill—such as driving a car, programming a computer, or playing a sport or a musical instrument—may be challenging. As diligent effort and consistent practice breeds success, the activity becomes more enjoyable. This text provides instruction and exercises created to assist you in developing the following essential mathematical practices (MPs).

- MP1** Persevere in problem solving.
- MP2** Use abstract reasoning.
- MP3** Construct logical arguments.
- MP4** Use mathematical models.
- MP5** Use appropriate tools.
- MP6** Use precise language.
- MP7** See and apply structure.
- MP8** Generalize patterns.

During your study of geometry, you will learn to apply formal logical thinking to prove statements and solve real-life problems. Each section contains clearly stated learning targets, thorough explanations of each concept, step-by-step examples, skill checks, and cumulative review exercises. QR codes link to tutorial videos and additional activities. *GEOMETRY* has been written and designed with student success in mind.

Enjoy making the most of this year learning more about geometry. The deductive reasoning and problem-solving skills you develop will serve you well whether you become an engineer, a pilot, or a cake designer.



2.6 PROOFS USING SEGMENTS

How is deductive reasoning used to prove a theorem?

What are the 3 parts of a typical 2-column proof?

- complete 2-column proofs about segments
- create 2-column proofs about segments

KEYWORD SEARCH

geometry proof strategies

Proving theorems can be an exciting, creative skill to learn. To become successful at this new skill, you often attempt to prove new theorems and accept the challenge that such success in proving theorems. At times, work can have negative connotations, but the Bible says that we are to work diligently at the tasks we believe in (Gen. 2:15; Eph. 4:28; Col. 3:23). These verses tell us that the development of good thinking and reasoning abilities will provide more benefits throughout your life, such as solving problems about where to go to college or which career to pursue.

While there is no set method of proving theorems, the following steps can assist you in the process.

1. Identify the premise (the given information).
2. Identify the conclusion (what you are trying to prove).
3. Make a sketch to ensure you understand the theorem.
4. Consider any related definitions, properties, postulates, or theorems.
5. Work backward from the conclusion if necessary.

In section 5.3 an informal paragraph proof was given for one first theorem: "A line and a point not on that line are contained in exactly 1 plane." The next common form for geometric proofs is the 2-column proof. A 2-column proof includes clear statements of the premise (Given) and the conclusion (Prove), a diagram, and the reasons for the statements and their supporting theorems.

The Given and Prove statements are more easily identified when the theorem is written as a conditional of the form $p \rightarrow q$. "If a line and a point are on that line exist, then they are contained in exactly 1 plane."

EXAMPLE 1: Writing a 2-Column Proof

Write a 2-column proof of Theorem 1.3.1:
A line and a point not on that line are contained in exactly 1 plane.

Answer:
Given: There exists a line k and a point P not on that line.
Prove: There is exactly 1 plane q containing k and P .

Statements	Reasons
1. There exists a line k and a point P not on that line.	1. Given
2. Line k contains 2 points, X and Y .	2. Expansion Postulate
3. X , Y , and P are noncollinear.	3. Definition of noncollinear points
4. Points X , Y , and P determine exactly 1 plane q .	4. Plane Postulate
5. Line k and point P are	



Line k and the point P are not on that line, so they are contained in exactly 1 plane.

Essential Question & Learning Targets

Start each section with a question about a key idea and a list of the skills you should expect to learn.

Keyword Searches

Quickly locate additional information, interactive activities, and supplementary resources online.

QR Codes

Link to free tutorial content located on AfterSchoolHelp.com.

Key Concepts

Read through explanations of key concepts with notes on related ideas within margin boxes.

Extra care must be taken when negating statements containing quantifiers.

- All roses are pink. (a false statement)
- Correct: "Not all roses are pink" or "Some roses are not pink." (true statements)
- Incorrect: "All roses are not pink" or "No roses are pink." (false statements)
- "Some ladies are tall." (a true statement)
- Correct: "All ladies are not tall" or "No ladies are tall." (false statements)
- Incorrect: "Not all ladies are tall" or "Some ladies are not tall." (true statements)

The arrows in the table indicate the negations of generalized quantified statements.

Original	Existential
All A are B .	Some A are B .
No A are B .	Some A are not B .

EXAMPLE 2: Negating Statements

Negate each statement.

- p : All pies are round.
 - q : There exists a student who gets straight A's.
 - r : No square is a circle.
 - s : Some angles are not acute.
- Answers:
- $\neg p$: Some pies are not round.
 - $\neg q$: No student gets straight A's.
 - $\neg r$: There exists a square that is a circle.
 - $\neg s$: All angles are acute.

Concept mapping must be built on a series of true statements. You can decide whether a statement is true by comparing it with Scripture or previously proved facts.

Truth tables provide a summary of all possible truth values for the given statements. The first column of the negation truth table shows the corresponding value of $\neg p$.

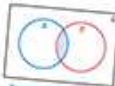
Single statements can be combined to form a compound statement. Disjunctions and conjunctions connect single statements in the same way they connect inequalities in algebra.

- A **conjunction** is a compound statement formed by connecting 2 statements with the word "and." The conjunction "p and q" is denoted $p \wedge q$.
- A **disjunction** is a compound statement formed by connecting 2 statements with the word "or." The disjunction "p or q" is denoted $p \vee q$.

Both single statements must be true for a conjunction to be true. The statement "Christmas is in December, and February has 31 days" is false because the second statement is false. Notice that the truth table lists all 4 possible combinations of truth values for p and q in the first 2 columns.

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CONJUNCTION
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Definitions, Postulates, Theorems & Proofs
 Focus on these important building blocks in geometry.

Examples & Skill Checks
 Study the step-by-step reasoning used to solve example problems; then check your understanding by completing exercises related to each example.

DEFINITION
 A conditional is logically equivalent to its contrapositive. (T.2.3.1)
 Combining a conditional and its converse with the word "and" forms a biconditional.

DEFINITION
 A biconditional is a conjunction of the form $p \leftrightarrow q$ or $p \leftrightarrow q$.
 Biconditionals are often written as "if and only if" or "p if and only if q" and denoted by \leftrightarrow .
 The Segment Addition Postulate can be stated as a biconditional: B is between A and C if $AB + BC = AC$.

EXAMPLE 4: Writing a Biconditional as a Conjunction
 Write the following biconditional as a conjunction of 2 conditionals.
 "Coplanar lines do not intersect if and only if they are parallel."
Answer
 If coplanar lines do not intersect, then they are parallel; and if the lines are parallel, then they are coplanar and do not intersect.

Recall that a good definition is reversible. In other words, it could be rewritten as a biconditional. Notice how the truth table is used to determine the truth values of a biconditional.

p	q	$p \leftrightarrow q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

Just as it can be helpful to explain something in your own words, it can be useful to express a compound statement in terms of another construct. The following theorem allows us to change a conditional to a disjunction and vice versa.

THEOREM
 The conditional $p \rightarrow q$ is logically equivalent to the disjunction $\neg p \vee q$. (T.2.2.2)

PROOF
 Find the truth values for $p \rightarrow q$ and $\neg p \vee q$.

p	q	$p \rightarrow q$	$\neg p \vee q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

The truth values for $p \rightarrow q$ and $\neg p \vee q$ are identical. Therefore, the two are equivalent.

A. EXERCISES
 Use the number line for exercises 1–5.

- Find each length.
 - AC
 - BC
 - BD
- Find each length.
 - CE
 - DE
 - AE
- State the coordinate of the midpoint of AB .
- State the coordinate of the midpoint of CE .
- Which point is the midpoint of CD ?

Determine whether each statement is true or false.

- R is the midpoint of DE , then S , T , and R are collinear points.
- R is the midpoint of PQ and $PQ = 24$, then $PR = 12$.
- If $AX = 20$, then X is the midpoint of AE .
- If $AX = \frac{1}{2}AE$, then X is the midpoint of AE .

Copy each segment and then construct the following:

- a bisector of AB
- a bisector of CD
- a bisector of EF

Use each angle and use it to construct an angle with each measure.

Construct the following:

- an angle congruent to $\angle STU$
- an angle bisector of $\angle JKL$
- an angle bisector of $\angle JKM$
- $\angle KLN$ congruent to $m\angle KJL$

Question: How can I use an 180° angle to find $\angle JLN$? Is it long work?

Point M is the midpoint of AB and $AM = 10$.

Point N is the midpoint of CD and the length of BN is 8.

Find AC .

Use the number line for exercises 10–13 to construct each segment.

- Construct AB with A at -5 and B at 3 .
- Construct CD with C at 2 and D at 8 .
- Construct EF with E at -2 and F at 6 .
- Construct GH with G at 1 and H at 9 .

CUMULATIVE REVIEW

State each postulate.

- Plane Intersection Postulate (I.1)
- Plane Separation Postulate (I.4)

Determine whether each statement is true or false.

- Inductive reasoning draws a conclusion from a pattern found in several examples. (I.1)
- The lack of a counterexample is a valid method of mathematical proof. (I.2)
- $3 + a = a + 3$. (I.3)
- $2(x + 3) = 3x + 6$. (I.3)
- If p is true and q is false, state the truth value of each expression. (I.2–I.3)
 - $p \wedge q$
 - $p \vee q$
 - $\neg p$
 - $\neg q$
 - $p \rightarrow q$
 - $\neg(p \rightarrow q)$
 - $p \leftrightarrow q$
 - $\neg(p \leftrightarrow q)$
- Name each statement related to the conditional $p \rightarrow q$. (I.3)
 - $\neg p \rightarrow \neg q$
 - $\neg q \rightarrow \neg p$
 - $p \rightarrow \neg q$
 - $\neg p \rightarrow q$
- Which of the following are equivalent to the conditional $r \rightarrow s$? List all correct responses. (I.3)
 - $\neg r \rightarrow \neg s$
 - $\neg s \rightarrow \neg r$
 - $s \rightarrow r$
 - $\neg r \vee s$
 - none of these
- Which conclusion is justified by the Segment Addition Postulate if A , B , and C are collinear? (I.3)
 - A , B , and C are collinear.
 - $AB + BC = AC$
 - $AB = BC = AC$
 - $BC = \frac{1}{2}(AB + BC)$
 - none of these

MIND OVER MATH
 Owen, Kevin, and Frank each have a different hobby: hang gliding, rappelling, or skydiving. They each also have a different favorite sport: basketball, hockey, or soccer. Frank likes soccer, the athlete likes basketball, hockey, or soccer, Frank likes soccer, the athlete likes hockey, and Kevin is the hang glider. Copy and use the grid to determine each man's hobby and favorite sport.

	Hobby	Sport
Owen		
Kevin		
Frank		

Exercises
 Build and maintain your skills with carefully sequenced exercises that emphasize the essential question and essential mathematical practices (MPs).

Cumulative Review
 Systematically review key concepts and practice strategies for standardized testing.

Mind over Math
 Challenge your critical thinking with this unique feature found in each chapter.

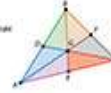
Technology Corner

Use dynamic geometry software applications to visualize and discover geometric concepts.

TECHNOLOGY CORNER CENTER OF GRAVITY

A. In Triangles

Recall that the centroid of a triangle of uniform density and thickness is its center of gravity. You can use dynamic geometry software to demonstrate this property.

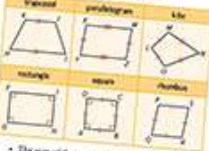


1. Draw $\triangle ABC$. Construct the midpoint of each side and label the midpoints D , E , and F . Construct the medians of the triangle and label the centroid G . Shade the interior of each of the smaller triangles by using the polygon tool to select the 3 vertices of each triangle and then choosing a different color for that region. Observe the shapes and sizes of the smaller triangles while:
 - a. Most any of the smaller triangles be congruent to each other?
 - b. Is there an easily recognized relationship between the triangles?
2. Display the area of each smaller triangle. Move the vertices and observe the areas of the triangles. What is true about each of the 6 triangular areas formed by the medians of a triangle?
3. What would happen if a triangle of uniform density and thickness were placed on a small wooden dowel aligned with one of the medians? What would happen if the triangle were placed on the end of the dowel at point G ?
4. Experiment. Place the pencil at step 3.

6 CHAPTER REVIEW

Key Concepts

Classifying Quadrilaterals (6.1)



- The sum of the interior angle measures of any convex quadrilateral is 360° (Quadrilateral Angle Sum Theorem).
- The sum of the interior angle measures of any convex n -gon is $(n - 2)180^\circ$ (Polygon Angle Sum Theorem).
- The sum of the measures of an exterior angle from each vertex of any convex n -gon is 360° .

Characteristics of Parallelograms (6.2)

- Parallelograms have:
 - opposite sides congruent
 - opposite angles congruent
 - diagonals that bisect each other
 - consecutive angles that are supplementary

- both pairs of opposite angles are congruent
- the diagonals bisect each other
- Two polygons are congruent if they can be divided into congruent corresponding triangles (Polygon Congruence Theorem)
- Parallelograms can be shown to be congruent with SAS.

Rectangles, Rhombi & Squares (6.4)

- The diagonals of a rectangle bisect each other.
- The diagonals of a rhombus:
 - bisect the opposite angles.
 - are perpendicular bisectors of each other.
- A parallelogram is a rectangle if its diagonals are congruent.
- A parallelogram is a rhombus if:
 - its diagonals are perpendicular.
 - a diagonal bisects a pair of opposite angles.
 - 2 consecutive sides are congruent.
- A square is both a rectangle and a rhombus.

Trapezoids & Kites (6.5)

- In an isosceles trapezoid, each pair of base angles is congruent.
- If a trapezoid is a kite, then it is a rhombus.

Biblical Worldview Shaping

Geometry & Design

Use dynamic geometry software to explore the properties of quadrilaterals and their relationships to each other.

KEY NOTES

List each type of quadrilateral that exhibits the stated characteristics.

(6.1-6.3)

- A parallelogram
- Rhombus
- Kite

- Rectangle
- Trapezoid

- Rhombus
- Isosceles trapezoid

- Square

- Rectangle
- Isosceles trapezoid
- Rhombus
- Kite

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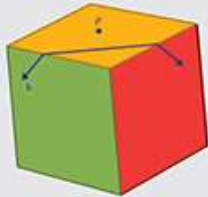
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STEM ACTIVITY

DO PARALLEL LINES EXIST?

What type of lines are used in Euclidean geometry? Can other 2-dimensional surfaces exist as a plane? If you draw a line on a sphere or other curved surface, it won't stay straight and perpendicular to a line. Can you construct a plane on a curved surface? Do these other surfaces have any other properties? Use the STEM activity you will investigate to explore these questions. Use the STEM activity you will investigate to explore these questions. Use the STEM activity you will investigate to explore these questions.



STEM Activity

Introduce each semester's STEM project located in Student Activities.

Chapter Review

Prepare for the chapter test with a summary of key concepts, a review of the biblical worldview theme, and review exercises.