

USING YOUR BOOK

CHAPTER 02 LINEAR RELATIONS

Chapter Introduction

Preview the basic outline of the chapter.

How are mathematical models useful to people?

Seven teenagers were backpacking in Denali National Park in July 2011 when their wind-expedited, a grisly bear got too close for comfort. The group, badly missing their tent, had to huddle in a tight circle in the woods, trying to stay warm.

The group set up an emergency camp, and one unpaired member activated an emergency radio beacon. The device sent out a distress signal to search-and-rescue satellites that relayed the signal to a local rescue center. The signal included the group's geographical and topographical coordinates, which helped guide the searchers to their location.

The COASTS CASSEAT system is a search-and-rescue network that uses radio receivers and transmitters. Modern day search efforts are aided by sophisticated technology as well as search efforts. We are able to pinpoint a satellite and use that to assist searchers in their efforts to find the group. The technology used in search efforts relies on mathematics to help manage such situations and assist others in need (Cal. 6.10).

CHAPTER OVERVIEW

- 2.1 Relations & Functions
 - 2.2 Graphing Linear Equations
 - 2.3 Writing Linear Equations
 - 2.4 Modeling Linear Data
 - 2.5 Piecewise Functions
 - 2.6 Graphing Inequalities
 - 2.7 Distances & Midpoints
- Application Problems—Linear Analysis of Climate Change
- Chapter 2 Review
- College Entrance Test Preparation—Strategy Coaching

ALGEBRA AROUND THE WORLD ARABIC MATH

The Arabic astronomer and mathematician Muhammad al-Khwarizmi (ca. AD 825) described one of his solutions as “completing the square” in the same process we use to solve quadratic equations and derive the quadratic formula. Al-Khwarizmi’s work notably lacks modern notations, using only words to describe numbers rather than symbols.

An example of his “completing the square” method could be written as follows: “A square and ten of its roots are equal to nine and thirty. Take half the number of roots, in this case five, then multiply this by itself and the result is five and twenty. Add this to nine and thirty, which gives four and sixty. Take the square root, which is eight, and subtract from it half the number of roots, namely five, and there remains three. This is the root.”

Al-Khwarizmi also wrote a book called *Kitab al-Jabr wa'l-Muqabala*. Our modern term algebra comes from the word *al-jabr* in that Arabic title, which is roughly translated “balancing” and refers to the balance that must be maintained when solving an equation.

Our numbers are called Hindu-Arabic numerals. Do you know why? Our numerals and place-value system originated in India. Brahmagupta (ca. AD 628) was the first mathematician to work extensively with negative numbers, introduced for debts in the marketplace. Indians were also the first to study irrational numbers such as $\sqrt{3}$ and $\sqrt{5}$.

Arabs adopted the Indian system and introduced it to Spain. This was the trade language of the Muslim world from Persia to Spain.

The transition of the Arabic numeral system to Europe during the Renaissance was the attention of Italian mathematician and poet, Omar Khayyam. Another Arabic mathematician and poet, Omar Khayyam (ca. AD 1100), believed he could justify the solution to any cubic equation using the geometric idea of conic sections. This connection between cubic equations and conic sections was probably the greatest Arabic contribution to algebra because it showed that geometry and algebra are intrinsically related. This idea eventually grew into the field of mathematics called analytic geometry.

Discussion

- Where did the term algebra come from? Explain what the term means.
- Why are our numbers called Hindu-Arabic numerals?
- Research Omar Khayyam’s solution to a cubic equation. Write out the cubic equation and sketch the graph that he used to solve the problem.

$$\begin{aligned} \text{Solve } x^2 + 10x + 39 &= 0 \\ x^2 + 10x + 25 - 25 + 39 &= 0 \\ (x + 5)^2 - 8 &= 0 \\ x + 5 &= \pm\sqrt{8} \\ x &= -5 \pm 2\sqrt{2} \end{aligned}$$

Arabic
Hindu-Arabic Numerals
1 2 3 4 5 6 7 8 9 0



Monument at the Old City of Jerusalem, Israel

APPLICATION PROBLEMS PLANNING A BUSINESS

Suppose you have moved some of your neighbors into a small business and are interested in expanding it. You know that you will need to invest in some new equipment to be successful.

You can purchase a reliable, rear-loading, self-propelled push mower for \$400. You estimate that it will cost \$1.50 per hour to use this mower. For \$200, you can purchase a zero-turn riding mower, which will allow you to mow an average lawn in 30 min using \$3 in gas. However, you are concerned that making such a large initial investment may not be a wise decision.

After talking with several businesspeople in your church, you learn that many small businesses fail due to the lack of a business plan. It is wise to seek advice from others (James 4:13–14). The business owners at church have advised you to answer your business so that you know how much you can charge \$30 per hour and \$150 for your labor.

Compare your labor cost per lawn and then write a function based on the number of lawns.

Write a function for the riding mower and the push mower. Label the variables and determine the domain for each function based on this information.

Write a function modeling the business profit (after gas and labor expenses) and state the profit per lawn that the business makes after breaking even. Explain why or why not.

Does the fact that one scenario provides a much higher profit for the business imply that the owner is charging an excessive price for that service? Explain why or why not.

Suppose gas prices skyrocket over the course of the summer. The average 30 min lawn now uses \$5 in gas. Should the change in gas prices affect the pricing in your business plan? Explain.



- How many weeks would it take for you to break even (assuming you are able to mow enough lawns to work a 20 hr week)?
 - using the push mower
 - using the riding mower

- How does this additional analysis affect your choice on which mower to buy?

- Write a system of inequalities modeling the business profit on the graph shown in exercise 1.
 - using the push mower
 - using the riding mower

- Write a function modeling the business profit (after gas and labor expenses) and state the profit per lawn that the business makes after breaking even.
 - using the push mower
 - using the riding mower

- Does the fact that one scenario provides a much higher profit for the business imply that the owner is charging an excessive price for that service? Explain why or why not.

- Suppose gas prices skyrocket over the course of the summer. The average 30 min lawn now uses \$5 in gas. Should the change in gas prices affect the pricing in your business plan? Explain.

Algebra around the World

Explore and discuss the historical development of mathematics across cultures.

Application Problems

Apply learned math skills and biblical world-view concepts to solve real-world problems.

Essential Question & Learning Targets

Start each section with a question about the key idea and a list of the skills you should expect to learn.

Keyword Searches

Locate additional information, interactive activities, and supplementary resources online.

Key Concepts

Read through explanations of key concepts and remember important ideas highlighted within margin boxes.

8.5 LOGARITHMIC FUNCTIONS

What does Scripture reveal about the pursuit of advances in human understanding?

I will be able to

- compare between exponential and logarithmic forms.
- evaluate logarithmic expressions.
- graph logarithmic functions.
- formulate a biblical view of the relationship between math and Scripture.

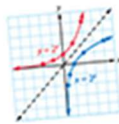


The Indonesian volcano Kawah Ijen (Green Crater) contains the world's largest acidic volcanic crater lake. Sulfuric acid gives the water its green color. The acidity of a solution depends on the concentration of hydrogen ions (H^+) ions, which can vary from 1 to 1×10^{-14} moles per liter. In order to conveniently measure this wide range of very small numbers, scientists use a logarithmic function, the pH scale.

Quote of sulfur on the right, fully water of the lake with pH = 1.3

Logarithmic functions are inverses of exponential functions. Consider the exponential function $y = 2^x$ and its inverse function $x = 2^y$. Recall that the inverse of a function is found by exchanging the x and y variables and that the graph of the inverse is the reflection of the function's graph across the line $y = x$.

$x = 2^y$	$x = 2^y$
x	y
-2	1
-1	2
0	3
1	4
2	5



KEYWORD SEARCH
logarithmic function
exponent

Previously we had no method of solving for y in $x = 2^y$. The logarithmic function provides the means for solving such an equation.

DEFINITION
Logarithmic function: Assuming b and x are positive numbers and $b \neq 1$, $y = \log_b x$ if and only if $x = b^y$.

EXAMPLE 2: Graphing Inequalities Using Slope-Intercept Form

Graph $f(x, y) | 3x - 4y < -8$.

Answer

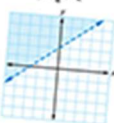
$$\begin{aligned} 3x - 4y < -8 \\ -4y < -3x - 8 \\ y > \frac{3}{4}x + 2 \end{aligned}$$

1. Solve the inequality for y .

2. Graph the boundary as a dashed line using the $(0, 2)$ and the slope of $\frac{3}{4}$.

3. Shade the half-plane above the line since there are y -coordinate greater than $\frac{3}{4}x + 2$.

4. Use the test point $(0, 0)$ to confirm that the half-plane below the line does not contain solutions to the inequality: $3(0) - 4(0) \not< -8$.



RECALL
Reverse the inequality symbol when dividing both sides by a negative number.

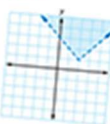
RECALL
When the inequality is $<$ or $>$, the boundary line should be dashed.

While 2-variable absolute value inequalities are not linear, they can be graphed using these same methods. Graph the boundary using a solid or dashed line and then shade the region containing the solutions.

EXAMPLE 3: Graphing Absolute Value Inequalities

Graph $y > |x - 2| + 1$.

Answer



1. Translate the graph of $y = |x|$ two units right and one unit up and graph the boundary $y = |x - 2| + 1$ as a dashed line.

2. Shade the region above the boundary since these points have a y -coordinate greater than $|x - 2| + 1$.

3. Confirm that the region containing $(0, 0)$ is not part of the solution: $0 > |0 - 2| + 1$.

Linear inequalities are often used to model real-world situations in which there are acceptable ranges of values. The domains and range of these applications are often limited to nonnegative numbers.

EXAMPLE 4: Solving Real-World Problems

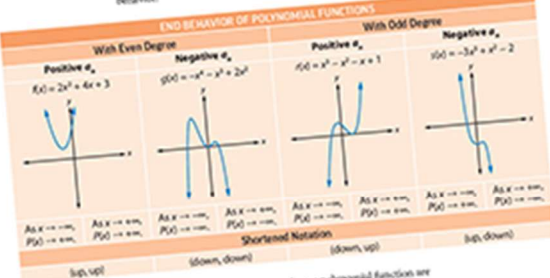
The athletic department is attempting to raise \$600 by selling T-shirts and sweatshirts featuring the school mascot. They make \$4 on each T-shirt and \$10 on each sweatshirt.

- Write a linear inequality modeling the funds raised by the sale and graph the inequality.
- What is the minimum number of sweatshirts that need to be sold if they have 9 dozen T-shirts?

Examples & Skill Checks

Study the step-by-step reasoning in solving example problems, and check your understanding by completing targeted exercises mapped to each example.

You can graph a polynomial function by understanding important characteristics of the function and plotting several key points. A function's **end behavior** describes its values on the left as x approaches negative infinity ($x \rightarrow -\infty$) and on the right as x approaches positive infinity ($x \rightarrow \infty$). The degree of the polynomial and the sign of the leading coefficient determine the function's end behavior.



The end behaviors of an even-degree polynomial function are similar. These functions have an even number of real roots. The fact that the graph of $g(x)$ is tangent to the x -axis at the origin indicates that 0 is a double root of the function. Polynomial functions of odd degree must have at least 1 real zero because their 2 end behaviors are opposite.

EXAMPLE 1: Describing a Polynomial

Given $P(x) = x^3 - 2x^2 - 5x + 6$,

- state the function's end behavior and
- describe the number of complex and real zeros.

Answers

- As $x \rightarrow -\infty$, $P(x) \rightarrow -\infty$; as $x \rightarrow \infty$, $P(x) \rightarrow \infty$. (down, up)
- The polynomial has 3 complex roots. One of these roots must be real. The other two may be imaginary or real.

- The degree of the polynomial is odd, and the leading coefficient is positive.
- The polynomial is of third degree.
- Odd-degree polynomials must have at least 1 real zero.

AFTERSCHOOLHELP



Graphing Polynomial Functions

QR Codes

Link to valuable tutorial content located on AfterSchoolHelp.com.

Exercises

Build and maintain your skills with carefully sequenced exercises that emphasize the essential question and essential mathematical practices (MPs).

Cumulative Review

Systematically review key concepts and practice strategies for standardized testing.

44. **MP4** Essential Question: Estimate the area of Yellowstone National Park by using determinants to find the area of the 3 triangles (1 unit = 3 mi). Write a system of 2 equations in 2 variables and solve using Cramer's rule.
45. **MP7** The general form of a parabola graphed on the coordinate plane is $y = ax^2 + bx + c$. What is the equation of the parabola containing $(-2, 3)$, $(-1, -2)$, and $(1, 4)$?
46. **MP4** The sons of 3 brothers' ages in 28, Jesse is as old as the sum of Sammy's and Billy's ages. Find each age if the difference of Sammy's and Billy's ages is 8 less than three times Jesse's age.

CUMULATIVE REVIEW

Solve for x . (4,2)

47. $x = \begin{bmatrix} -1 & 12 \\ 10 & 9 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

Determine the dimensions of each matrix product, if the product is undefined, state so. (4,3)

48. $\begin{bmatrix} 2 & 1 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Find each matrix product. (4,3)

49. $\begin{bmatrix} -4 & 9 \\ 1 & 12 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$

Find the distance between each pair of points; then find the midpoint of the segment connecting them. (2,7)

50. $(2, 5)$ and $(7, -3)$; $d = \dots$; $M = \dots$

Determine which value is in the solution set. List all that apply. (3,8)

51. $2x + 7 = 5$

- A -10
B -6
C -5
D 0
E none of these

PROGRAMMING CHALLENGE

System Solver

Write a program that solves a system of 3 linear equations in 3 variables. The program should prompt the user to enter each element of the coefficient and the constant matrices and then use Cramer's rule to solve the system. If the system does not have a unique solution, the program should output the message "cannot be solved."

Programming Challenge

Challenge yourself by writing programs for the TI-84 Plus as described in this unique feature in each chapter.

A. EXERCISES

Solve by factoring.

- $x^2 + 6x + 8 = 0$
- $x^2 - x - 42 = 0$
- $x^2 - 6x - 45 = 0$
- $x^2 + 3 = 2 = 0$
- $2x^2 + 7x = 0$
- $5x^2 + 4x = 0$
- $2x^2 - 9x - 5 = 0$
- $8x^2 - 34x - 8 = 0$
- $6x^2 + 37x + 56 = 0$
- $4x^2 - 15x + 9 = 0$
- $5x^2 - 32x - 21 = 0$
- $3x^2 - 20x - 32 = 0$
- $x^2 - 4x = 0$
- $x^2 - 9x^2 + 18y = 0$
- $x^2 + 6x + 9 = 0$

B. EXERCISES

Solve by factoring. Remember to write each equation in standard form first.

- $y^2 = 4y - 4$
- $x^2 - 49 = 0$
- $18. 7x = 8 - x^2$
- $x^2 + 27 = 12x$
- $5 = 63y + 79$
- $21. 7x + 30 = 2x^2$
- $22. 3x^2 + 7x + 11 = x^2 - 5x - 7$
- $23. 8x^2 + 8g = 3 - 13g$
- $24. x^2 + 6x^2 - 65 = 0$
- $25. x^2 + 81 = -16x^2$
- $26. 10x^2 + x^2 = 3x$
- $27. 36x^2 + 5x - 12x^3$
- MP4** Read solves $x^2 - x = 6$ by factoring it as $(x - 3)(x + 2)$. He states that $x = 6$ or $x = 1 = 6$ and determines that $x = 6, 7$. Describe and correct his error.
- $31. x = -\frac{1}{2}, -2$

Write a standard-form quadratic equation with the given solutions.

29. $x = 2, 11$
30. $x = -6, 9$
32. $x = -\frac{1}{2}, \frac{1}{3}$
33. $x = -\frac{1}{2}, \frac{1}{3}$

Write a quadratic equation and solve.

34. A number sums to 33 and whose product is 266.
35. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

36. A number sums to 33 and whose product is 266.
37. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

38. A number sums to 33 and whose product is 266.
39. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

40. A number sums to 33 and whose product is 266.
41. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

42. A number sums to 33 and whose product is 266.
43. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

44. A number sums to 33 and whose product is 266.
45. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

46. A number sums to 33 and whose product is 266.
47. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

48. A number sums to 33 and whose product is 266.
49. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

50. A number sums to 33 and whose product is 266.
51. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

52. A number sums to 33 and whose product is 266.
53. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

54. A number sums to 33 and whose product is 266.
55. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

56. A number sums to 33 and whose product is 266.
57. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

58. A number sums to 33 and whose product is 266.
59. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

60. A number sums to 33 and whose product is 266.
61. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

62. A number sums to 33 and whose product is 266.
63. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

64. A number sums to 33 and whose product is 266.
65. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

66. A number sums to 33 and whose product is 266.
67. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

68. A number sums to 33 and whose product is 266.
69. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

70. A number sums to 33 and whose product is 266.
71. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

72. A number sums to 33 and whose product is 266.
73. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

74. A number sums to 33 and whose product is 266.
75. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

76. A number sums to 33 and whose product is 266.
77. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

78. A number sums to 33 and whose product is 266.
79. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

80. A number sums to 33 and whose product is 266.
81. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

82. A number sums to 33 and whose product is 266.
83. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

84. A number sums to 33 and whose product is 266.
85. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

86. A number sums to 33 and whose product is 266.
87. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

88. A number sums to 33 and whose product is 266.
89. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

90. A number sums to 33 and whose product is 266.
91. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

92. A number sums to 33 and whose product is 266.
93. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

94. A number sums to 33 and whose product is 266.
95. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

96. A number sums to 33 and whose product is 266.
97. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

98. A number sums to 33 and whose product is 266.
99. A number sums to 28 and whose product is 180.

Write a quadratic equation and solve.

100. A number sums to 33 and whose product is 266.
101. A number sums to 28 and whose product is 180.

COLLEGE ENTRANCE TEST PREPARATION STRATEGY: ELIMINATING CHOICES

Eliminating choices by working backwards is often an effective strategy on questions where a straightforward solution is not evident, when numerical choices are arranged in ascending order, trying the middle value can be extremely efficient. Consider the following question.

Which of the following is a possible value for x if $x^2 = 2x + 35$?

- A 2 D 5
B 3 E 6
C 4

You may have forgotten how to solve quadratic equations. $x^2 = 2x + 35$ yields $x^2 - 2x - 35 = 0$, which is too small. You could eliminate 2 and 3 as solutions as well. Substituting 5 for correct answer is D.

Another way to eliminate choices is to be alert for this answer. Consider the question above rearranged as $x^2 - 2x - 35 = 0$. The correct response is E, since $x = 5$ is $5^2 - 2(5) - 35 = 25 - 10 - 35 = -40$. Use the strategy of eliminating choices by working backwards in the following exercises.

1. Which of the following is a possible value for x if $x^2 - 2x = 235$?

- A 17 D 23
B 19 E 25
C 21

2. Which of the following is a solution to both $x^2 - x^2 = 12$ and $25x = x^2 - 18$?

- A 1 D 4
B 2 E 5
C 3

3. Which of the following is a solution to $3(x - 5) = 19$?

- A 4 D 7
B 5 E 8
C 6

4. The sum of the digits of a 3-digit number is 17. The units digit is twice the hundreds digit. The tens digit is one more than the hundreds digit. What is the number?

- A 234 D 436
B 346 E 537
C 396

5. What are the zeros of the polynomial function $f(x) = x^2 + 2x^2 - 5x + 6$?

- A $(-3, -2, 1)$ D $(-1, 2, 3)$
B $(-2, -1, 3)$ E $(1, 2, 3)$
C $(-2, 1, 3)$

6. Which of the following is a solution to $x^2 + x = 2750$?

- A -54 D -51
B -53 E -49
C -52

7. Bob's age in fifteen years less than half his father's age. In six years, his father's age will be eleven years more than triple Bob's age. What is his father's age now?

- A 7 D 44
B 13 E 50
C 28

8. What is the area of a semicircle with a radius of 5 ft?

- A $31\frac{1}{2}$ ft² D $75\frac{1}{2}$ ft²
B $41\frac{1}{2}$ ft² E 314 ft²
C $39\frac{1}{2}$ ft²

College Entrance Test Preparation

Learn content and test-taking strategies useful for successfully completing college entrance exams.



Where does our knowledge of mathematics come from?

On January 12, 2010, a devastating earthquake along with many serious aftershocks rocked Haiti's capital, Port-au-Prince. The Haitian government estimated 300,000 people dead, 300,000 injured, and 1.5 million homeless. This earthquake remains one of the deadliest in recent history. It also shook the world into action.

Humanitarian aid and donations began pouring in from countries around the world. Survivors were rescued from the rubble, the injured were treated, and the dead were buried. In the months that followed, the people of Haiti began to find a new way of life amidst the rubble. But the need for aid wasn't over.

In October 2010 cholera broke out north of Port-au-Prince because of poor water and sanitation after the earthquake. By March 2011 cholera had killed over 4000 people and infected at least 250,000. How can we predict and prevent such loss in the future?

Analysis of climate conditions before cholera epidemics like the one in Haiti led scientists and mathematicians to a stunning conclusion. It may be possible to determine when epidemics of cholera and other similar diseases are more likely to occur by monitoring local weather conditions, including temperature changes. An increase in rainfall as well as an increase in the average monthly minimum temperature of just 1°C could signal an outbreak of cholera in the next 4 months. Models like this could provide early warning signs to those at risk and help prevent future outbreaks. Because we are created in God's image (Gen. 1:27), we have the ability to model and interpret God's world, allowing us to help those in need (Heb. 13:16).

CHAPTER OVERVIEW

1.1 Real Number Operations

1.2 Simplifying Algebraic Expressions

Algebra around the World—Ancient Numeration

1.3 Solving Equations

1.4 Applying Equations

1.5 Solving Inequalities

1.6 Compound Inequalities

1.7 Absolute Value Equations

Technology Corner—Replicating Formulas

1.8 Absolute Value Inequalities

Application Problems—Climate & Disease

Chapter 1 Review

College Entrance Test Preparation—Content: Arithmetic

1.2 SIMPLIFYING ALGEBRAIC EXPRESSIONS

How can I know an algebraic expression is simplified?

I will be able to

- simplify algebraic expressions.
- classify polynomials by their number of terms and degree.
- add and subtract polynomials.



Expressions containing powers can often be simplified using the properties of exponents. Reviewing these familiar properties and definitions will build a strong foundation for working with more complicated expressions.

Engineers use mathematics to design complex but beautiful designs like the Falkirk Wheel.

PROPERTIES OF EXPONENTS		
Property or Definition		Example
Product	$x^a \cdot x^b = x^{a+b}$	$x^3 \cdot x^4 = x^7$
Quotient	$\frac{x^a}{x^b} = x^{a-b}; x \neq 0$	$\frac{x^7}{x^2} = x^5$
Power	$(x^a)^b = x^{ab}$	$(x^3)^4 = x^{12}$
Power of a Product	$(xy)^a = x^a y^a$	$(2x)^3 = 8x^3$
Power of a Quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}; y \neq 0$	$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$
Zero Exponent	$x^0 = 1; x \neq 0$	$5^0 = 1$
Negative Exponent	$x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n; x \neq 0$	$3^{-2} = \frac{1}{9}$ and $\left(\frac{1}{b}\right)^{-2} = b^2$

A term is generally considered to be simplified when each base appears only once, all fractions have been reduced, and there are no parentheses or negative exponents.

EXAMPLE 1: Using Properties of Exponents

Simplify each term.

a. $(4x^2y)(-3x^{-2}y^{-4})$ b. $\frac{r^4st^7}{r^{-1}s^3t^5}$ c. $\left(\frac{-2ac^4}{5b^3}\right)^2$ d. $(3x^{2n}y^2)^3(2x^3y^{-2m})^{-2}$

Answers

a. $(4x^2y)(-3x^{-2}y^{-4})$
 $= 4(-3)x^{2+(-2)}y^{1+(-4)}$
 $= -12x^0y^{-3}$
 $= -\frac{12}{y^3}$

1. Apply the Product Property.

2. Apply the definitions of a negative exponent and a zero exponent.

b. $\frac{r^4st^7}{r^{-1}s^3t^5}$
 $= r^{4-(-1)}s^{1-3}t^{7-5}$
 $= r^5s^{-2}t^2$
 $= \frac{r^5t^2}{s^2}$

1. Apply the Quotient Property.

2. Apply the definition of a negative exponent.

c. $\frac{(-2ac^4)^2}{(5b^3)^2} = \frac{(-2)^2a^2(c^4)^2}{5^2(b^3)^2}$
 $= \frac{4a^2c^8}{25b^6}$

1. Apply the Power of a Quotient and Power of a Product Properties.

2. Apply the Power Property.

d. $(3x^{2n}y^2)^3(2x^3y^{-2m})^{-2}$
 $= (3^3x^{6n}y^6)(2^{-2}x^{-6}y^{4m})$
 $= \frac{27}{4}x^{6n-6}y^{6+4m}$

1. Apply the Power of a Product Property.

2. Apply the Product Property.

SKILL CHECK EXERCISES 15, 21

Algebraic expressions are classified in many ways. A *monomial* is a single-term expression. The numerical factor in a monomial is called the **coefficient**. In $4x^2$ the coefficient is 4, and in x^3 the coefficient is 1. *Polynomials* are classified by the number of terms (or monomials) they contain.

DEFINITIONS

A **monomial** is a single-term expression containing a constant, a variable with a whole number exponent, or the product of a constant and 1 or more such variables.

A **polynomial** is an algebraic expression of 1 or more monomials. Each monomial is a term of the polynomial.

A **binomial** is a polynomial with exactly 2 terms.

A **trinomial** is a polynomial with exactly 3 terms.

Polynomials are also classified by their degree. The **degree of a monomial** is the sum of the exponents of its variables. For example, the degree of $7x^2y^3$ is $2 + 3 = 5$. The **degree of a polynomial** is equal to the degree of its highest-degree term. It is customary to write the terms in order of descending degree of a variable.

AFTERSCHOOLHELP



Simplifying Algebraic Expressions

EXAMPLE 2: Finding the Degree of a Polynomial

Determine the degree of each expression.

- a. $2xy$ b. $5x^2y$ c. 7 d. $4x^3y + 7x^2y^3 - 15y^4$

Answers

- | | | |
|------------------------------|--|---|
| a. $2xy$ | $1 + 1 = 2$ | Add the exponents of each variable. |
| b. $5x^2y$ | $2 + 1 = 3$ | |
| c. 7 | 0 | Since 7 can be thought of as $7x^0$, its degree is 0. |
| d. $4x^3y + 7x^2y^3 - 15y^4$ | $4x^3y: 3 + 1 = 4$
$7x^2y^3: 2 + 3 = 5$
$15y^4: 4$ | 1. Determine the degree of each term.
2. Select the degree of the highest-degree term. |
- degree of the polynomial: 5

SKILL CHECK EXERCISES 1, 3

The Distributive Property is often used to simplify expressions containing polynomials. Notice how it is used to remove the parentheses from the following expression.

$$-2x(5y - 7) = -10xy + 14x$$

Recall that this property is also the basis for combining like terms (terms that contain the same variables with the same exponents).

$$3z + 5z = (3 + 5)z = 8z$$

Polynomial expressions are considered simplified when each term is simplified and all like terms are combined.

EXAMPLE 3: Combining Like Terms

Simplify each expression.

- a. $(3x^2 + 2x - 8) + (5x^2 - 3x + 9)$ b. $2a(2a^2b + ab - 4b^3) - (5a^3b + 3ab^2 - 4ab^3)$

Answers

- | | |
|---|---|
| a. $(3x^2 + 2x - 8) + (5x^2 - 3x + 9)$
$= (3x^2 + 5x^2) + (2x - 3x) + (-8 + 9)$
$= 8x^2 - x + 1$ | 1. Identify like terms. This regrouping may be done mentally.
2. Combine like terms. |
| b. $2a(2a^2b + ab - 4b^3) - (5a^3b + 3ab^2 - 4ab^3)$
$= 4a^3b + 2a^2b - 8ab^3 - 5a^3b - 3ab^2 + 4ab^3$
$= 4a^3b + 2a^2b - 8ab^3 - 5a^3b - 3ab^2 + 4ab^3$
$= -a^3b + 2a^2b - 3ab^2 - 4ab^3$ | 1. Remove parentheses. Be sure to distribute -1 to each term in the second expression.
2. Identify like terms.
3. Combine like terms. |

SKILL CHECK EXERCISE 29

VOCABULARY

- binomial
- coefficient
- degree of a monomial
- degree of a polynomial
- monomial
- polynomial
- trinomial

A. EXERCISES

State the degree of each polynomial. Then classify each as a monomial, binomial, trinomial, or other polynomial.

1. $4xy$ 2. $3x^2y + 4xy - 5xy^3$ 3. $5a^3b^2c - 7abc + 3a^4b$
 4. $5c^2d + 25cd^2$ 5. 9 6. $4a^2bc + 3x^2y - 5d + 6c^2d^4$

Simplify each expression.

7. $(2x^2 + 3x - 8) + (5x^2 - 9x + 2)$
 8. $(5a^3 - b) - (4a^3 + 2b)$
 9. $(12c^2 + 3cd - 5d^2) + (6cd + 2d^2)$
 10. $(7a^2 - 9b^2) + (3a^2 + 4ab + 7b^2)$
 11. $6a^2b \cdot 9ab^2$
 12. $5x^2 \cdot 3x^3 \cdot 2x^0$
 13. $(2a^3b)^2$
 14. $(7x^4)^0$
 15. $(8a^2b)(4ab^3)$
 16. $\left(\frac{2x}{y}\right)^3$

B. EXERCISES

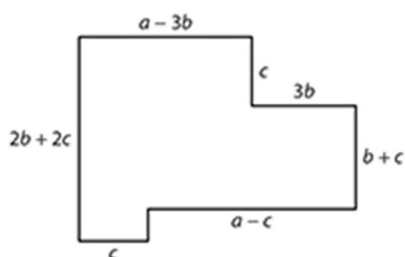
Simplify each expression.

17. $4x^n \cdot 5x^m$
 18. $(x^2)^{n+1}$
 19. $\left(\frac{x^2}{b^3}\right)^{2n}$
 20. $\left(\frac{a^{2x}}{b^y}\right)^{3x}$
 21. $\frac{(x^2y)^3}{x^3y^{-5}}$
 22. $\frac{c^2d^3(c^{-3}d)^2}{(cd^{-3})^{-2}}$
 23. $(2x^2y^5)^2 + 6x^4y^{10}$
 24. $(3x^3y^6)^2 + (2x^2y^4)^3$
 25. $3a^2(2a + 4x - 9)$
 26. $5xy^2(x^2y^3 - 14xy)$
 27. $a^2cd^3(a^5 - 3acd + 7a)$
 28. $2(3c^2 + 4c - d^2) + (6c^2 - 5c + 6d^2) - \frac{1}{3}(6c^2 - 3d^2)$
 29. $(3a + 4b - 9c) + 3(7a - 3b + c) - \frac{1}{2}(4a - 6b + 2c)$
 30. $\left(\frac{2}{3}x^2y - \frac{3}{4}xy^2\right) + \left(\frac{1}{4}xy - \frac{1}{2}xy^2\right) - \left(\frac{1}{6}x^2y - \frac{1}{3}xy\right)$
 31. $(c^2d + cd^2) - \left(\frac{3}{5}c^2d + \frac{1}{4}cd^2\right)$

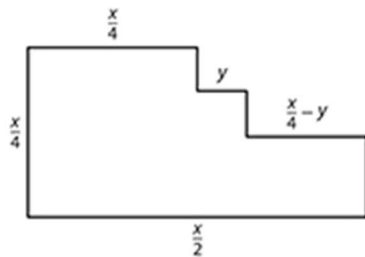
32. **Essential Question:** When is an algebraic expression fully simplified?

Write a simplified polynomial expression for the perimeter of each figure.

33.



34.



35. A polygon has sides with lengths of $11x$, $9x + 2$, $x - 3$, $10x - 7$, and $2x + 1$. Write a simplified polynomial expression for its perimeter.

C. EXERCISES

Simplify each expression.

36. $\left(\frac{2}{3}x^2 - \frac{5}{7}x + \frac{1}{2}\right) - \left(\frac{1}{4}x^2 + \frac{2}{5}x - \frac{1}{4}\right)$
 37. $(4.75x^2 - 32.6x + 87.5) - (12.8x^2 - 1.79x + 1.74)$

Find the values of a , b , and c that make each statement true.

38. $(5x^2 - bx + c) + (ax^2 - 7x + 8) = 7x^2 - 12x + 5$
 39. $(ax^2 - 9x - c) - (3x^2 + bx - 8) = 7x^2 - 12x + 5$
 40. $\frac{x^ay^bz^8}{x^5y^az^c} = \frac{y^4}{x^2}$

CUMULATIVE REVIEW

Determine whether each statement is *always*, *sometimes*, or *never* true. [1.1]

41. The product of 2 whole numbers is a whole number.
42. The difference of 2 whole numbers is rational.
43. The product of 2 rational numbers is irrational.
44. The quotient of 2 real numbers is rational.

Simplify each expression. [1.1]

45. $|-9 - 25| + 3 - |-5| - (-11 + 7)^2$

46. $[3 - (4 + |-2|)^2] - 4^2$

Evaluate when $x = -3$ and $y = 5$. [1.1]

47. $\frac{|x| - (2 - y)}{(-x + y)^2}$

48. $\frac{(-x + y^2) - |x|}{-|x - y|}$

49. Which subset of the real numbers does not include 7? [1.1]

- A \mathbb{N}
- B \mathbb{W}
- C \mathbb{Z}
- D \mathbb{Q}
- E All of these include 7.

50. To which subsets of the real numbers does $\sqrt{7}$ belong? List all that apply. [1.1]

- A \mathbb{N}
- B \mathbb{Z}
- C \mathbb{Q}
- D \mathbb{Q}'
- E \mathbb{R}


ALGEBRA AROUND THE WORLD

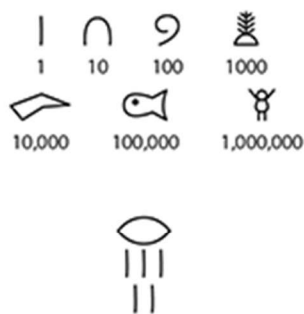
ANCIENT NUMERATION

Egypt, Babylon, and China, three of the world's oldest cultures, each developed a numeration system. These numeration systems permitted each culture to develop mathematical skills.

Egyptian

Egyptian numeration symbols are hieroglyphics, or picture symbols, and are expressed in multiples of 10 but without place values. The Rhind Papyrus, the best-preserved Egyptian mathematical papyrus (ca. 1700 BC), contains about 85 mathematical problems and their solutions.

Historical records show that the Egyptians knew enough algebra to use fractions, solve simple quadratics, and calculate areas and volumes. Their formula for the area of a circle was $A = \left(\frac{8d}{9}\right)^2$, where d is the diameter. This gave a very good approximation of 3.1605 for π . Their fractions, however, were always unit fractions (with 1 as the numerator). They used the symbol  for this numerator. The diagram shows the symbols equivalent to $\frac{1}{5}$.

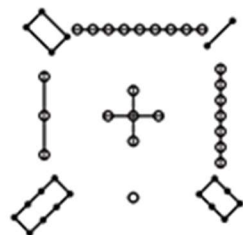


Chinese

The most significant ancient Chinese mathematical treatise is *Jiuzhang suanshu*, or *The Nine Chapters on the Mathematical Art*. This book included area formulas for triangles and trapezoids and introduced negative numbers. For instance, if T indicated 6, then \mathcal{Y} represented -6 .

The diagram illustrates ancient Chinese numeration in a magic square, called the Lo Shu. A magic square is an arrangement of numbers such that the sum of the rows, columns, or diagonals is always constant.

In the thirteenth century the Chinese invented a way to solve equations numerically. Two works by Zhu Shijie, *Introduction of Mathematical Science* and *Precious Mirror of Four Elements*, set forth Chinese algebra. As early as AD 906 the Chinese had also used matrices to find solutions to systems of equations, a method frequently used today by computers.



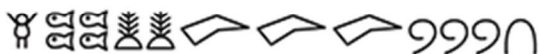
The seated colossi of Pharaoh Rameses II at Abu Simbel in Egypt.



Sculpted lions guard the entrance to China's Forbidden City.

Discussion

1. Joseph used the ancient Egyptian numeration system in tracking the storage and distribution of grain and in the construction of storage facilities. If Joseph were looking at the following values for units of grain, what would he get as the total?



2. Identify each Egyptian fraction.



3. If the grid represents the Chinese Lo Shu pictured earlier and the upper left number is 4, complete the magic square. What is the constant sum?

4		

4. Research ancient Babylonian numerals to answer the following questions.
- What base did the Babylonians use for their number system?
 - Write the Babylonian representations for numbers 1–12.
 - Write the Babylonian representations for the numbers 20, 30, 40, and 50.

When an equation contains more than 1 operation, these operations are “undone” by performing the inverse operations in the reverse order of operations on both sides of the equation.

EXAMPLE 1: Solving a 2-Step Equation

Solve $5x + 12 = -18$.

Answer

$$5x + 12 - 12 = -18 - 12$$

$$5x = -30$$

$$\frac{5x}{5} = \frac{-30}{5}$$

$$x = -6$$

$$5(-6) + 12 = -18$$

1. Subtract 12 from both sides.
2. Divide both sides by 5.
3. Check mentally.

SKILL CHECK EXERCISES 1, 11 ✓

The Distributive Property is often used to simplify both sides of more complicated equations. If the simplified equation has the same variable on both sides of the equation, the Addition Property of Equality is used to move all the terms with the variable to the same side of the equation and all the constants to the other side.

EXAMPLE 2: Solving a Multistep Equation

Solve $-3(x + 2) = 4(x + 18) - 1$.

Answer

$$-3x - 6 = 4x + 71$$

$$-6 = 7x + 71$$

$$-77 = 7x$$

$$-11 = x$$

$$x = -11$$

$$-3[(-11) + 2] = 4[(-11) + 18] - 1$$

$$27 = 27$$

1. Simplify both sides.
2. Add 3x to both sides and subtract 71 from both sides to collect the terms containing the variable on one side of the equation and constant terms on the other side.
3. Divide both sides by 7.
4. Apply the Symmetric Property of Equality.
5. Check mentally.

SKILL CHECK EXERCISE 15 ✓

An equation can be cleared of fractions by multiplying both sides by a common denominator, preferably the least common denominator (LCD). Decimals can be eliminated by multiplying by a power of 10.

AFTERSCHOOLHELP



Solving Equations