USING YOUR BOOK

Essential Question & Learning Targets

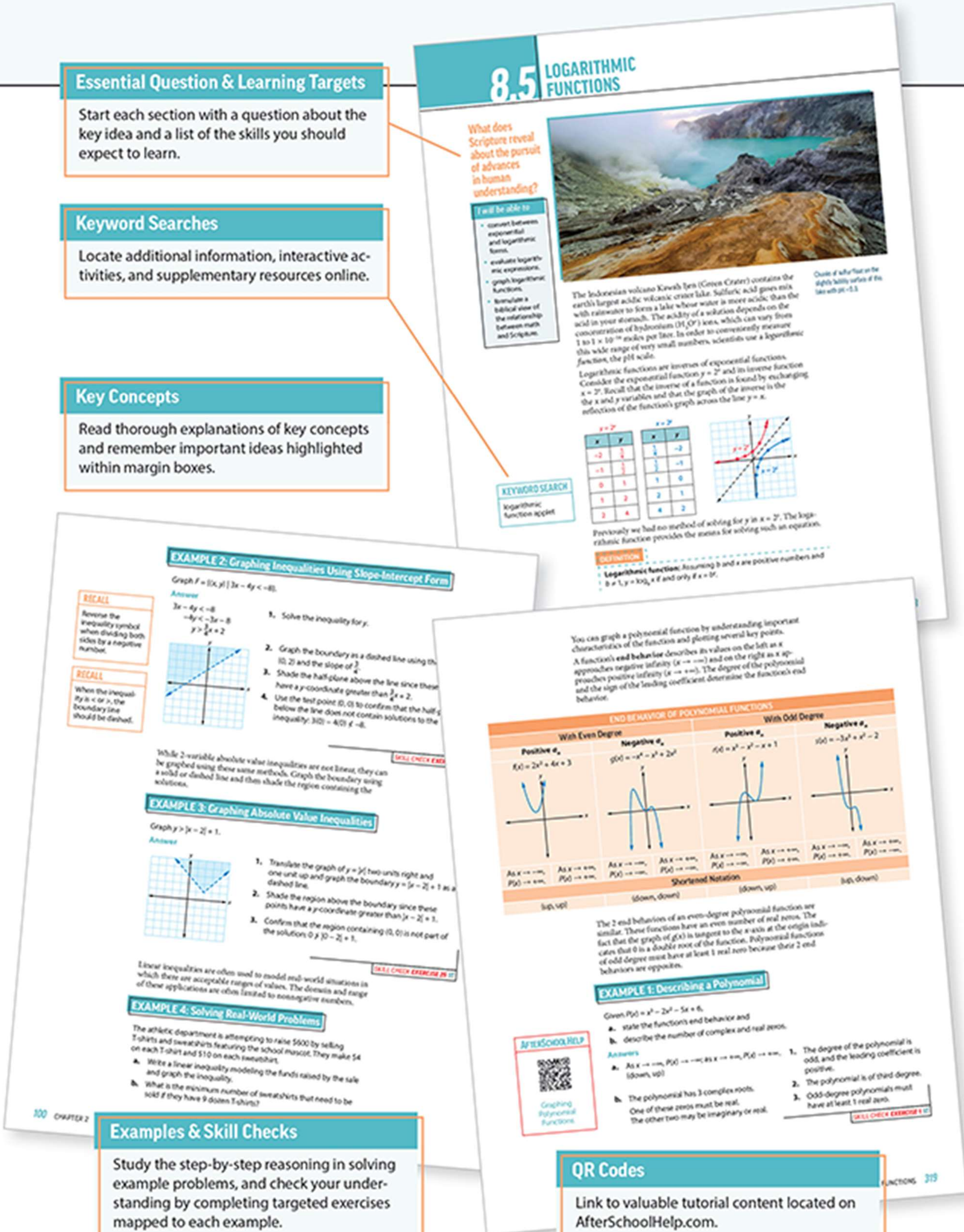
Start each section with a question about the key idea and a list of the skills you should expect to learn.

Keyword Searches

Locate additional information, interactive activities, and supplementary resources online.

Key Concepts

Read thorough explanations of key concepts and remember important ideas highlighted within margin boxes.



Examples & Skill Checks

Study the step-by-step reasoning in solving example problems, and check your understanding by completing targeted exercises mapped to each example.

QR Codes

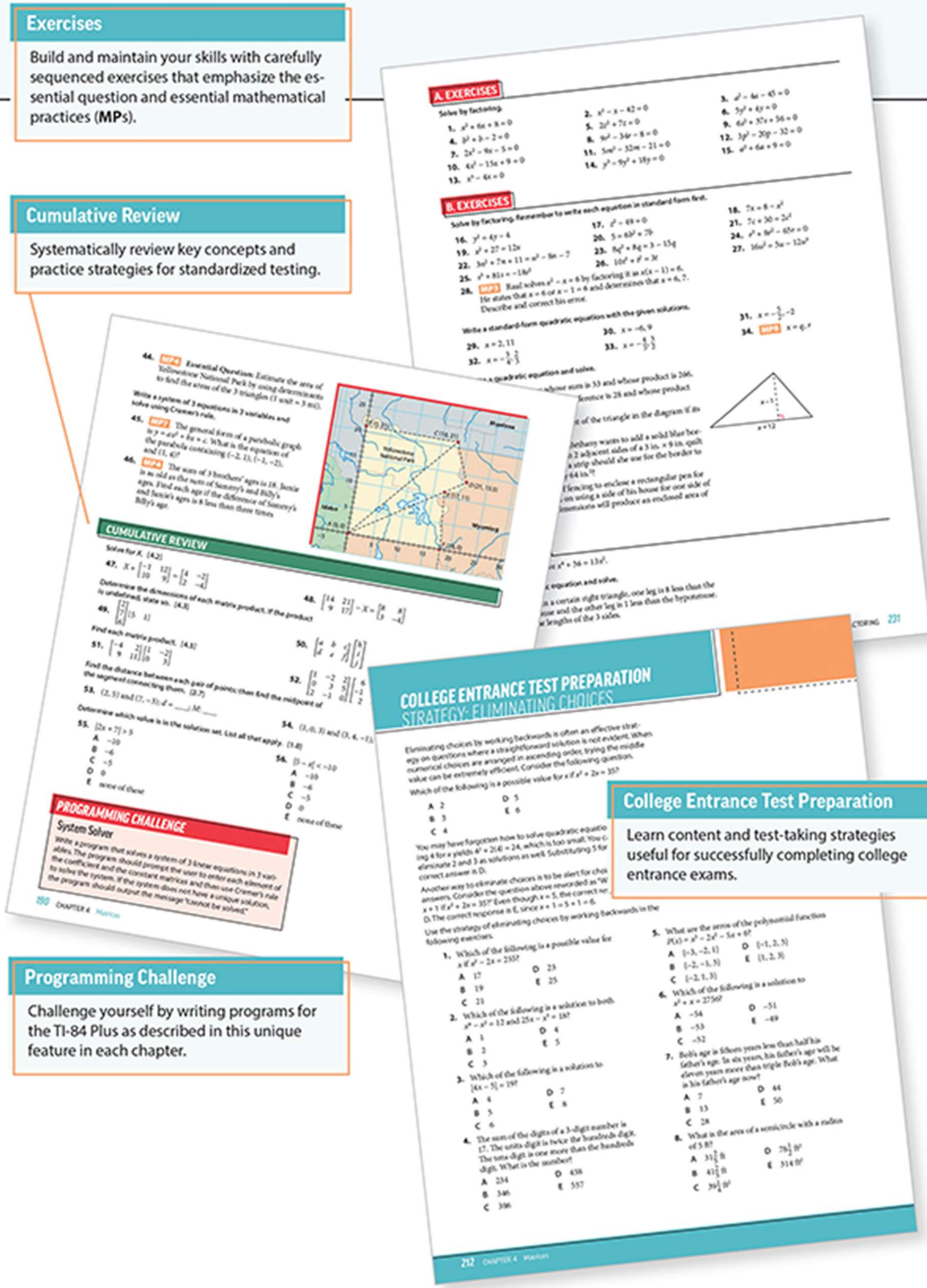
Link to valuable tutorial content located on AfterSchoolHelp.com.

Exercises

Build and maintain your skills with carefully sequenced exercises that emphasize the essential question and essential mathematical practices (**MP**s).

Cumulative Review

Systematically review key concepts and practice strategies for standardized testing.



College Entrance Test Preparation

Learn content and test-taking strategies useful for successfully completing college entrance exams.

Programming Challenge

Challenge yourself by writing programs for the TI-84 Plus as described in this unique feature in each chapter.

Where does our knowledge of mathematics come from?

On January 12, 2010, a devastating earthquake along with many serious aftershocks rocked Haiti's capital, Port-au-Prince. The Haitian government estimated 300,000 people dead, 300,000 injured, and 1.5 million homeless. This earthquake remains one of the deadliest in recent history. It also shook the world into action.

Humanitarian aid and donations began pouring in from countries around the world. Survivors were rescued from the rubble, the injured were treated, and the dead were buried. In the months that followed, the people of Haiti began to find a new way of life amidst the rubble. But the need for aid wasn't over.

In October 2010 cholera broke out north of Port-au-Prince because of poor water and sanitation after the earthquake. By March 2011 cholera had killed over 4000 people and infected at least 250,000. How can we predict and prevent such loss in the future?

Analysis of climate conditions before cholera epidemics like the one in Haiti led scientists and mathematicians to a stunning conclusion. It may be possible to determine when epidemics of cholera and other similar diseases are more likely to occur by monitoring local weather conditions, including temperature changes. An increase in rainfall as well as an increase in the average monthly minimum temperature of just 1°C could signal an outbreak of cholera in the next 4 months. Models like this could provide early warning signs to those at risk and help prevent future outbreaks. Because we are created in God's image (Gen. 1:27), we have the ability to model and interpret God's world, allowing us to help those in need (Heb. 13:16).

CHAPTER OVERVIEW

1.2 SIMPLIFYING ALGEBRAIC EXPRESSIONS

How can I know an algebraic expression is simplified?

I will be able to

- simplify algebraic expressions.
- classify polynomials by their number of terms and degree.
- add and subtract polynomials.

Engineers use mathematics to design complex but beautiful designs like the Falkirk Wheel.

Expressions containing powers can often be simplified using the properties of exponents. Reviewing these familiar properties and definitions will build a strong foundation for working with more complicated expressions.

PROPERTIES OF EXPONENTS		
Property or Definition		**Example**
Product	$x^a \cdot x^b = x^{a+b}$	$x^3 \cdot x^4 = x^7$
Quotient	$\frac{x^a}{x^b} = x^{a-b}; x \neq 0$	$\frac{x^7}{x^2} = x^5$
Power	$(x^a)^b = x^{ab}$	$(x^3)^4 = x^{12}$
Power of a Product	$(xy)^a = x^a y^a$	$(2x)^3 = 8x^3$
Power of a Quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}; y \neq 0$	$\left(\frac{z}{3}\right)^2 = \frac{z^2}{9}$
Zero Exponent	$x^0 = 1; x \neq 0$	$5^0 = 1$
Negative Exponent	$x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n; x \neq 0$	$3^{-2} = \frac{1}{9}$ and $\left(\frac{1}{b}\right)^{-2} = b^2$

A term is generally considered to be simplified when each base appears only once, all fractions have been reduced, and there are no parentheses or negative exponents.

EXAMPLE 1: Using Properties of Exponents

Simplify each term.

a. $(4x^2y)(-3x^{-2}y^{-4})$ **b.** $\frac{r^4st^7}{r^{-1}s^3t^5}$ **c.** $\left(\frac{-2ac^4}{5b^3}\right)^2$ **d.** $(3x^{2n}y^2)^3(2x^3y^{-2m})^{-2}$

Answers

a. $(4x^2y)(-3x^{-2}y^{-4})$

$= 4(-3)x^{2+(-2)}y^{1+(-4)}$

$= -12x^0y^{-3}$

$= -\frac{12}{y^3}$

1. Apply the Product Property.
2. Apply the definitions of a negative exponent and a zero exponent.

b. $\frac{r^4st^7}{r^{-1}s^3t^5}$

$= r^{4-(-1)}s^{1-3}t^{7-5}$

$= r^5s^{-2}t^2$

$= \frac{r^5t^2}{s^2}$

1. Apply the Quotient Property.
2. Apply the definition of a negative exponent.

c. $\frac{(-2ac^4)^2}{(5b^3)^2} = \frac{(-2)^2a^2(c^4)^2}{5^2(b^3)^2}$

$= \frac{4a^2c^8}{25b^6}$

1. Apply the Power of a Quotient and Power of a Product Properties.
2. Apply the Power Property.

d. $(3x^{2n}y^2)^3(2x^3y^{-2m})^{-2}$

$= (3^3x^{6n}y^6)(2^{-2}x^{-6}y^{4m})$

$= \frac{27}{4}x^{6n-6}y^{6+4m}$

1. Apply the Power of a Product Property.
2. Apply the Product Property.

SKILL CHECK **EXERCISES 15, 21**

Algebraic expressions are classified in many ways. A *monomial* is a single-term expression. The numerical factor in a monomial is called the **coefficient**. In $4x^2$ the coefficient is 4, and in x^3 the coefficient is 1. *Polynomials* are classified by the number of terms (or monomials) they contain.

DEFINITIONS

A **monomial** is a single-term expression containing a constant, a variable with a whole number exponent, or the product of a constant and 1 or more such variables.

A **polynomial** is an algebraic expression of 1 or more monomials. Each monomial is a term of the polynomial.

A **binomial** is a polynomial with exactly 2 terms.

A **trinomial** is a polynomial with exactly 3 terms.

Polynomials are also classified by their degree. The **degree of a monomial** is the sum of the exponents of its variables. For example, the degree of $7x^2y^3$ is $2 + 3 = 5$. The **degree of a polynomial** is equal to the degree of its highest-degree term. It is customary to write the terms in order of descending degree of a variable.

EXAMPLE 2: Finding the Degree of a Polynomial

Determine the degree of each expression.

a. $2xy$ **b.** $5x^2y$ **c.** 7 **d.** $4x^3y + 7x^2y^3 - 15y^4$

Answers

a.	$2xy$	$1 + 1 = 2$	Add the exponents of each variable.
b.	$5x^2y$	$2 + 1 = 3$	
c.	7	0	Since 7 can be thought of as $7x^0$, its degree is 0.
d.	$4x^3y + 7x^2y^3 - 15y^4$	$4x^3y$: $3 + 1 = 4$ $7x^2y^3$: $2 + 3 = 5$ $15y^4$: 4	**1.** Determine the degree of each term.
	degree of the polynomial: 5		**2.** Select the degree of the highest-degree term.

SKILL CHECK **EXERCISES 1, 3**

AFTERSCHOOLHELP

Simplifying Algebraic Expressions

The Distributive Property is often used to simplify expressions containing polynomials. Notice how it is used to remove the parentheses from the following expression.

$$-2x(5y - 7) = -10xy + 14x$$

Recall that this property is also the basis for combining like terms (terms that contain the same variables with the same exponents).

$$3z + 5z = (3 + 5)z = 8z$$

Polynomial expressions are considered simplified when each term is simplified and all like terms are combined.

EXAMPLE 3: Combining Like Terms

Simplify each expression.

a. $(3x^2 + 2x - 8) + (5x^2 - 3x + 9)$ **b.** $2a(2a^2b + ab - 4b^3) - (5a^3b + 3ab^2 - 4ab^3)$

Answers

a.	$(3x^2 + 2x - 8) + (5x^2 - 3x + 9)$ $= (3x^2 + 5x^2) + (2x - 3x) + (-8 + 9)$	**1.** Identify like terms. This regrouping may be done mentally.
	$= 8x^2 - x + 1$	**2.** Combine like terms.
b.	$2a(2a^2b + ab - 4b^3) - (5a^3b + 3ab^2 - 4ab^3)$ $= 4a^3b + 2a^2b - 8ab^3 - 5a^3b - 3ab^2 + 4ab^3$	**1.** Remove parentheses. Be sure to distribute –1 to each term in the second expression.
	$= 4a^3b + 2a^2b - 8ab^3 - 5a^3b - 3ab^2 + 4ab^3$	**2.** Identify like terms.
	$= -a^3b + 2a^2b - 3ab^2 - 4ab^3$	**3.** Combine like terms.

SKILL CHECK **EXERCISE 29**

VOCABULARY

- binomial
- coefficient
- degree of a monomial
- degree of a polynomial
- monomial
- polynomial
- trinomial

A. EXERCISES

State the degree of each polynomial. Then classify each as a monomial, binomial, trinomial, or other polynomial.

1. $4xy$ **2.** $3x^2y + 4xy - 5xy^3$ **3.** $5a^3b^2c - 7abc + 3a^4b$

4. $5c^2d + 25cd^2$ **5.** 9 **6.** $4a^2bc + 3x^2y - 5d + 6c^2d^4$

Simplify each expression.

7. $(2x^2 + 3x - 8) + (5x^2 - 9x + 2)$

8. $(5a^3 - b) - (4a^3 + 2b)$

9. $(12c^2 + 3cd - 5d^2) + (6cd + 2d^2)$

10. $(7a^2 - 9b^2) + (3a^2 + 4ab + 7b^2)$

11. $6a^2b \cdot 9ab^2$

12. $5x^2 \cdot 3x^3 \cdot 2x^0$

13. $(2a^3b)^2$

14. $(7x^4)^0$

15. $(8a^2b)(4ab^3)$

16. $\left(\frac{2x}{y}\right)^3$

B. EXERCISES

Simplify each expression.

17. $4x^n \cdot 5x^m$

18. $(x^2)^{n+1}$

19. $\left(\frac{x^2}{b^3}\right)^{2n}$

20. $\left(\frac{a^{2x}}{b^y}\right)^{3x}$

21. $\frac{(x^2y)^3}{x^3y^{-5}}$

22. $\frac{c^2d^0(c^{-3}d)^2}{(cd^{-3})^{-2}}$

23. $(2x^2y^5)^2 + 6x^4y^{10}$

24. $(3x^3y^6)^2 + (2x^2y^4)^3$

25. $3a^2(2a + 4x - 9)$

26. $5xy^2(x^2y^3 - 14xy)$

27. $a^2cd^3(a^5 - 3acd + 7a)$

28. $2(3c^2 + 4c - d^2) + (6c^2 - 5c + 6d^2) - \frac{1}{3}(6c^2 - 3d^2)$

29. $(3a + 4b - 9c) + 3(7a - 3b + c) - \frac{1}{2}(4a - 6b + 2c)$

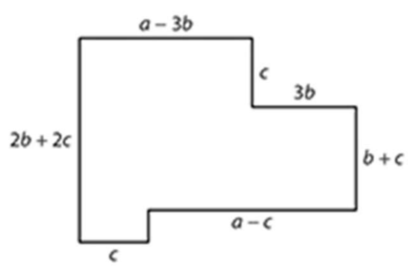
30. $\left(\frac{2}{3}x^2y - \frac{3}{4}xy^2\right) + \left(\frac{1}{4}xy - \frac{1}{2}xy^2\right) - \left(\frac{1}{6}x^2y - \frac{1}{3}xy\right)$

31. $(c^2d + cd^2) - \left(\frac{3}{5}c^2d + \frac{1}{4}cd^2\right)$

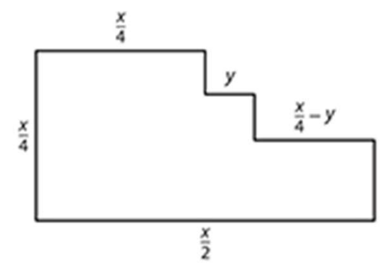
32. **Essential Question:** When is an algebraic expression fully simplified?

Write a simplified polynomial expression for the perimeter of each figure.

33.



34.



35. A polygon has sides with lengths of $11x$, $9x + 2$, $x - 3$, $10x - 7$, and $2x + 1$. Write a simplified polynomial expression for its perimeter.

C. EXERCISES

Simplify each expression.

36. $\left(\frac{2}{3}x^2 - \frac{5}{7}x + \frac{1}{2}\right) - \left(\frac{1}{4}x^2 + \frac{2}{5}x - \frac{1}{4}\right)$

37. $(4.75x^2 - 32.6x + 87.5) - (12.8x^2 - 1.79x + 1.74)$

Find the values of *a*, *b*, and *c* that make each statement true.

38. $(5x^2 - bx + c) + (ax^2 - 7x + 8) = 7x^2 - 12x + 5$

39. $(ax^2 - 9x - c) - (3x^2 + bx - 8) = 7x^2 - 12x + 5$

40. $\frac{x^a y^b z^8}{x^5 y^a z^c} = \frac{y^4}{x^2}$

CUMULATIVE REVIEW

Determine whether each statement is *always*, *sometimes*, or *never* true. [1.1]

41. The product of 2 whole numbers is a whole number.

42. The difference of 2 whole numbers is rational.

43. The product of 2 rational numbers is irrational.

44. The quotient of 2 real numbers is rational.

Simplify each expression. [1.1]

45. $|-9 - 25| + 3 - |-5| - (-11 + 7)^2$

46. $[3 - (4 + |-2|)^2] - 4^2$

Evaluate when $x = -3$ and $y = 5$. [1.1]

47. $\frac{|x| - (2 - y)}{(-x + y)^2}$

48. $\frac{(-x + y^2) - |x|}{-|x - y|}$

49. Which subset of the real numbers does not include 7? **[1.1]**

A $\mathbb{N}$

B $\mathbb{W}$

C $\mathbb{Z}$

D $\mathbb{Q}$

E All of these include 7.

50. To which subsets of the real numbers does $\sqrt{7}$ belong? List all that apply. **[1.1]**

A $\mathbb{N}$

B $\mathbb{Z}$

C $\mathbb{Q}$

D $\mathbb{Q}'$

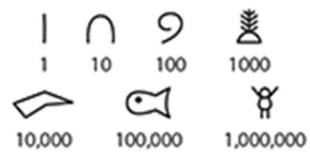
E $\mathbb{R}$

ALGEBRA AROUND THE WORLD
ANCIENT NUMERATION

Egypt, Babylon, and China, three of the world's oldest cultures, each developed a numeration system. These numeration systems permitted each culture to develop mathematical skills.

Egyptian

Egyptian numeration symbols are hieroglyphics, or picture symbols, and are expressed in multiples of 10 but without place values. The Rhind Papyrus, the best-preserved Egyptian mathematical papyrus (ca. 1700 BC), contains about 85 mathematical problems and their solutions.

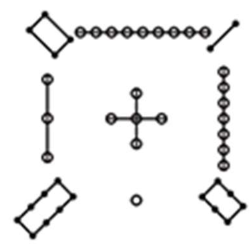


Historical records show that the Egyptians knew enough algebra to use fractions, solve simple quadratics, and calculate areas and volumes. Their formula for the area of a circle was $A = \left(\frac{8d}{9}\right)^2$, where d is the diameter. This gave a very good approximation of 3.1605 for π. Their fractions, however, were always unit fractions (with 1 as the numerator). They used the symbol ⬭ for this numerator. The diagram shows the symbols equivalent to $\frac{1}{5}$.

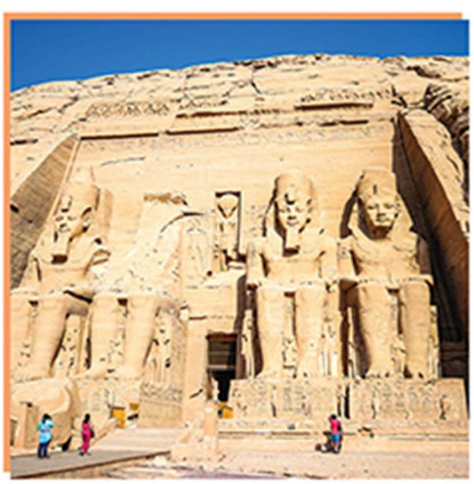
Chinese

The most significant ancient Chinese mathematical treatise is *Jiuzhang suanshu*, or *The Nine Chapters on the Mathematical Art*. This book included area formulas for triangles and trapezoids and introduced negative numbers. For instance, if T indicated 6, then $\not{T}$ represented –6.

The diagram illustrates ancient Chinese numeration in a magic square, called the Lo Shu. A magic square is an arrangement of numbers such that the sum of the rows, columns, or diagonals is always constant.



In the thirteenth century the Chinese invented a way to solve equations numerically. Two works by Zhu Shijie, *Introduction of Mathematical Science* and *Precious Mirror of Four Elements*, set forth Chinese algebra. As early as AD 906 the Chinese had also used matrices to find solutions to systems of equations, a method frequently used today by computers.



The seated colossi of Pharaoh Rameses II at Abu Simbel in Egypt.

Sculpted lions guard the entrance to China's Forbidden City.

Discussion

1. Joseph used the ancient Egyptian numeration system in tracking the storage and distribution of grain and in the construction of storage facilities. If Joseph were looking at the following values for units of grain, what would he get as the total?

2. Identify each Egyptian fraction.

3. If the grid represents the Chinese Lo Shu pictured earlier and the upper left number is 4, complete the magic square. What is the constant sum?

4		

4. Research ancient Babylonian numerals to answer the following questions.
    - **a.** What base did the Babylonians use for their number system?
    - **b.** Write the Babylonian representations for numbers 1–12.
    - **c.** Write the Babylonian representations for the numbers 20, 30, 40, and 50.

When an equation contains more than 1 operation, these operations are "undone" by performing the inverse operations in the reverse order of operations on both sides of the equation.

EXAMPLE 1: Solving a 2-Step Equation

AfterSchoolHelp

Solving Equations

Solve $5x + 12 = -18$.

Answer

$5x + 12 - 12 = -18 - 12$ $5x = -30$	**1.** Subtract 12 from both sides.
$\frac{5x}{5} = \frac{-30}{5}$ $x = -6$	**2.** Divide both sides by 5.
$5(-6) + 12 = -18$	**3.** Check mentally.

SKILL CHECK ***EXERCISES 1, 11***

The Distributive Property is often used to simplify both sides of more complicated equations. If the simplified equation has the same variable on both sides of the equation, the Addition Property of Equality is used to move all the terms with the variable to the same side of the equation and all the constants to the other side.

EXAMPLE 2: Solving a Multistep Equation

Solve $-3(x + 2) = 4(x + 18) - 1$.

Answer

$-3x - 6 = 4x + 71$	**1.** Simplify both sides.
$-6 = 7x + 71$ $-77 = 7x$	**2.** Add $3x$ to both sides and subtract 71 from both sides to collect the terms containing the variable on one side of the equation and constant terms on the other side.
$-11 = x$	**3.** Divide both sides by 7.
$x = -11$	**4.** Apply the Symmetric Property of Equality.
$-3[(-11) + 2] = 4[(-11) + 18] - 1$ $27 = 27$	**5.** Check mentally.

SKILL CHECK ***EXERCISE 15***

An equation can be cleared of fractions by multiplying both sides by a common denominator, preferably the least common denominator (LCD). Decimals can be eliminated by multiplying by a power of 10.