

# **START MATH RIGHT**

## **THE JOURNEY TO MATHEMATICS EXCELLENCE**

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**WITH KATHLEEN COTTER CLAYTON**

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