

LESSON
49
Writing Equations in Slope-Intercept Form
Warm Up

1. **Vocabulary** Which ordered pair could be the coordinates of the ⁽³⁵⁾ y -intercept of a graph?
 A (3, 0) B (0, 4) C (1, 2) D (-1, -6)

Solve for y .

2. ⁽²⁹⁾ $x + 3y - 4 = 0$

3. ⁽²⁹⁾ $3x - 2y - 7 = 0$

Find the y -value given the x -value.

4. ⁽³⁰⁾ $y = 2x + 10$; $x = 4$

5. ⁽³⁰⁾ $y = x^2$; $x = -9$

New Concepts

The slope-intercept form of a linear equation can be used to graph lines quickly because it gives information about the characteristics of the graph of the equation.

Math Language

The **y -intercept** is the y -value where a graph intersects the y -axis, and is usually represented by the variable b . The coordinate is $(0, b)$.

Slope-Intercept Form of an Equation

The **slope-intercept form** is $y = mx + b$, where the value of m is the slope of the line and the value of b is the y -intercept.

Example 1 **Determining the Slope and y -Intercept of a Line**

Determine the slope and the y -intercept of each equation.

a. $y = 3x - 4$

SOLUTION The equation is already written in slope-intercept form. The slope (m) is the coefficient of x , and the y -intercept is the constant value in the equation. Write the equation so that the operation with the constant is addition.

$$y = 3x + (-4)$$

slope: 3 y -intercept: -4

b. $2x + 3y - 9 = 0$

SOLUTION Isolate the variable y to write the equation in slope-intercept form.

$$2x + 3y - 9 = 0$$

$$\underline{-2x}$$

$$\underline{-2x}$$

Subtraction Property of Equality

$$3y - 9 = -2x$$

$$\underline{+9}$$

$$\underline{+9}$$

Addition Property of Equality

$$\frac{3y}{3} = \frac{-2x + 9}{3}$$

Division Property of Equality

$$y = -\frac{2}{3}x + 3$$

slope: $-\frac{2}{3}$ y -intercept: 3



Online Connection

www.SaxonMathResources.com

The **solution of an equation in two variables** is an ordered pair or set of ordered pairs that satisfies the equation. Solutions of equations in two variables can be represented in a table of values or as a graph on a coordinate plane.

Example 2 Graphing an Equation of a Line in Slope-Intercept Form

Graph each line using the equation that is in slope-intercept form.

a. $y = \frac{2}{3}x$

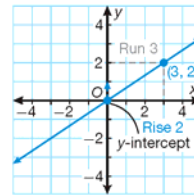
SOLUTION

Identify the slope and y -intercept from the equation. There is no constant, so the value of b is 0.

slope: $\frac{2}{3}$ y -intercept: 0

Graph the y -intercept on the coordinate plane at point $(0, 0)$.

Use the value of the slope to plot another point on the line. The slope is $\frac{2}{3}$, so this means a rise of 2 over a run of 3. Starting at the y -intercept, $(0, 0)$, move 2 units up and 3 units to the right. A second point on the line is $(3, 2)$.



Draw a line through the two points.

b. $2x + y + 5 = 0$

SOLUTION

Write the equation in slope-intercept form.

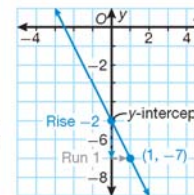
$y = -2x - 5$

Identify the slope and y -intercept from the equation.

slope: -2 y -intercept: -5

Graph the y -intercept on the coordinate plane at point $(0, -5)$.

Use the value of the slope to plot another point on the line. The slope is -2 , or $-\frac{2}{1}$, so this means a negative rise of 2 over a run of 1. Starting at the y -intercept, $(0, -5)$, move 2 units down and 1 unit to the right. A second point on the line is $(1, -7)$.



Draw a line through the two points.

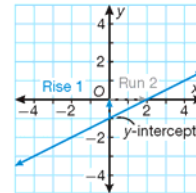
Hint

Slope is often written as $\frac{\text{rise}}{\text{run}}$ or "rise over run." When the value of m is a whole number, think about it in rational form. For example, $6 = \frac{6}{1}$.

You can use a graph to write an equation of the line in slope-intercept form.

Example 3 Writing the Equation of a Line from a Graph

- a. Write the equation of the graphed line in slope-intercept form.



SOLUTION

Identify the y -intercept from the graph by identifying the y -value where the line crosses the y -axis.

y -intercept: -1

Identify the slope by determining how to move from one point to another.

rise = 1

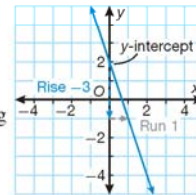
run = 2

slope: $\frac{1}{2}$

Write the equation in slope-intercept form.

$$y = \frac{1}{2}x - 1$$

- b. Write the equation of the graphed line in slope-intercept form.



SOLUTION

Identify the y -intercept from the graph by identifying the y -value where the line crosses the y -axis.

y -intercept: 2

Identify the slope by determining how to move from one point to another.

rise = -3

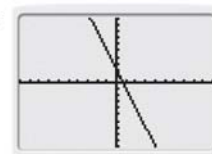
run = 1

slope: -3

Write the equation in slope-intercept form.

$$y = -3x + 2$$

Check Verify the solution by graphing the equation on a graphing calculator and comparing it to the original graph.



Math Reasoning

Generalize What is the slope of a horizontal line? What is the slope-intercept form of the equation of a horizontal line?

When working with a real-world application of linear equations, it is important to correctly identify all of the values. The y -intercept is often a starting value. The slope of a line is the rate of change, so the value that is the rate of change in the problem will be substituted for m .

Example 4 Calculating Rental Rates

Monica is helping prepare a budget for her family vacation. The family has decided to rent canoes for a day on the lake. The rental for a canoe is a \$25 flat fee plus \$10 per hour. Write an equation in slope-intercept form to represent this situation and then graph it.

SOLUTION

Define the variables.

Let y represent the total rental cost and x represent the number of hours of canoe rental.

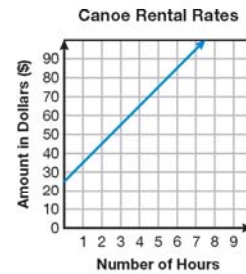
Identify the slope and y -intercept from the information in the problem.

The slope is 10 because it is the rate of change based on the number of hours the canoe is rented. The y -intercept is 25, the flat fee or cost, of renting the canoe for 0 hours.

Write the equation in slope-intercept form.

$$y = 10x + 25$$

Graph the equation of the line.



Math Reasoning

Analyze Why is the graph of the line shown only in the first quadrant?

Caution

Even though a linear equation is often used to represent a real-world problem, pay attention to restrictions on the domain and range.

Lesson Practice

Determine the slope and the y -intercept of the equation.

(Ex 1)

a. $y = 0.7x - 4.9$

b. $-9x + 3y = 12$

Graph each line using the equation that is in slope-intercept form.

(Ex 2)

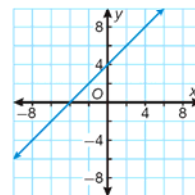
c. $y = \frac{3}{5}x$

d. $x - 4y - 20 = 0$

Write the equation of the graphed line in slope-intercept form.

(Ex 3)

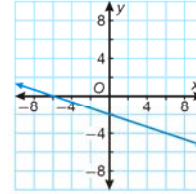
e. Write the equation of the graphed line in slope-intercept form.



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f. Write the equation of the graphed line in slope-intercept form

g. (Ex 4) Monica's family will be renting a car on their vacation. The car rents for an initial fee of \$50 plus \$0.50 per mile. Write a linear equation in slope-intercept form to represent this situation and then graph it.



Practice Distributed and Integrated

Expand each expression by using the Distributive Property.

1. $x^2y^3(3xy - 5y)$
(15)

2. $-2x^3y^3(4x^2y - 3xy)$
(15)

Evaluate each expression for the given values.

3. x^2y^3z if $x = 3$, $y = -2$, and $z = 4$
(16)

4. $-x^2 - y^3$ if $x = -3$ and $y = -2$
(16)

5. Find the x -intercept of the line $3x + 2y - 10 = 0$.
(35)

*6. Identify the slope and y -intercept of the line $2x - 5y - 6 = 0$.
(49)

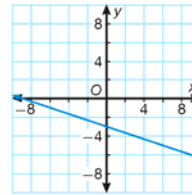
*7. **Multiple Choice** What is the equation of the graphed line?
(49)

A $y = -\frac{1}{3}x + 3$

B $y = -\frac{1}{3}x - 3$

C $y = -3x + 3$

D $y = -3x - 3$



*8. **Multi-Step** The directions on a box of frozen biscuits state to cook one biscuit for 90 seconds on high and to add 15 seconds of cooking time for each additional biscuit.
(49)

a. Write a linear equation in slope-intercept form to represent the situation.

Identify what the variables y and x represent.

b. **Analyze** What is the y -intercept of the graph? Does it have any meaning? Why or why not?

*9. **Measurement Conversion** The formula for converting from Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$, where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit. Identify the slope and the y -intercept of the equation and then graph it.
(49)

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- *10. **Pet Care** ⁽⁴⁹⁾ The table shows a feeding chart on the bag of a certain brand of cat food.

Maximum Weight of Cat	Amount of Food Per Day
6 lb	$\frac{3}{4}$ cup
9 lb	$1\frac{1}{4}$ cups
12 lb	$1\frac{3}{4}$ cups

a. Write an equation in slope-intercept form.

-  b. **Write** The three points in the table show that this is a linear equation. Do you think this relationship would be a linear equation? Explain.

11. Find the mean, median, and mode of the set of data below.
⁽⁴³⁾ number of pockets in 8 pairs of pants: 4, 2, 0, 4, 5, 4, 2, 3

12. **Error Analysis** ⁽⁴⁸⁾ Two students studied the table of data.
Student A says that the median lowest temperature per month for San Diego, California, is 38.5°F. Student B says that the median is 52.5°F. Which student is correct? Explain the error.

Lowest Recorded Temperature in San Diego (in °F)			
July 2006	68	Jan 2007	35
Aug 2006	63	Feb 2007	45
Sep 2006	61	Mar 2007	45
Oct 2006	55	Apr 2007	50
Nov 2006	42	May 2007	55
Dec 2006	42	Jun 2007	58

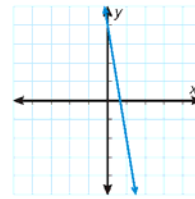
-  *13. **Probability** ⁽⁴⁸⁾ Hari surveyed the first 15 students who came to class about the number of pets they owned. According to the data he collected, is the next person he surveys likely to have more than 4 pets? Explain.

2, 1, 3, 1, 1, 0, 2, 2, 3, 7, 0, 4, 2, 1, 1

14. Simplify $\frac{x^2 - 12x}{2x^2 - x}$.
⁽⁴³⁾

15. Find 22% of 80 using a proportion.
⁽⁴²⁾

16. **Estimate** ⁽⁴¹⁾ Approximate the slope of the line shown in the graph.



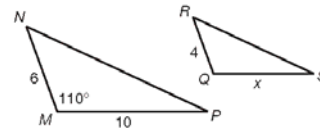
17. **Travel** ⁽²⁸⁾ Quick Cab charges a \$5 fee for a ride, plus \$0.15 for every block.
Speedy Cab charges \$7 for a ride, but only \$0.05 every block. For what number of blocks is the cost of travel the same?

18. A deck of cards contains 15 cards, five of each number 1, 2, and 3. What are the odds of getting a 1? What are the odds against getting a 3?
⁽³³⁾

19. Write a recursive formula for the arithmetic sequence with $a_1 = -3$ and common difference $d = 9$. Then find the first four terms of the sequence.
⁽³⁴⁾

20. **Multi-Step** ⁽³⁶⁾ Use the similar triangles shown.

- a. What angle corresponds with $\angle M$?
b. Find $m\angle Q$.
c. What is the scale factor of the triangles?
d. Use the scale factor to find the value of x .



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21. **Tourism** ⁽⁴⁸⁾ The number of visitors to national parks in the United States each year for the years 1990 to 2005 (in millions) are as follows: 258.7, 267.8, 274.7, 273.1, 268.6, 269.6, 265.8, 275.3, 286.7, 287.1, 285.9, 279.9, 277.3, 266.1, 276.9, 273.5. Find the range of the data.

22. **Geometry** ⁽⁴⁸⁾ Ferdinand found the areas of seven triangles. This was his data set: 7 cm^2 , 8 cm^2 , 12 cm^2 , 6 cm^2 , 7 cm^2 , 10 cm^2 , 6 cm^2 . If the triangle at right has the same area as the mean value in the data set, what is the height (h) of the triangle?

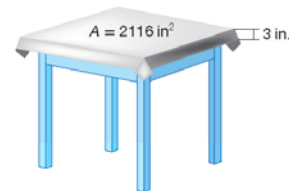


23. **Error Analysis** ⁽⁴⁷⁾ A box of chocolates sells for \$12.50. The price of the chocolates decreased by 15% from last year. Student A and Student B tried to find the price from last year. Which student is correct? Explain the error.

Student A	Student B
$0.15n = \$12.50$	$0.85n = \$12.50$
$\frac{0.15n}{0.15} = \frac{\$12.50}{0.15}$	$\frac{0.85n}{0.85} = \frac{\$12.50}{0.85}$
$n = \$83.33$	$n = \$14.71$

- *24. **Multi-Step** ⁽⁴⁶⁾ A square table has an area of 2116 square inches. A tablecloth hangs 3 inches over each end of the table.

- What is the length of one side of the table?
- What is the area of the tablecloth?
- What is the area of the part of the tablecloth that hangs over the table?



- *25. **Analyze** ⁽⁴⁶⁾ Copy and complete the statement with $\sqrt[4]{\sqrt{256}}$ $___$ $\sqrt[4]{\sqrt{256}}$ $>$, $<$, or $=$.

26. ⁽⁴⁷⁾ Find the percent of increase or decrease to the nearest percent from an original price of \$20 to a new price of \$23.

27. ⁽⁴⁵⁾ Write an inequality for the following sentence: The sum of twice a number and $\frac{1}{3}$ is less than $1\frac{2}{3}$.

- *28. **Remote Sensing** ⁽⁴⁴⁾ Satellites perform sophisticated data collection, but over time their orbits decay, affecting their measurements. In 1980 the NOAA-06 satellite was holding an orbit of 826 km. By 1996 the orbit had decayed to 804 km. What was the average rate of orbital decay for NOAA-06?

29. **Analyze** ^(39 & 4) Determine if the following statement uses inductive or deductive reasoning. Explain your choice.

"If Sharon made at least one goal at each of her last soccer games, then she will make at least one goal at her next soccer game."

30. **Multi-Step** ⁽⁴⁰⁾ A leather footstool in the shape of a cube is being made for a living room. The length of one side of the footstool is $2x$ inches.

- What is the area of one side of the footstool?
- What is the surface area of leather needed to make the footstool?

LESSON
59

Solving Systems of Linear Equations by Substitution

Warm Up

1. **Vocabulary** A (n) _____ to a system of linear equations is an ordered pair or set of ordered pairs that satisfies all the equations in the system.

2. Is $(-1, 3)$ a solution to the system below?

$$3x + 2y = 3$$

$$x - 3y = -10$$

Solve each equation.

3. $3x + 7 = 5x - 28$

4. $5x + 12 = 3x + 36$

New Concepts

To be a solution to a system of equations, an ordered pair must satisfy both equations. One method for finding solutions to systems of equations is to use the substitution method.

Steps for Solving by Substitution

1. Rearrange one of the equations so that it is of the form $y = mx + b$, or $x = my + b$, if necessary.
2. Substitute the equivalent expression for the variable from the first step into the second equation of the system. The result is an equation with one unknown.
3. Solve the resulting equation from the second step for the variable.
4. Substitute the value of the variable from the third step into one of the original equations to find the value of the other unknown.
5. Write the values of the unknowns as an ordered pair.

Example 1 Using Substitution

Solve the system of equations by substitution.

$$y = 2x - 5$$

$$y = 5x + 7$$

SOLUTION

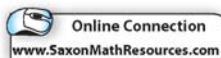
$$y = 5x + 7 \quad \text{Write the second equation.}$$

$$2x - 5 = 5x + 7 \quad \text{Substitute } 2x - 5 \text{ for } y.$$

$$-5 = 3x + 7 \quad \text{Subtract } 2x \text{ from both sides.}$$

$$-12 = 3x \quad \text{Subtract 7 from both sides.}$$

$$-4 = x \quad \text{Divide both sides by 3.}$$



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The value of x in the solution is -4 . The y -value can be found by substituting -4 into either of the original equations.

First Equation	Second Equation
$y = 2x - 5$	$y = 5x + 7$
$y = 2(-4) - 5$	$y = 5(-4) + 7$
$y = -8 - 5$	$y = -20 + 7$
$y = -13$	$y = -13$

The solution to the system is $(-4, -13)$.

Hint

You can substitute for x or y . Choose the equation that is already solved for a variable.

Example 2 Using the Distributive Property

Solve the system of equations by substitution. Check your answer.

$$\begin{aligned} 12x - 6y &= 12 \\ x &= -2y + 11 \end{aligned}$$

SOLUTION

$12x - 6y = 12$	Write the first equation.
$12(-2y + 11) - 6y = 12$	Substitute $-2y + 11$ for x .
$-24y + 132 - 6y = 12$	Distribute.
$-30y + 132 = 12$	Combine like terms.
$-30y = -120$	Subtract 132 from both sides.
$y = 4$	Divide both sides by -30 .

The value of y in the solution is 4. The x -value can be found by substituting 4 into either of the original equations.

First Equation	Second Equation
$12x - 6y = 12$	$x = -2y + 11$
$12x - 6(4) = 12$	$x = -2(4) + 11$
$12x - 24 = 12$	$x = -8 + 11$
$12x = 36$	$x = 3$
$x = 3$	

The solution to the system is $(3, 4)$.

Check To determine whether the ordered pair satisfies both of the original equations, substitute the x -value and y -value into the original equations.

First Equation	Second Equation
$12x - 6y = 12$	$x = -2y + 11$
$12(3) - 6(4) \stackrel{?}{=} 12$	$3 \stackrel{?}{=} -2(4) + 11$
$36 - 24 \stackrel{?}{=} 12$	$3 \stackrel{?}{=} -8 + 11$
$12 = 12 \quad \checkmark$	$3 = 3 \quad \checkmark$

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Hint

Use the variable with the coefficient of 1 whenever possible.

Caution

Be sure to distribute the negative sign with the number.

Hint

When neither variable has a coefficient of 1, choose a variable with a small coefficient or one that easily divides into the other coefficients and constants in the equation.

So far, every system of equations has had at least one equation of the form $y = mx + b$ or $x = my + b$. If neither equation is in this form, the first step is to rearrange one of the equations.

Example 3 Rearrange Before Substitution

- a. Solve the system of equations by substitution. Check your answer.

$$2x + y = -4$$

$$5x - 2y = -1$$

SOLUTION

Because neither equation is in a form that can be used for substitution, one equation will need to be rearranged. The first equation can be rearranged to be in the form $y = mx + b$ easily because the coefficient of y is 1.

$$2x + y = -4$$

Write the first equation.

$$y = -2x - 4$$

Subtract $2x$ from both sides.

Now substitute $-2x - 4$ for y in the second equation.

$$5x - 2y = -1$$

Write the second equation.

$$5x - 2(-2x - 4) = -1$$

Substitute $-2x - 4$ for y .

$$5x + 4x + 8 = -1$$

Distribute.

$$9x + 8 = -1$$

Combine like terms.

$$9x = -9$$

Subtract 8 from both sides.

$$x = -1$$

Divide both sides by 9.

The value of x in the solution is -1 . The y -value can be found by substituting -1 into either of the original equations.

First Equation

$$2x + y = -4$$

$$2(-1) + y = -4$$

$$-2 + y = -4$$

$$y = -2$$

Second Equation

$$5x - 2y = -1$$

$$5(-1) - 2y = -1$$

$$-5 - 2y = -1$$

$$-2y = 4$$

$$y = -2$$

The solution to the system is $(-1, -2)$.

Check To check that this ordered pair satisfies both of the original equations, substitute the x -value and y -value into the original equations.

First Equation

$$2x + y = -4$$

$$2(-1) + (-2) \stackrel{?}{=} -4$$

$$-2 + (-2) \stackrel{?}{=} -4$$

$$-4 = -4 \quad \checkmark$$

Second Equation

$$5x - 2y = -1$$

$$5(-1) - 2(-2) \stackrel{?}{=} -1$$

$$-5 + 4 \stackrel{?}{=} -1$$

$$-1 = -1 \quad \checkmark$$

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Math Reasoning

Write When substituting the value of one variable to find the value of the other, only one equation needs to be used. So, why must both equations be used to check the answer?

- b. Solve the system of equations by substitution. Check your answer.

$$4x + 7y = 43$$

$$2x - 3y = -11$$

SOLUTION

Again, one equation will need to be rearranged so that it can be used for substitution. None of the variables have a coefficient of 1, so solve the second equation for x .

$$2x - 3y = -11$$

Write the second equation.

$$2x = 3y - 11$$

Add $3y$ to both sides.

$$x = \frac{3}{2}y - \frac{11}{2}$$

Divide both sides by 2.

Now substitute $\frac{3}{2}y - \frac{11}{2}$ for the x in the first equation.

$$4x + 7y = 43$$

Write the first equation.

$$4\left(\frac{3}{2}y - \frac{11}{2}\right) + 7y = 43$$

Substitute $\frac{3}{2}y - \frac{11}{2}$ for x .

$$\frac{12}{2}y - \frac{44}{2} + 7y = 43$$

Distribute.

$$6y - 22 + 7y = 43$$

Simplify.

$$13y - 22 = 43$$

Combine like terms.

$$13y = 65$$

Add 22 to both sides.

$$y = 5$$

Divide both sides by 13.

The value of y in the solution is 5. Substitute 5 into either of the original equations to find x .

First Equation

Second Equation

$$4x + 7y = 43$$

$$2x - 3y = -11$$

$$4x + 7(5) = 43$$

$$2x - 3(5) = -11$$

$$4x + 35 = 43$$

$$2x - 15 = -11$$

$$4x = 8$$

$$2x = 4$$

$$x = 2$$

$$x = 2$$

The solution to the system is $(2, 5)$.

Check To determine whether the ordered pair satisfies both of the original equations, substitute the x -value and the y -value into the original equation.

First Equation

Second Equation

$$4x + 7y = 43$$

$$2x - 3y = -11$$

$$4(2) + 7(5) \stackrel{?}{=} 43$$

$$2(2) - 3(5) \stackrel{?}{=} -11$$

$$8 + 35 \stackrel{?}{=} 43$$

$$4 - 15 \stackrel{?}{=} -11$$

$$43 = 43 \quad \checkmark$$

$$-11 = -11 \quad \checkmark$$

Hint

To determine how to define the variables, look at the question. The question will indicate what needs to be found; that is, what is unknown. Variables represent the unknown.

Example 4 Application: Play Tickets

A school play charged adults \$8 and students \$5 for tickets. There were 75 people who attended the play. The box office collected \$444. How many adults and how many students attended the play? Use substitution to solve.

SOLUTION

First define the variables.

Let x = number of adults.

Let y = number of students.

Translate the situation into a system of equations.

$$x + y = 75$$

Adults plus students equals the total number of people.

$$8x + 5y = 444$$

The total amount of money collected can be found by multiplying the cost of the ticket by the number of people buying that kind of ticket.

Now use substitution to solve the system.

$$y = -x + 75$$

Solve the first equation for y .

$$8x + 5(-x + 75) = 444$$

Substitute $-x + 75$ for y in the second equation.

$$8x - 5x + 375 = 444$$

Distribute.

$$3x + 375 = 444$$

Combine like terms.

$$3x = 69$$

Subtract 375 from both sides.

$$x = 23$$

Divide both sides by 3.

$$23 + y = 75$$

Substitute $x = 23$ into the first equation.

$$y = 52$$

Subtract 23 from both sides.

The solution is (23, 52). There were 23 adults and 52 students at the play.

Check

First Equation

$$x + y = 75$$

$$23 + 52 \stackrel{?}{=} 75$$

$$75 = 75 \quad \checkmark$$

Second Equation

$$8x + 5y = 444$$

$$8(23) + 5(52) \stackrel{?}{=} 444$$

$$184 + 260 \stackrel{?}{=} 444$$

$$444 = 444 \quad \checkmark$$

Lesson Practice

Solve each system of equations by substitution. Check your answer.

a. $y = 4x - 3$
(Ex 1) $y = 3x - 5$

c. $4x + 3y = 2$
(Ex 3) $2x + y = 6$

b. $x = 3y - 11$
(Ex 2) $5x + 2y = -4$

d. $4x + 3y = 19$
(Ex 3) $7x - 6y = -23$

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- e. **Finance** (Ex 4) The cost of 5 books and 10 pencils is \$36. The cost of 2 books and 40 pencils is \$18. How much do books cost and how much do pencils cost?

Practice Distributed and Integrated

Solve.

1. (28) $-[-(-k)] - (-2)(-2 + k) = -k - (4k + 3)$

2. (26) $\frac{1}{3} + 5\frac{1}{3}k + 3\frac{2}{9} = 0$

3. (13) Simplify $\sqrt{9} + \sqrt{16} - \sqrt{225}$.

4. (1) Give an example of a rational number that is not an integer.

5. (13) True or False: The number $\sqrt{49}$ is a rational number.

*6. (59) Solve by substitution: $y = 3x - 5$
 $y = -2x + 15$.

*7. (59) Solve by substitution: $y = -8x + 21$
 $y = -3x + 6$.

 *8. **Write** (59) How do you know if a point is a solution to a system of equations?

*9. **Multiple Choice** (59) Which ordered pair is a solution to the system of equations?

$$4x + 9y = 75$$

$$8x + 6y = 66$$

A (3, 7)

B (0, 11)

C (10, 4)

D (12, -5)

*10. (59) The sum of two numbers is 64. Their difference is 14. Find each of the numbers.

11. **Error Analysis** (58) Students were asked to find the product of $2b^2(b^3 + 4)$. Which student is correct? Explain the error.

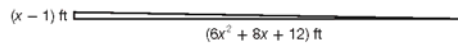
Student A	Student B
$2b^2(b^3 + 4)$	$2b^2(b^3 + 4)$
$2b^2(b^3) + 2b^2(4)$	$2b^2(b^3) + 2b^2(4)$
$2b^5 + 8b^2$	$2b^6 + 8b^2$

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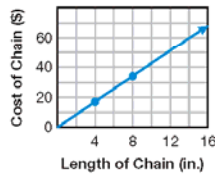
- *12. Geometry** ⁽⁵⁸⁾ The length of a rectangular pool is four times the width. A four-foot-wide deck surrounds the pool. Write a polynomial expression for the area of the pool and deck. Use the Distributive Property and write your answer in standard form.



- *13. Multi-Step** ⁽⁵⁸⁾ Henry has a game that includes a number cube. The side length of the cube is $(5x + 1)$ inches. Find the volume of the number cube.
- Find the area of the base. Multiply the length times the width.
 - Find the volume by multiplying the product found in part **a** by the height.
- *14. Measurement** ⁽⁵⁸⁾ Tim has a garden in the shape of a right triangle. The triangle has a base of $6x^2 + 8x + 12$ feet and a height of $(x - 1)$ feet. What is the area of Tim's garden?



15. Find the product of $4x(x^2 + 2x - 9)$. ⁽⁵⁸⁾
16. Write an equation of the line in slope-intercept form. The slope is 2; the y -intercept is -1 . ⁽⁴⁹⁾
17. What is the degree of $12x^4x^3 + 6xy + 41x^2y^3$? ⁽⁵³⁾
18. **Baseball** ⁽⁵⁴⁾ The percentage of games won is used to determine a team's standing in the league. In the American League, the following percentages were recorded:
- 0.604, 0.540, 0.505, 0.465, 0.380, 0.600, 0.584, 0.505, 0.446,
0.430, 0.580, 0.545, 0.475, 0.451
- Make a box-and-whisker plot of these percentages and determine if there is an outlier.
- *19. Estimate** ⁽⁵⁶⁾ The cost of a chain is directly proportional to its length. Use the graph to estimate the cost of a chain that is 18 inches long.



20. **Multiple Choice** ⁽⁵⁶⁾ Which point represents the same direct variation as $(3, -9)$?
- A $(4, -8)$ B $(4, -7)$ C $(4, -12)$ D $(4, -16)$

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21. Find the LCM of $16c^6$ and $24c^3$.
(57)

*22. **Games** Every $20x^3y$ turns, you win \$500. Every $12xy^3c$ turns, you get to roll again.
(57) How many turns do you take before you win \$500 and get to roll again on the same turn?

23. Find the LCM of $300d^2$ and $90d^4$.
(57)

24. **Formulate** The equations $y = 3x - 24$ and $y = 24x + 9$ define two lines. Write a rational expression that represents the ratio of the first line to the second line. Simplify the expression, if possible.
(43)

25. Determine the slope of the line that goes through the points $(-1, 1)$ and $(1, -1)$.
(44)

26. **Biology** Write a counterexample for the following statement: If an animal has wings, then the animal is an insect.
(Inv 4)

27. **Multi-Step** A canister of oatmeal has a height of 7 inches. Its volume is 28π cubic inches.
(46)

- Write an equation you can use to find the radius of the cylinder.
- Find the radius of the canister.



28. **Astronomy** The relative gravity on Jupiter is 2.34. This means that the weight of an object on Jupiter is 2.34 times greater than its weight on Earth. Identify the slope and the y -intercept of the equation representing this relationship and then write an equation for the situation in slope-intercept form.
(49)

29. **Verify** Write the converse of the following statement: If a number is an integer, then it is a rational number. Give an example to show that the converse is false.
(Inv 5)

 30. **Write** What is an excluded value for a rational expression? Why is it excluded?
(51)

LESSON
100

Solving Quadratic Equations by Graphing

Warm Up

1. **Vocabulary** The U-shaped curve that results from graphing a quadratic function is called a(n) _____.

Evaluate each expression for the given values.

2. $3(x - y)^2 - 4y^2$ for $x = -5$ and $y = -2$

3. $-x^2 - 3xy + y$ for $x = 3$ and $y = -1$

Determine the direction that the parabola opens.

4. $f(x) = 3x^2 + x - 4$

5. $f(x) = -2x^2 + x + 1$

New Concepts

The solution(s) of a quadratic equation, $0 = ax^2 + bx + c$, can be found by graphing the related function, $f(x) = ax^2 + bx + c$. The U-shaped graph of a quadratic function is called a parabola. The solutions of the equation are called roots and can be found by determining the x -intercepts or zeros of the quadratic function. These zeros can be found by graphing the related function to see where the parabola intersects the x -axis.

Math Language

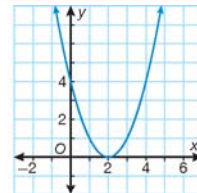
The same function is described by $y = 3x^2 - 5$ and $f(x) = 3x^2 - 5$.

The **function notation** for y is $f(x)$. It is read, "f of x."

Graphical Solutions

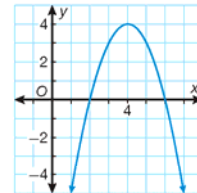
One Real Solution

The graph intersects the x -axis at the vertex.



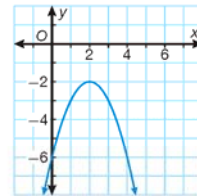
Two Real Solutions

The graph intersects the x -axis at two distinct points.



No Real Solutions

The graph does not intersect the x -axis.



Online Connection

www.SaxonMathResources.com

Hint

When the coefficient of the x^2 -term is positive, the parabola will open upward.

When the coefficient of the x^2 -term is negative, the parabola will open downward.

Math Reasoning

Write Why are the x -intercepts substituted into the original equation?

Example 1 Solving Quadratic Equations by Graphing

Solve each equation by graphing the related function.

a. $x^2 - 36 = 0$

SOLUTION

Step 1: Find the axis of symmetry.

$$x = -\frac{b}{2a} \quad \text{Use the formula.}$$

$$x = -\frac{0}{2(1)} = 0 \quad \text{Substitute values for } a \text{ and } b.$$

The axis of symmetry is $x = 0$.

Step 2: Find the vertex.

$$f(x) = x^2 - 36$$

$$f(0) = (0)^2 - 36 \quad \text{Evaluate the function for } x = 0 \text{ to find the vertex.}$$

The vertex is $(0, -36)$.

Step 3: Find the y -intercept.

The y -intercept is c , or -36 .

Step 4: Find two more points that are not on the axis of symmetry.

$$f(5) = 5^2 - 36$$

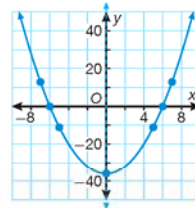
$$(5, -11)$$

$$f(7) = 7^2 - 36$$

$$(7, 13)$$

Step 5: Graph.

Graph the axis of symmetry $x = 0$, the vertex and the y -intercept, both at coordinate $(0, -36)$. Reflect the points $(5, -11)$ and $(7, 13)$ over the axis of symmetry and graph the points $(-5, -11)$ and $(-7, 13)$. Connect the points with a smooth curve.



From the graph, the x -intercepts appear to be 6 and -6 .

Check Substitute the values for x in the original equation.

$$x^2 - 36 = 0; x = 6$$

$$(6)^2 - 36 \stackrel{?}{=} 0$$

$$36 - 36 \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

$$x^2 - 36 = 0; x = -6$$

$$(-6)^2 - 36 \stackrel{?}{=} 0$$

$$36 - 36 \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

The solutions are 6 and -6 .

b. $-x^2 - 2 = 0$

SOLUTION

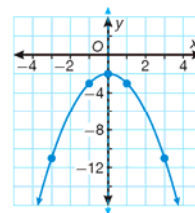
Graph the related function $f(x) = -x^2 - 2$.

axis of symmetry: $x = 0$

vertex: $(0, -2)$

y-intercept: $(0, -2)$

two additional points: $(1, -3)$ and $(3, -11)$



Reflect these two points across the axis of symmetry and connect them with a smooth curve.

From the graph, it can be seen that there is no x -intercept because the graph does not intersect the x -axis.

There is no real-number solution.

c. $x^2 + 16 = 8x$

SOLUTION

Write the equation in standard form.

$$x^2 - 8x + 16 = 0$$

Graph the related function $f(x) = x^2 - 8x + 16$.

axis of symmetry: $x = 4$

vertex: $(4, 0)$

y-intercept: $(0, 16)$

two additional points: $(2, 4)$ and $(6, 16)$

Reflect these two points across the axis of symmetry and connect them with a smooth curve.

From the graph, the x -intercept appears to be 4.

Check Substitute 4 for x in the original equation.

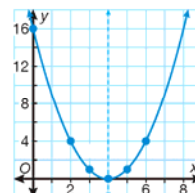
$$x^2 - 8x + 16 = 0; x = 4$$

$$(4)^2 - 8(4) + 16 \stackrel{?}{=} 0$$

$$16 - 32 + 16 \stackrel{?}{=} 0$$

$$0 = 0 \quad \checkmark$$

The solution is 4.



Caution

When a parabola does not cross the x -axis, there is no real-number solution to the quadratic equation.

Hint

For help with graphing quadratic functions, see Graphing Calculator Lab 8: Characteristics of Parabolas on p. 583.

Example 2 Solving Quadratic Equations Using a Graphing Calculator

Solve each equation by graphing the related function on a graphing calculator.

a. $-6x - 9 = x^2$

SOLUTION

Write the equation in standard form.

$$-x^2 - 6x - 9 = 0$$

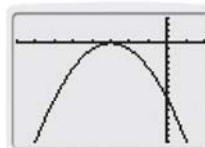
Graph the related function

$$f(x) = -x^2 - 6x - 9.$$

The graph appears to have an x -intercept at -3 .

Use the Table function to determine the zeros of this function.

The solution is -3 .



b. $-6x = -x^2 - 13$

SOLUTION

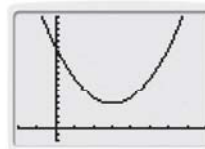
Write the equation in standard form.

$$x^2 - 6x + 13 = 0.$$

Graph the related function $f(x) = x^2 - 6x + 13$.

The graph opens upward and does not intersect the x -axis.

There is no solution.



c. $-3x^2 + 5x = -7$

Round to the nearest tenth.

SOLUTION

Write the equation in standard form.

$$-3x^2 + 5x + 7 = 0$$

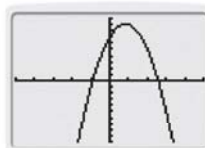
Graph the related function

$$f(x) = -3x^2 + 5x + 7.$$

The graph appears to have x -intercepts at 3 and -1 .

Use the Zero function to determine the zeros of this function. Round to the nearest tenth.

The solutions are $x = 2.6$ and -0.9 .



Hint

The time t is plotted on the x -axis. The height h is plotted on the y -axis.

Example 3 Application: Physics

Gill drops a baseball from the top of a platform 64 feet off the ground. The height of the baseball is described by the quadratic equation $h = -16t^2 + 64$, where h is the height in feet and t is the time in seconds. Find the time t when the ball hits the ground.

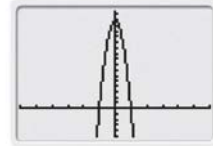
SOLUTION

Graph the related function $h(t) = -16t^2 + 64$ on a graphing calculator.

Height h is zero when the ball hits the ground. Use the Zero function of the graphing calculator to determine the zeros of this function.

There are two zeros for the given parabola: $t = 2$ and $t = -2$. Only values greater than or equal to zero are considered. So, $t = 2$ is the only solution.

The baseball hits the ground in 2 seconds.



Lesson Practice

Solve each equation by graphing the related function.

(Ex 1)

- $3x^2 - 147 = 0$
- $5x^2 + 6 = 0$
- $x^2 - 10x + 25 = 0$

Solve each equation by graphing the related function on a graphing calculator.

d. $x^2 + 64 = 16x$

(Ex 2)

e. $x^2 + 4 = 2x$

(Ex 2)

f. Round to the nearest tenth: $-7x^2 + 3x = -7$.

(Ex 2)

- g. Marcus shot an arrow while standing on a platform. The path of its movement formed a parabola given by the quadratic equation $h = -16t^2 + 2t + 17$, where h is the height in feet and t is the time in seconds. Find the time t when the arrow hits the ground. Round to the nearest hundredth.

Practice Distributed and Integrated

Solve.

1. $x(2x - 11) = 0$

(98)

2. $\frac{12}{x-6} = \frac{4}{x}$

(99)

- *3. **Generalize** Using the path of a ball thrown into the air as an example, describe in mathematical terms each part of the graph the path of the ball creates.

(100)

- *4. **Generalize** What does the graph of a quadratic equation look like when there is no solution? one solution? two solutions?

(100)

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5. Given that y varies directly with x , identify the constant of variation such that when $x = 15$, $y = 30$.



- *6. **Basketball** Ramero shoots a basketball into the air. The ball's movement forms a parabola given by the quadratic equation $h = -16t^2 + 7t + 7$, where h is the height in feet and t is the time in seconds. Find the maximum height of the path the basketball makes and the time t when the basketball hits the ground. Round to the nearest hundredth.

- *7. **Multiple Choice** What is the equation of the axis of symmetry of the parabola defined by $y = \frac{1}{4}(x - 4)^2 + 5$?
A $x = 1$ **B** $x = 4$ **C** $x = 5$ **D** $x = -4$

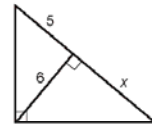
- *8. Solve $-7x^2 - 10 = 0$ by graphing.

- *9. Solve $\frac{6}{x} = \frac{8}{x + 7}$.

10. A deck of ten cards has 5 red and 5 black cards. Cards are replaced in the deck after each draw. Use an equation to find the probability of drawing a black card twice and rolling a 6 on a number cube.



11. **Geometry** The altitude of the right triangle divides the hypotenuse into segments of lengths x units and 5 units. To find x , solve the equation $\frac{x+5}{6} = \frac{6}{x}$.



- *12. **Multi-Step** Henry starts working a half-hour before Martha. He can complete the job in 4 hours. Martha can complete the same job in 3 hours.
 a. Let t represent the total time they work together. In terms of t , how long does Henry work?
 b. Use an equation to find how long they work together to complete the job.
 c. How long does Henry work?

13. Find the quotient of $\frac{a^2 + 10a - 24}{a - 2}$. 14. Simplify $\sqrt{49y^5}$.

15. **Profit** An entrepreneur makes \$3 profit on each object sold. She would like to make \$270 plus or minus \$30 total. What is the minimum and maximum number of objects she needs to sell?



16. **Data Analysis** A student knows there will be 4 tests that determine her semester grade. She wants her average to be an 85, plus or minus 5 points. What is the minimum and maximum number of points she needs to earn during the semester?

17. Solve the equation $|10x| - 3 = 87$.

18. **Exercise** Tom ran a total of $\frac{7x}{x^2 + 3x - 18}$ miles in August and $\frac{2x + 1}{7x + 42}$ miles in September. How many more miles did he run in August?

19. Graph the function $y = 5x^2 - 10x + 5$.

- *20. **Verify** A boundary line is a vertical line. The inequality contains a $<$ symbol. Which half-plane should be shaded on the graph?

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21. **Multiple Choice** Which point does not satisfy the inequality $x + 2y < 5$?
 (97) A (0, 0) B (2, 1) C (3, -4) D (-1, 3)

- *22. **Ages** A boy is b years old. His father is 23 years older than the boy. The product of their ages is 50. How old is each person?
 (98)

- *23. **Error Analysis** Two students find the roots of $3x^2 - 6x = 24$. Which student is correct? Explain the error.
 (98)

Student A	Student B
$3x^2 - 6x = 24$	$3x^2 - 6x = 24$
$3x(x - 2) = 24$	$3x^2 - 6x - 24 = 0$
$3x = 0$ $x - 2 = 0$	$3(x^2 - 2x - 8) = 0$
$x = 0$ $x = 2$	$3(x - 4)(x + 2) = 0$
	$x - 4 = 0$ $x + 2 = 0$
	$x = 4$ $x = -2$

24. Does the graph of $y + 2x^2 = 12 + x$ open upward or downward?
 (84)
25. Do the side lengths 18, 80, and 82 form a Pythagorean triple?
 (85)
26. **Multi-Step** The volume of a prism is $3x^3 + 12x^2 + 9x$. What are the possible dimensions of the prism?
 (87)
- Factor out common terms.
 - Factor completely.
 - Find the dimensions.
27. **Travel** The Jackson family drove 480 miles on Saturday and 300 miles on Sunday. Their average rate on Sunday was 10 miles per hour less than their rate was on Saturday. Write a simplified expression that represents their total driving time.
 (90)
28. **Multi-Step** At the carnival, a man says that he will guess your weight within 5 pounds.
 (91)
- You weigh 120 pounds. Write an absolute-value inequality to show the range of acceptable guesses.
 - Solve the inequality to find the actual range of acceptable guesses.
29. **Verify** If the numerator of a rational expression is a polynomial and the denominator of the rational expression is a different polynomial, will factoring the polynomials always provide a way to simplify the expression? Verify your answer by giving an example.
 (92)
30. If a 9% decrease from the original price resulted in a new price of \$227,500, what was the original price?
 (47)